

Mathematics

Class 8

Government of Nepal

Ministry of Education, Science and Technology

Curriculum Development Centre

Sanothimi, Bhaktapur

Publisher:

Government of Nepal
Ministry of Education, Science and Technology
Curriculum Development Centre
Sanothimi, Bhaktapur

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First Edition:

2023

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Preface

School education is the foundation for preparing the citizen who are loyal to the nation and nationality, committed to the norms and values of federal democratic republic, self-reliant and respecting the social and cultural diversity. It is also remarkable for developing a good moral character with the practical know-how of the use of ICT along with the application of scientific concept and positive thinking. It is also expected to prepare the citizens who are moral and ethical, disciplined, social and human value sensitive with the consciousness about the environmental conversation and sustainable development. Moreover, it should be helpful for developing the skills for solving the real life problems. This textbook 'Mathematics, Grade 8' is fully aligned with the intent carried out by the National Curriculum Framework for School Education, 2076 and is developed fully in accordance with the new Basic Level Curriculum, Grade 9-10, 2078.

This textbook is initially written by Mr. Tikaram Pokharel, Mr. Raju Kant Acharya, Mr. Som Kumar Shrestha, Mrs. Reetu Shrestha and Mr. Ram Chandra Dhakal. It has been translated by Mr. Navin Poudel, Mr. Uttam Prasad Panta, Mrs. Anu Shrestha, Mr. Prem K.C., Mr. Shakti Prasad Acharya and Mr. Ram Chandra Dhakal. The contribution made by Director General Mr. Baikuntha Prasad Aryal, Prof. Dr. Hari Prasad Upadhyay, Mr. Gyanendra Ban, Mrs. Anupama Sharma, Mr. Navin Poudel, Mr. Satya Narayan Maharjan, Mrs. Pramila Bhakati and Mrs. Ritu Shrestha is remarkable in bringing the book in this form. The language of the book has been edited by Mr. Nabin Khadka. Art editing of this book was done by Mr. Jayaram Kuikel by making it four colour. The Curriculum Development Centre extends sincere gratitude to all of them.

The textbook is a primary resource for classroom teaching. Considerable efforts have been made to make the book helpful in achieving the expected competencies of the curriculum. Curriculum Development Centre always welcomes constructive feedback for further betterment of its publications.

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1.0 Review

If the Universal set $(U) = \{\text{counting numbers up to } 10\}$, then construct the following sets using the listing method and discuss the following questions.

- $A = \{\text{prime numbers}\}$
- $B = \{\text{odd numbers}\}$
- $C = \{\text{multiples of } 3\}$
- $D = \{\text{factors of } 8\}$
- $E = \{\text{composite numbers}\}$

Are the elements of the sets given above also the elements of Universal Set U ?
What is the relationship between Sets A, B, C, D, E with Set U ?

1.1 Disjoint and overlapping sets

Activity 1

Study the situation given below and discuss it in your group.

There are eight SAARC nations. Some of these nations are landlocked, while others are coastal. In order to organize football games between the SAARC countries, all the countries were divided into two groups.

Countries on the first set = Nepal, Pakistan, Bhutan, Sri-Lanka

Countries on the second set = India, Afghanistan, Maldives, Bangladesh

- How can we represent the given information using set notation?
- Can we represent them in more than one way in the set?
- If yes, what are they?

Here,

The given sets using the listing method can be represented as,

The set of SAARC nation is universal set and represented by U , so,

$U = \{\text{Nepal, Pakistan, Bhutan, Sri-Lanka, India, Afghanistan, Maldives, Bangladesh}\}$

If the first set of nations is represented by A, then,

$A = \{\text{Nepal, Pakistan, Bhutan, Sri Lanka}\}$

If the second set of nations is represented by B, then,

$B = \{\text{India, Afghanistan, Maldives, Bangladesh}\}$

The relationship between the sets A, B, and U can be shown in a diagram:



Figure (A)

If the set of landlocked countries is represented by L, then,

$L = \{\text{Nepal, Bhutan, Afghanistan}\}$

We can represent the relationship between U, A, and L as :

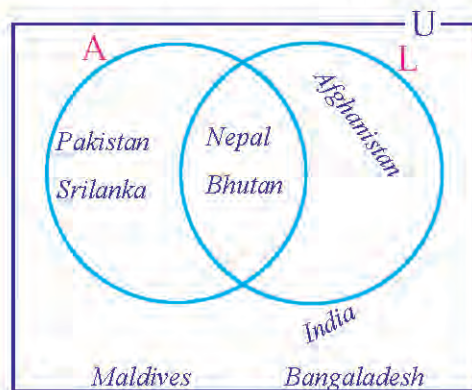


Figure (B)

The relationship between these sets is shown in Figures A and B. This visual representation of the set is known as a Venn diagram.

A Venn diagram is a visual representation of the relationship between the sets using figures. A rectangle is used to symbolize the universal set, while various subsets of the universal set are represented by circles or ellipses. The Venn diagram is first used by Mathematician John Venn in 1880. The name of the figure is given from his name so as to honour him.

What is the difference between the two Venn diagrams above? Discuss in pairs. In Figure A, is there any country that lies in both Set A and Set B? No, there is not. The countries in the sets are different. So, the sets A and B are disjoint sets. Similarly, In Figure B, is there any country that lies on both set A and set L? Yes, there is Nepal and Bhutan lie on both sets. So, sets A and L are overlapping sets.

If there is at least one element common and one element distinct in two or more sets, then the sets are said to be overlapping sets. If there is no element in common, then the sets are said to be disjoint sets.

Example 1

Study the Venn diagram alongside and answer the following questions.

- Write the set M and N by description method and tabulation method.
- Write the universal set U.
- State whether set M and N are overlapping or disjoint. And why?

Solution:

- Descriptive method:

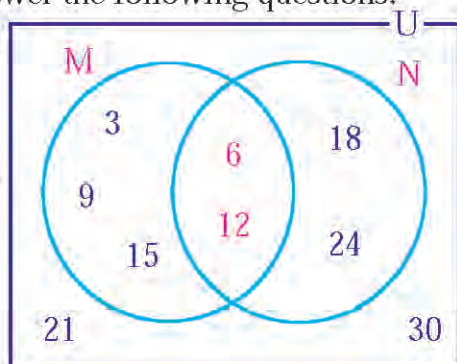
$M = \{ \text{Multiple of 3 up to 15} \}$ or $\{ \text{first five multiples of 3} \}$

$N = \{ \text{Multiple of 6 up to 24} \}$ or $\{ \text{first four multiples of 6} \}$

Listing Method: $M = \{3, 6, 9, 12, 15\}$

$N = \{6, 12, 18, 24\}$

- Universal set $U = \{3, 6, 9, 12, 15, 18, 21, 24, 27, 30\}$
- M and N are overlapping sets because 6 and 12 are two common elements of both sets. M and n has at least one distinct elements.



Example 2

P, Q, and R are the subsets of Universal Set U. If $U = \{\text{multiples of 2 upto 20}\}$, $P = \{\text{factors of 10}\}$, $Q = \{\text{multiples of 5}\}$ and $R = \{\text{multiples of 6}\}$, then represent the relationship between P and Q, and Q and R in Venn diagram. Also state the relationship between P and Q, and Q and R.

Solution

Here,

$$U = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$$

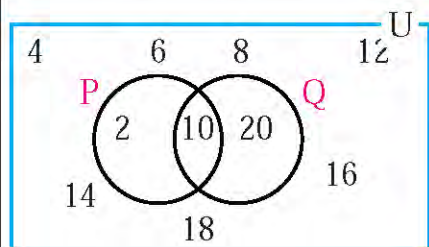
$$P = \{2, 10\}$$

$$Q = \{10, 20\}$$

$$R = \{6, 12, 18\}$$

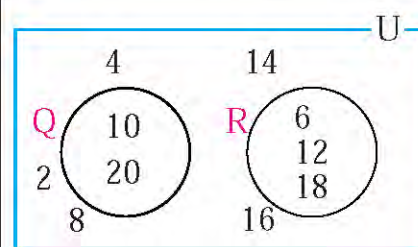
Now,

Representing the relationship between P and Q in Venn Diagram



$\therefore$ P and Q are overlapping sets.

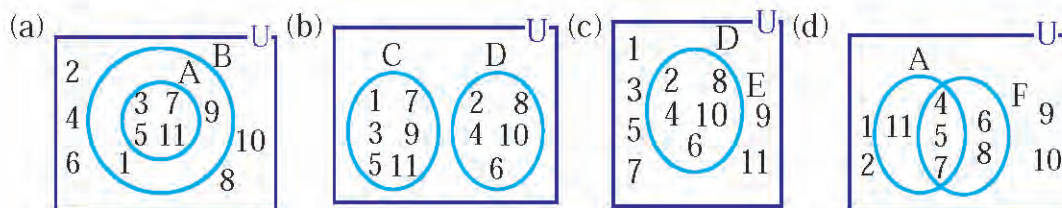
Representing the relationship between Q and R in Venn Diagram



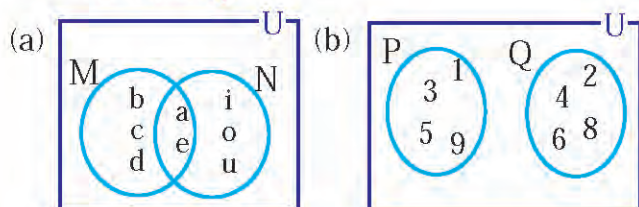
$\therefore$ Q and R are Disjoint sets.

Exercise 1.1

- Study the venn diagrams given below and write each set by discriptive method and tabulation method. Also, write the Universal set U.



- State whether the following sets are overlapping or disjoint. And, why?



- State whether the following pair of sets are overlapping or Disjoint.

$$A = \{2, 4, 6, 8, 10, 12\}, \quad B = \{1, 3, 5, 7, 9, 11\},$$

$$C = \{0, 1, 3, 5, 7, 11\}, \quad D = \{5, 6, 7, 8, 10\}, \quad E = \{0, 2, 9, 12\}$$

- (a) A and B (b) A and C (c) B and C (d) A and D (e) A and E
(f) B and D (g) B and E (h) C and D (i) C and E (j) D and E

- If $U = \{a, b, c, d, e, f, g, h, i, j\}$, $P = \{a, b, c, d, e, i\}$, $Q = \{a, e, i\}$, then represent the following pair of sets in a separate venn diagram. Identify the pairs of overlapping and disjoint sets.

- (a) P and Q (b) Q and R (c) Q and S (d) R and T

- If $A = \{0, 1, 2, 3, 4, 5\}$ list out the elements of the following sets by the listing method.

a. Set B is formed by adding 1 to each element of the set A.

- b. Set C is formed by multiplying each element of set A by 2.
- c. Set D is formed by collecting all the odd numbers of set A.
- d. Set E is formed by collecting the factors of 10 from Set A.
- e. Write the Universal set for the sets given above.
- f. Identify overlapping and disjoint sets among sets A, B, C, D, and E.

6. If $M = \{x: x \text{ is multiple of 5 less than } 40\}$, list out the elements of the following sets by Listing method.

- a. Set N is formed by adding 3 to each element of the set M.
- b. Set P is formed by multiplying each element of set M by 2.
- c. Set Q is formed by collecting all the odd numbers of set M.
- d. Set R is formed by collecting the Multiples of 10 from set M.
- e. Write the Universal set for the sets given above.

Identify the overlapping and disjoint sets among sets M, N, P, Q, and R.

Project work

- a. Write any two characteristics of domestic animals found in your community. Make two different sets of domestic animals based on these characteristics. Construct a Venn diagram to represent the relationship between these two sets whether the sets are overlapping or disjoint. Present your project in your class.
- b. Consider all the members of your class as universal set U. Based on this universal set, do the following work:
 - i. Prepare two different sets of students who like milk and curd. Represent the relationship between two sets in a Venn diagram.
 - ii. Identify whether the sets are overlapping or disjoint.

Answers

Show to your teacher

1.2 Subset

Activity 2

Shanti called her three friends Dolma, Rakesh, and Laxmi to celebrate her birthday. Construct all possible sets of her friends having;

- a. Three friends
- b. Two friends

A and B are any two sets. If every element of A are the elements of B, then A is called a subset of B. Symbolically, $A \subseteq B$ or $B \supseteq A$

1.2.1 (Proper and Improper Subsets)

Activity 3

The set of fruits, F is given in the figure.



- a. Make all possible subsets from set F.
- b. Which are the subsets of F formed by taking some elements of F and which is the subset formed by taking all the members of the set F?
- c. How do we represent the relationship between a subset having some elements of F with F?
- d. How do we represent the relationship between the subset having all elements of F with F?

If the set B is made by taking some elements of set A, then set B is called the proper subset of set A. Symbolically, $B \subset A$ and $A \supset B$.

If set B is the subset of set A and the number of elements of set B is less than that of A then set B is called proper subset. Hence, empty set is a proper subset of all sets (except itself).

If set B is made by taking all elements of set A, then set B is called an improper subset of set A.

Symbolically, $B \subseteq A$ and $A \supseteq B$. So, equal sets are the improper subsets of one another.

Example 1

Make all possible subsets of set $M = \{s, k, y\}$ and name them. Classify them as proper and improper subsets.

Solution:

The possible subsets of set $M = \{s, k, y\}$

Subset having no element, $A = \{\}$

Subset having only one element, $B = \{s\}$, $C = \{k\}$, $D = \{y\}$

Subset having only two elements $E = \{s, k\}$, $F = \{s, y\}$, $G = \{k, y\}$

Subset having all elements $H = \{s, k, y\}$

Here, proper subsets are, $A = \{\}$, $B = \{s\}$, $C = \{k\}$, $D = \{y\}$,

$E = \{s, k\}$, $F = \{s, y\}$, $G = \{k, y\}$

Here, improper subsets are $H = \{s, k, y\}$

Example 2

X and Y are a subset of the universal set U . If $U = \{y: y \text{ is a multiple of 2 up to } 20\}$ $X = \{\text{factors of } 20\}$ and $Y = \{\text{multiple of } 5\}$ then show the relationship of X and Y in the Venn diagram. Also, state the relationship between X and Y .

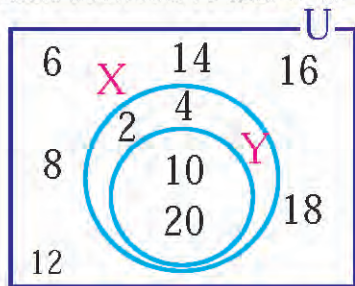
solution

Here $U = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$

$X = \{2, 4, 10, 20\}$

$Y = \{10, 20\}$

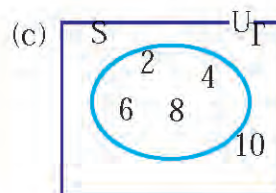
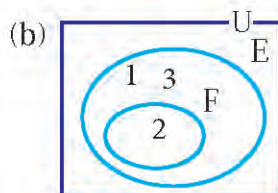
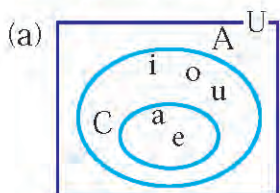
Now, representing the relation between X and Y in the Venn diagram



$\therefore Y$ is the proper subset of X . symbolically $Y \subset X$

Exercise 1.2

1. Study the venn diagrams given below. Identify proper and improper subsets in each case. Write the relationship using the symbol.



2. $N = \{x : x \text{ is counting numbers up to } 5\}$, express set N by the listing method. Make the following subset from the given set and name them.
- Subset that has only one element.
 - Subset that has two elements.
 - Subset that has three elements.
 - Subset that has four elements.
 - Subset that has five elements.
 - Subset having no elements. Write the number of subsets formed from the given set.
3. Make all possible subsets that can be made from the following set and name them.
- $F = \{\text{mango}\}$
 - $V = \{\text{radish, pumpkin}\}$
 - $T = \{\text{bus, jeep, aeroplane}\}$
 - $A = \{\text{dog, cat, horse, cow}\}$
 - $B = \{\text{pigeon, crow, eagle, parrot, crane}\}$
4. Identify proper and improper subsets in each case of the sets in Q.N.3

5. Copy the given table in your notebook and fill it out.

S.N	Set	Number of elements	Subset	Number of subsets
1.	{mango}	1		
2.	{radish, pumpkin}	2		
3.	{bus, jeep, aeroplane}			
4.	{ dog, cat, horse, cow}			
5.	{pegeon, crow, eagle, parrot, crane}			

From the table above, derive the formula for the total number of possible subsets that can be made from the given set.

6. Prepare three proper subsets from set $E = \{2, 3, 5, 7\}$ and represent them in the separate venn diagram.

Project work

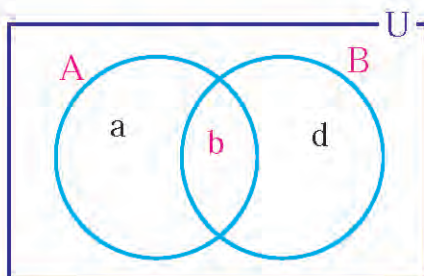
- a. Prepare a universal set consisting of your family members and do the following work:
 - i. Set of members of your family who likes bread as their breakfast.
 - ii. Prepare the subsets of the family members who like wheat bread, cornbread, or millet bread and represent them on chart paper. Present it to the class.

Answer

Show your teacher.

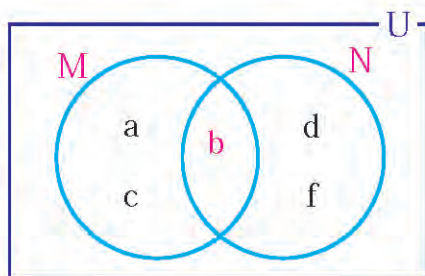
Mixed Exerciese

1. Prepare all possible subsets from set $P = \{2, 3, 5\}$ and classify them as a proper and improper subset.
2. The set $A = \{2, 4, 6\}$ and $B = \{1, 2, 3\}$ are the two subsets of universal set U
 - a. Specify, if the sets are disjoint or overlap. State the reason.
 - b. Write improper subset from set A .
 - c. Which subset of set A and set B are common?
3. Given that set $A = \{3, 5\}$, and $B = \{2, 4, 7\}$,
 - a. Write the proper subsets of set A .
 - b. What member needs to be added to set A in order to make A and B overlapping sets?
4. The set $Q = \{1, 3, 5\}$ and $R = \{1, 2, 3\}$ are the two subsets of universal set $U = \{1, 2, 3, 4, 5\}$.
 - a. Write down the proper and improper subsets of set Q .
 - b. Which proper subset of set R and set Q are common?
 - c. Represent the set Q and R in the venn diagram
5. If $U = \{\text{the set of natural number up to } 20\}$, $A = \{\text{the set of even numbers up to } 10\}$, $B = \{\text{the set of odd numbers up to } 10\}$, $C = \{\text{the set of prime numbers up to } 10\}$ and $D = \{\text{multiples of } 2\}$ Specify if the given sets are disjoint or overlap. Represent them in a venn diagram.
 - a. A and B
 - b. A and C
 - c. A and D
 - d. B and C
 - e. C and D
 - f. B and D
6. The elements of set A and B are shown in the venn diagram alongside.
 - a. Specify the kind, if the given sets A and B are disjoint or overlapping.
 - b. Write the two proper subsets of set A .
 - c. What needs to be done to make the given sets disjoint?



7. The elements of sets M and N are shown in the venn diagram alongside.

- Write all possible proper and improper subsets of set M and N.
- Are the subsets of the given sets M and N equal in number?
- Which member of set M must be removed to make M and N disjoint sets?



8. If $S = \{s, o, n, y\}$, then prepare 5 proper subsets; s_1, s_2, s_3, s_4 , and s_5 that can be made from the set S. Represent the relationship between the following pairs of sets in venn diagram.

- | | | |
|--------------------|--------------------|--------------------|
| a. S_1 and S_2 | b. S_1 and S_3 | c. S_1 and S_4 |
| d. S_1 and S_5 | e. S_2 and S_3 | f. S_2 and S_4 |
| g. S_2 and S_5 | h. S_3 and S_4 | i. S_3 and S_5 |
| j. S_4 and S_5 | | |

9. If Set $A = \{0, 2\}$, $B = \{1, 2\}$ and $C = \{\text{the square number between 2 and 5}\}$ are the subset of universal set U, then,

- Write down the proper and improper subsets that can be made from set C.
- Represent set A and B in the Venn diagram.
- Which of the sets A or C has more numbers of subsets? Write reason.

Lesson 2

Whole Numbers

2.0 Review

Study the situation given below and discuss.

The place value table of number 1256 is given in the table below:

Thousand	Hundred	Ten	one
10^3	10^2	10^1	10^0
1	2	5	6

Why one, ten, hundred and thousand are written as 10^0 , 10^1 , 10^2 , 10^3 in the table?



Writing 1256 in expanded form,

$$1256 = 1 \times 1000 + 2 \times 100 + 5 \times 10 + 6 \times 1$$

$$= 1 \times 10^3 + 2 \times 10^2 + 5 \times 10^1 + 6 \times 10^0$$

Any number can be written in expanded form using 10 and its powers. In the place value table of a number if there is one, ten, hundred, and thousand, .. or 10^0 , 10^1 , 10^2 , and 10^3 , then the number system is the decimal number system. In this number system, we use ten digits 0,1,2,3,4,5,6,7,8, and 9. This number system is also called Hindu Arabic Numeration System.

For example: $24 = 2 \times 10 + 4 = 2 \times 10^1 + 4 \times 10^0$,

$$576 = 5 \times 100 + 7 \times 10 + 6 = 5 \times 10^2 + 7 \times 10^1 + 6 \times 10^0$$

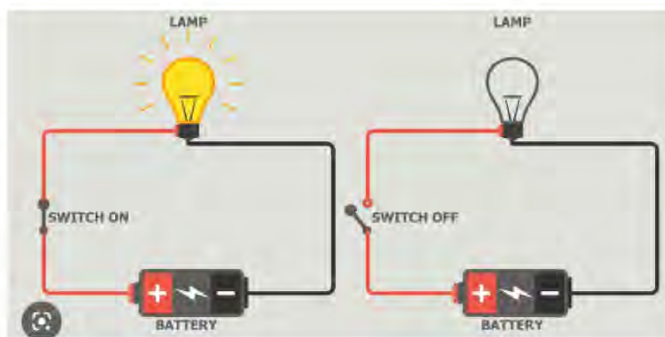
2.1 Binary Number System

2.1.1 Introduction to binary number system

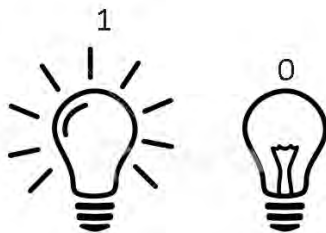
Activity 1

The figure of the electric circuit is given below. Study the figure and discuss the questions below.

- How many processes are there in switching an electric circuit? What are they?
- How is the domestic electric circuit related to the circuit given in the figure?



Here are two processes of switching in the electric circuit, which are turning it ON and turning it OFF. The light bulb illuminates when you turn it ON. The light bulb stops glowing when you switch it OFF. It can be represented in symbol 1 or 0



Activity 2

List the numbers made up of digits 0 and 1 between the decimal numbers 0 and 150. Arrange them in ascending order. Compare them with the numbers of the decimal number system.

Here, the numbers in the decimal system are formed by digits 0 and 1 only.
For example,

0, 1, 10, 11, 100, 101, 110, 111

Now, count these numbers as 1, 2, 3, and so on, and compare them as shown in the table below.

Binary number system	1	10	11	100	101	110	111			
Decimal number system	1	2	3	4	5	6	7	8	9	10

In the Decimal system, we use 10 digits whereas in the binary system, we use only two digits namely 0 and 1. We count numbers in the binary system as {1,10,11,100,101,110,}

2.1.2 Binary number system and its expansion

Activity 3

How can we represent the following binary numbers in the place value table?

1_2 , 10_2 , 11_2 , 100_2 , 100101_2 ,

The place value table for the binary number system is represented as:

Sixteen	Eight	Four	Two	one
2^4	2^3	2^2	2^1	2^0

We can represent the binary number system in expanded form using the table above.

$$1_2 = 1 \cdot 2^0$$

$$10_2 = 1 \cdot 2^1 + 0 \cdot 2^0$$

$$11_2 = 1 \cdot 2^1 + 1 \cdot 2^0$$

$$100_2 = 1 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0$$

$$101_2 = 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$$

Similar to how we write in decimal number system in the place value table as one, ten, hundred, thousand $\dots$ $10^0, 10^1, 10^2, 10^3$.
 in the place value table we can write binary number system by writing one, two, eight, sixteen $\dots$ $2^0, 2^1, 2^2, 2^3$. We express the binary number system as the power of 2. A number from the binary number system is explained using the suffix 2.
 For example 102, 10012

2.1.3 Conversion from Decimal number system to Binary

Activity 4

Express 27 and 29 as the sum of the power of 2 and compare the process.

Here,

Place value table

$$27 = 16 \quad 8 \quad 2 \quad 1$$

$$= 1 \quad 2^4 \quad 1 \quad 2^3 \quad 1 \quad 2^1 \quad 1 \quad 2^0$$

$$\therefore 27 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 11011_2$$

sixteen	eight	four	two	one
2^4	2^3	2^2	2^1	2^0
1	1	0	1	1

Place value table

$$29 = 16 \quad 8 \quad 4 \quad 1$$

$$= 2^4 \quad 2^3 \quad 2^2 \quad 1$$

$$= 1 \quad 2^4 \quad 1 \quad 2^3 \quad 1 \quad 2^2 \quad 0 \quad 2^1 \quad 1 \quad 2^0$$

$$\therefore 29 = 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 11101_2$$

sixteen	eight	four	two	one
2^4	2^3	2^2	2^1	2^0
1	1	1	0	1

Here, in both processes, 27 and 29 are written as the sum of the numbers that can be expressed as a power of 2.

Based on the above activity, discuss how can we express the given decimal numbers as binary numbers.

Writing the remainder from bottom to top it will be 11011.

2	27	
2	13	1
2	6	1
2	3	0
2	1	1
0		1

-In base 10 the remainder is 0 to 9

-In base 2 remainder is 0 to 1

Hence, to convert a decimal number into binary we should divide them by 2 continuously and record the remainder.

Thus, $27 = 11011_2$

Similarly convert 29 into binary number system.

While converting from a decimal number system to a binary number system, we divide the decimal number and its quotient continuously by 2 until the quotient is zero and record the remainder on the right side. Finally, write the remainder from bottom to top to find the number in the binary system.

Example 1

Express 43 in the power of 2 and write in expanded form.

Example 1

Expressing 43 as the sum of the power of 2.

$$43 = 32 + 8 + 2 + 1$$

$$= 2^5 + 2^3 + 2^1 + 2^0$$

$$43 = 1 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$

Example 1

Convert 25 in the binary system.

Example 1

Here converting 25 into binary we get,

$$\therefore 25 = 11001_2$$

2	25	
2	12	... 1
2	6	... 0
2	3	... 0
2	1	... 1
0		... 1

2.1.4 Conversion from binary to decimal number system

Activity

The binary number 100101_2 is shown in the place value table. Study the table and discuss the following questions.

Base 10	32	16	8	4	2	1
Base 2	2^5	2^4	2^3	2^2	2^1	2^0
	1	0	0	1	0	1

- Can we write 100000_2 equal to 32 in the decimal number system? If yes how?
- What is the value of 101_2 in the decimal number system?
- Can we write 00101_2 in a binary number system? What is the value of it in the decimal number system?
- What is the value of 100101_2 in the decimal number system?

Here, expanding 100101_2 in the power of 2,

$$\begin{aligned} 100101_2 &= 1 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 \\ &= 1 \cdot 32 + 0 \cdot 16 + 0 \cdot 8 + 1 \cdot 4 + 0 \cdot 2 + 1 \cdot 1 \end{aligned}$$

Now, writing it in the decimal number system,

$$32 + 4 + 1 = 37$$

Note: We do not write base 10, as it is understood just by 37.

To convert the binary number system into a decimal number system we have to write the given number in the expanded form in the power of 2 and then simplify.

Example 3

Convert the following binary numbers into decimal numbers.

- (a) 1001_2 (b) 1111_2 (c) 1001011_2 (d) 1100101_2

Solution

$$\begin{aligned} \text{(a) } 1001_2 &= 1 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 \\ &= 8 + 0 + 0 + 1 \\ &= 9 \end{aligned}$$

$$\therefore 1001_2 = 9$$

$$\begin{aligned} \text{(b) } 1111_2 &= 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 \\ &= 8 + 4 + 2 + 1 \\ &= 15 \end{aligned}$$

$$\therefore 1111_2 = 15$$

$$\begin{aligned} \text{(c) } 1001011_2 &= 1 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 \\ &= 64 + 8 + 2 + 1 \\ &= 75 \end{aligned}$$

$$\therefore 1001011_2 = 75$$

$$\begin{aligned} \text{(d) } 1100101_2 &= 1 \cdot 2^6 + 1 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 \\ &= 64 + 32 + 4 + 1 \\ &= 101 \end{aligned}$$

$$\therefore 1100101_2 = 101$$

2.2 Quinary Number System

2.2.1. Introduction to quinary number system

Activity 6

The numbers from 0 to 59 in the decimal number system are given in the table below. Circle the numbers that are made up of using digits 0,1,2,3, and 4.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59

Now, Arrange them in ascending order. Compare them with the numbers of the decimal number system.

Here, we count these numbers as 1, 2, 3, 4, 10, 11, and so, on and compare them as shown in the table below.

Quinary Number system	1	2	3	4	10	11	12	13	14	20			
Decimal Number system	1	2	3	4	5	6	7	8	9	10			

In the decimal (base 10) system, we use 10 digits whereas in the quinary number system, we use only five digits namely 0, 1, 2, 3, and 4. We count numbers in the binary system as {0, 1, 2, 3, 4, 10, 11, 12, 13, 14, 100, 101, }

2.2.2. Quinary number system and its expansion

Activity 7

Form different groups within yourself as required. Collect 27 pens from the class. Arrange the pens in a group of 5. Express the group of 5 as the power of 5. In this manner how would you group 27 pens as the power of 5? Discuss with your friends.



Here,

$$\therefore 27 = 5 + 5 + 5 + 5 + 5 + 2$$

$$\begin{aligned}
 &= 5 \quad 5 \quad 2 \\
 &= 5^2 \quad 2 \quad 1 \\
 &= 1 \quad 5^2 \quad 2 \quad 5^0 \\
 &= 1 \quad 5^2 \quad 0 \quad 5^1 \quad 2 \quad 5^0
 \end{aligned}$$

As in the above discussion, expand 179 as the power of 5.

Here,

$$\begin{aligned}
 179 &= 125 \quad 50 \quad 4 \\
 &= 125 \quad 2 \quad 25 \quad 4 \\
 &= 5 \quad 5 \quad 5 \quad 2 \quad 5 \quad 5 \quad 4 \quad 1 \\
 &= 5^3 \quad 2 \quad 5^2 \quad 4 \quad 5^0 \\
 &= 1 \quad 5^3 \quad 2 \quad 5^2 \quad 0 \quad 5^1 \quad 4 \quad 5^0
 \end{aligned}$$

Example 8

How can we express given quinary numbers in the place value table? Discuss with your friend.

$$1_5, 43_5, 104_5, 1340_5$$

We can express the quinary number system in expanded form using the table below.

One hundred twenty five	Twenty five	Five	one
5^3	5^2	5^1	5^0

We can use the table to write the expansion of the numbers.

$$\begin{aligned}
 1_5 &= 1 \quad 5^0 \\
 43_5 &= 4 \quad 5^1 \quad 3 \quad 5^0 \\
 104_5 &= 1 \quad 5^2 \quad 0 \quad 5^1 \quad 4 \quad 5^0 \\
 1340_5 &= 1 \quad 5^3 \quad 3 \quad 5^2 \quad 4 \quad 5^1 \quad 0 \quad 5^0
 \end{aligned}$$

Expressing 13405 in place value table,

one hundred twenty five	Twenty five	Five	one
1	3	4	0

Similar to how we write in decimal number system in the place value table as one, ten, hundred, thousand . . . $10^0, 10^1, 10^2, 10^3$. . . in the place value table, we can write quinary number system by writing one, five, twenty-five, one hundred twenty-five, Or $5^0, 5^1, 5^2, 5^3$..

We express the quinary number system as the power of 5. A number from the quinary number system is explained using the suffix 5.

2.2.3 Conversion from decimal number system into quinary number

Activity 9

How can we write the decimal number 432 into the quinary number system? Discuss with your friends.

Here, converting 432 into the quinary number

$$\begin{array}{r}
 5 \overline{) 432} \\
 \underline{5 \quad 86} \quad 2 \\
 5 \overline{) 17} \quad 1 \\
 \underline{5 \quad 3} \quad 2 \\
 0 \quad 3 \\
 432 = 3212_5
 \end{array}$$

To Convert a decimal number into a quinary number, we keep on dividing by 5 continuously until the quotient is 0. The remainder should be written on the right-hand side of each step of the division. Finally, the remainders are written from bottom to top to get the final answer.

Example 4

Convert the following decimal numbers into quinary.

(a) 512

$$\begin{array}{r}
 5 \overline{) 512} \\
 \underline{5 \quad 102} \quad 2 \\
 5 \overline{) 20} \quad 2 \\
 \underline{5 \quad 4} \quad 0 \\
 0 \quad 4
 \end{array}$$

$$512 = 4022_5$$

(b) 7521

$$\begin{array}{r}
 5 \overline{) 7521} \\
 \underline{5 \quad 1504} \quad 1 \\
 5 \overline{) 300} \quad 4 \\
 \underline{5 \quad 60} \quad 0 \\
 5 \overline{) 12} \quad 0 \\
 \underline{5 \quad 2} \quad 2 \\
 0 \quad 2
 \end{array}$$

$$7521 = 220041_5$$

2.2.4 Conversion from quinary

Activity 10

The number 142013_5 is shown in the place value table. Study the place value table and discuss on following questions.

Base ten	3125	625	125	25	5	1
Base two	5^5	5^4	5^3	5^2	5^1	5^0
	1	4	2	0	1	3

- Can we write 100000_5 as 3125? If yes, how do you relate them?
- How can we write 2013_5 in the decimal number system?
- Is the value of 013_5 equal to 6 in the decimal number system?
- How can we write 142013_5 in the decimal number system?

Here, expanding 142013_5 as the power of 5

$$1 \quad 3125 \quad 4 \quad 625 \quad 2 \quad 125 \quad 0 \quad 25 \quad 1 \quad 5 \quad 3 \quad 1$$

$$\text{or, } 1 \quad 5^5 \quad 4 \quad 5^4 \quad 2 \quad 5^3 \quad 0 \quad 5^2 \quad 1 \quad 5^1 \quad 3 \quad 5^0$$

Now, expressing the value in the decimal system,

$3125 \quad 2500 \quad 250 \quad 0 \quad 5 \quad 3 = 5883$. But, base 10 is not written in the suffix as the decimal numbers are understood without it. $\therefore 142013_5 = 5883$

To convert from the quinary system to the decimal system, we express the number as a power of 5 and simplify it to obtain the final result.

Example 5

Convert 234_5 into a decimal number system.

Solution

$$\begin{aligned} 234_5 &= 2 \quad 5^2 \quad 3 \quad 5^1 \quad 4 \quad 5^0 \\ &= 2 \quad 25 \quad 3 \quad 5 \quad 4 \quad 1 \\ &= 50 \quad 15 \quad 4 = 69 \end{aligned}$$

Example 6

Convert the following numbers as indicated.

(A) 43_5

(B) 10011_2

Solution

(A)	(B)
$43_5 = 4 \cdot 5^1 + 3 \cdot 5^0$ $= 4 \cdot 5 + 3 \cdot 1$ $= 20 + 3$ $= 23$ Converting 23 in the binary number system $\begin{array}{r l} 2 & 23 \\ \hline 2 & 11 \quad \dots 1 \\ \hline 2 & 5 \quad \dots 1 \\ \hline 2 & 2 \quad \dots 1 \\ \hline 2 & 1 \quad \dots 0 \\ \hline & 0 \quad \dots 1 \end{array}$ $43_5 = 10111_2$ $\therefore 43_5 = 10111_2$	$10011_2 = 1 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$ $= 1 \cdot 16 + 0 \cdot 0 + 1 \cdot 2 + 1 \cdot 1$ $= 16 + 2 + 1$ $= 19$ Converting 19 in the quinary number system $\begin{array}{r l} 5 & 19 \\ \hline 5 & 3 \quad \dots 4 \\ \hline & 0 \quad \dots 3 \end{array}$ $19 = 34_5$ $\therefore 10011_2 = 34_5$

Converting to and from binary to quinary, we first convert them into a decimal number system.

Exercise 2.1

1. Write the following numbers in expanded form.

(a) 546

(b) 6542

(c) 1234

(d) 45872

(e) 258963

(f) 97332

2. Express the following numbers as the power of 2 and 5.

(a) 10

(b) 25

(c) 59

(d) 402

(e) 805

(f) 932

3. State the number system of the following numbers.

- (a) 10011_2 (b) 350 (c) 1001_2
(d) 42_5 (e) 555 (f) 77532
(g) 10010011_2 (h) 257903 (i) 4023_5
(j) 5321 (k) 1234_5 (l) 101011_2

4. Convert the following numbers from the decimal number system to the binary number system.

- (a) 4 (b) 9 (c) 12 (d) 25 (e) 35
(f) 79 (g) 94 (h) 100 (i) 104 (j) 135
(k) 190 (l) 22 (m) 250 (n) 275 (o) 366
(p) 512 (q) 530 (r) 600

5. Convert the following numbers from the binary number system to the decimal number system.

- (a) 11_2 (b) 101_2 (c) 111_2 (d) 1100_2
(e) 10101_2 (f) 11001_2 (g) 10010_2 (h) 11110_2
(i) 100001_2 (j) 11111_2 (k) 1100011_2 (l) 1110011_2
(m) 1100110011_2 (n) 1010101110_2 (o) 100001000_2
(p) 101110111_2 (q) 11011011001_2 (r) 1111111110_2

6. Convert the following numbers to the quinary number system.

- (a) 9 (b) 13 (c) 21 (d) 26 (e) 45 (f) 55
(g) 86 (h) 112 (i) 194 (j) 404 (k) 497 (l) 650
(m) 1128 (n) 1234 (o) 2125 (p) 2536 (q) 3000 (r) 3650

7. Convert the following numbers from the quinary number system to the decimal number system.

- (a) 21_5 (b) 24_5 (c) 34_5 (d) 101_5 (e) 123_5
(f) 300_5 (g) 343_5 (h) 441_5 (i) 2023_5 (j) 1234_5
(k) 2113_5 (l) 3313_5 (m) 2014_5 (n) 4201_5 (o) 4321_5
(p) 12304_5 (q) 10123_5 (r) 21432_5

8. Convert the following to binary if it is in quinary and quinary if it is in binary number system.

- (a) 40_5 (b) 10101_2 (c) 31_5 (d) 101_2 (e) 144_5
(f) 101110_2 (g) 23_5 (h) 23_5 (i) 10011_2
(j) 110111_2 (k) 313_5

9. (a) Which of the numbers 11012 and 245 is larger? Also, find their difference.
- (b) Which of the numbers 1100112 and 1445 is larger? Also, find their difference.

Project work

- Write any two whole numbers you like. Convert them into binary and quinary numbers and present them in your class.
- Write the total number of students in your school and the students in class 8. Convert these numbers of students into binary and quinary number system and present it in your class.

Answers

- (a) $5 \cdot 10^2 \cdot 4 \cdot 10^1 \cdot 6 \cdot 10^0$ (b) $6 \cdot 10^3 \cdot 5 \cdot 10^2 \cdot 4 \cdot 10^1 \cdot 2 \cdot 10^0$

(c) $1 \cdot 10^3 \cdot 2 \cdot 10^2 \cdot 3 \cdot 10^1 \cdot 4 \cdot 10^0$

(d) $4 \cdot 10^4 \cdot 5 \cdot 10^3 \cdot 8 \cdot 10^2 \cdot 7 \cdot 10^1 \cdot 2 \cdot 10^0$

(e) $2 \cdot 10^5 \cdot 5 \cdot 10^4 \cdot 8 \cdot 10^3 \cdot 9 \cdot 10^2 \cdot 6 \cdot 10^1 \cdot 3 \cdot 10^0$

(f) $9 \cdot 10^4 \cdot 7 \cdot 10^3 \cdot 3 \cdot 10^2 \cdot 3 \cdot 10^1 \cdot 2 \cdot 10^0$
- (a) $1 \cdot 2^3 \cdot 0 \cdot 2^2 \cdot 1 \cdot 2^1 \cdot 0 \cdot 2^0$ $2 \cdot 5^1 \cdot 0 \cdot 5^0$

(b) $1 \cdot 2^4 \cdot 1 \cdot 2^3 \cdot 0 \cdot 2^2 \cdot 0 \cdot 2^1 \cdot 1 \cdot 2^0 \cdot 1 \cdot 5^2 \cdot 0 \cdot 5^1 \cdot 0 \cdot 5^0$

(c) $1 \cdot 2^5 \cdot 1 \cdot 2^4 \cdot 1 \cdot 2^3 \cdot 0 \cdot 2^2 \cdot 1 \cdot 2^1 \cdot 1 \cdot 2^0 \cdot 2 \cdot 5^2 \cdot 1 \cdot 5^1 \cdot 4 \cdot 5^0$

(d) $1 \cdot 2^8 \cdot 1 \cdot 2^7 \cdot 0 \cdot 2^6 \cdot 0 \cdot 2^5 \cdot 1 \cdot 2^4 \cdot 0 \cdot 2^3 \cdot 0 \cdot 2^2 \cdot 1 \cdot 2^1 \cdot 0 \cdot 2^0 \cdot 3 \cdot 5^3$
 $1 \cdot 5^2 \cdot 0 \cdot 5^1 \cdot 2 \cdot 5^0$

(e) $1 \cdot 2^9 \cdot 1 \cdot 2^8 \cdot 0 \cdot 2^7 \cdot 0 \cdot 2^6 \cdot 1 \cdot 2^5 \cdot 0 \cdot 2^4 \cdot 0 \cdot 2^3 \cdot 1 \cdot 2^2 \cdot 0 \cdot 2^1 \cdot 1 \cdot 2^0$
 $1 \cdot 5^4 \cdot 1 \cdot 5^3 \cdot 2 \cdot 5^2 \cdot 1 \cdot 5^1 \cdot 0 \cdot 5^0$

(f) $1 \cdot 2^9 \cdot 1 \cdot 2^8 \cdot 1 \cdot 2^7 \cdot 0 \cdot 2^6 \cdot 1 \cdot 2^5 \cdot 0 \cdot 2^4 \cdot 0 \cdot 2^3 \cdot 1 \cdot 2^2 \cdot 0 \cdot 2^1 \cdot 0 \cdot 2^0$
 $1 \cdot 5^4 \cdot 2 \cdot 5^3 \cdot 2 \cdot 5^2 \cdot 1 \cdot 5^1 \cdot 2 \cdot 5^0$
- (a) binary (b) decimal (c) binary (d) quinary (e) decimal

(f) decimal (g) binary (h) decimal (i) quinary (j) decimal

(k) quinary (l) binary

4. (a) 1002 (b) 10012 (c) 11002 (d) 110012 (e) 1000112
 (f) 10011112 (g) 10111102 (h) 11001002 (i) 11010002 (j) 100001112
 (k) 101111102 (l) 110111002 (m) 111110102 (n) 1000100112
 (o) 1011011102 (p) 10000000002 (q) 10000100102
 (r) 10010110002
5. (a) 3 (b) 5 (c) 7 (d) 12 (e) 21 (f) 25
 (g) 18 (h) 30 (i) 33 (j) 63 (k) 99 (l) 115
 (m) 819 (n) 686 (o) 264 (p) 375 (q) 1753 (r) 1022
6. (a) 145 (b) 235 (c) 415 (d) 1015 (e) 1405 (f) 2105
 (g) 3215 (h) 4225 (i) 12345 (j) 31045 (k) 34425 (l) 101005
 (m) 140035 (n) 144145 (o) 320005 (p) 401215 (q) 440005
 (r) 1041005
7. (a) 11 (b) 14 (c) 19 (d) 26 (e) 38 (f) 75
 (g) 98 (g) 121 (h) 263 (i) 194 (j) 283 (k) 458
 (l) 259 (m) 551 (n) 586 (o) 954 (p) 663 (q) 1492

Lesson 3

Rational and Irrational Numbers

3.0 Review

Discuss the following questions in your groups.

- (i) What are rational numbers? Which of the following numbers are rational?

Answer with reasons.

(a) The average height of Nepalese people is $\frac{21}{4}$ feet.

(b) The freezing point of water at the sea level is 0 °C.

(c) A runner can run at the speed of 200 m per minute.

(d) $\frac{4}{7}$

(e) 1

(f) $\frac{4}{5}$

- (ii) Which of the following fractions can be changed into terminating or non-terminating recurring decimals?

(a) $\frac{4}{5}$

(b) $\frac{5}{3}$

(c) $\frac{4}{7}$

(d) $\frac{7}{2}$

(e) $\frac{5}{9}$

- (iii) Change the following decimals into fractions.

(a) 0.75

(b) 0.05

(c) $0.\overline{3}$

(d) $0.\overline{65}$

(e) $0.\overline{285714}$

Are the decimal numbers in question number (iii) rational or not?

3.1 Irrational Number

Activity 1

- (i) Find the value of following and identify which of them are rational numbers or which are not.

(a) square root of 4

(b) square root of $\frac{4}{9}$

(c) 0.25

(d) 4.6666

(e) square root of 2

(f) π

In the above cases,

(a) The square root of 4 is 2. The number 2 can be written in the form $\frac{a}{b}$ like $\frac{2}{1}$, $\frac{4}{2}$, $\frac{6}{3}$.. So, the square root of 4 is a rational number..

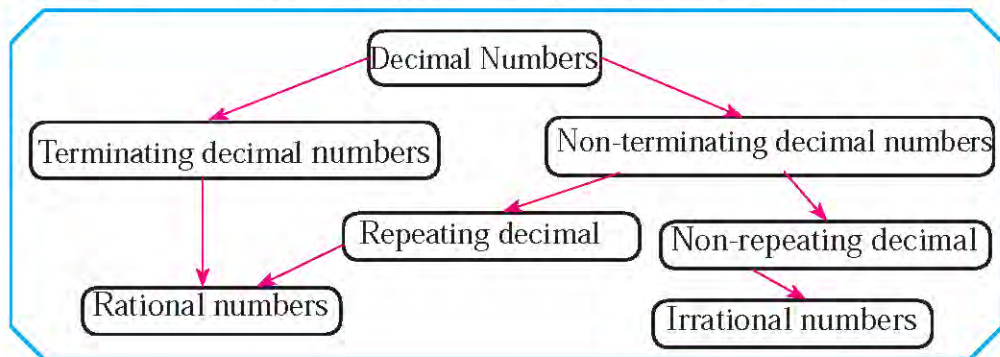
(b) The square root of $\frac{4}{9}$ is $\frac{2}{3}$ and it can be written in the form $\frac{a}{b}$.

So, it is a rational number.

- (c) 0.25 is a terminating decimal number. It can be written as a fraction $\frac{25}{100}$, which is in the form $\frac{a}{b}$. So, 0.25 is a rational number.
- (d) 4.66 is a non-terminating recurring decimal number.
 $4.66 \dots = \frac{42}{9} = \frac{14}{3}$ is written in the form $\frac{a}{b}$. So, 4.66 is rational.
- (e) The square root of 2 is 1.41421..., which is a non-terminating and non-recurring decimal number. It cannot be reduced to the fraction. So, it cannot be represented in the form $\frac{a}{b}$.
Hence, the square root of 2 is not a rational number.
- (f) The value of π is 3.1415926..., which is a non-terminating and non-recurring decimal number. It cannot be reduced to a fraction and so cannot be written in the form $\frac{a}{b}$. Hence, the value of π is not a rational number.

A number which can be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$, is called rational number. For example : 2, 5, 7, $\frac{5}{8}$, 2.13, $1.\bar{6}$... are rational numbers. The numbers which cannot be expressed in the form $\frac{a}{b}$ are irrational numbers. So, the numbers which are not rational are irrational. The terminating and non-terminating decimals are rational numbers as they can be reduced to fractions. On the other hand, the non-terminating non-recurring decimals are irrational numbers as they cannot be reduced to fractions. For example: $\sqrt{2}$, $\sqrt{5}$, $\sqrt[3]{10}$, $\sqrt{\frac{1}{3}}$, $\sqrt{7}$, 2.134..., π ... are irrational numbers.

The decimal numbers can be presented by the following chart.



Question to think $\frac{22}{7}$ is a rational number and π is an irrational number. But we take the value of $\pi = \frac{22}{7}$ while solving problems. Why?

Activity 2

We have already discussed about the natural numbers, whole numbers, integers, rational numbers and irrational numbers. Now, discuss and find the answer to the following.

The relationship between these numbers.

How can the numbers be presented in chart?

What will be the union of the sets of these numbers?

The counting numbers are called natural numbers. $N = \{1, 2, 3, \dots\}$

The set of counting numbers including zero becomes the set of whole numbers $W = \{0, 1, 2, 3, \dots\}$

The whole numbers as well as their negatives are integers. $Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

The integers and fractional numbers form the set of rational numbers. The decimal numbers are of three types: Terminating, non-terminating and recurring, and non-terminating and non-recurring. The terminating and non-terminating recurring decimal numbers are rational numbers as they can be reduced to fractions.

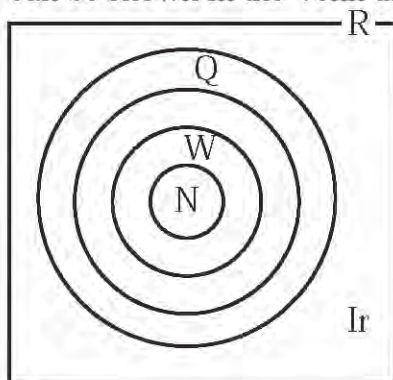
The numbers $\frac{4}{7}$, 1 , 0 , 1 , 0.75 , $0.\overline{3}$ are rational numbers.

The non-terminating and non-recurring decimal numbers cannot be reduced to fractions. Such numbers are irrational. The union of set of rational (Q) and set of irrational numbers (Ir) is the set of real numbers. The set of rational and irrational numbers are disjoint sets.

The real numbers can be presented in a real line. The relationship between these different numbers can be shown in the set relation as follows.

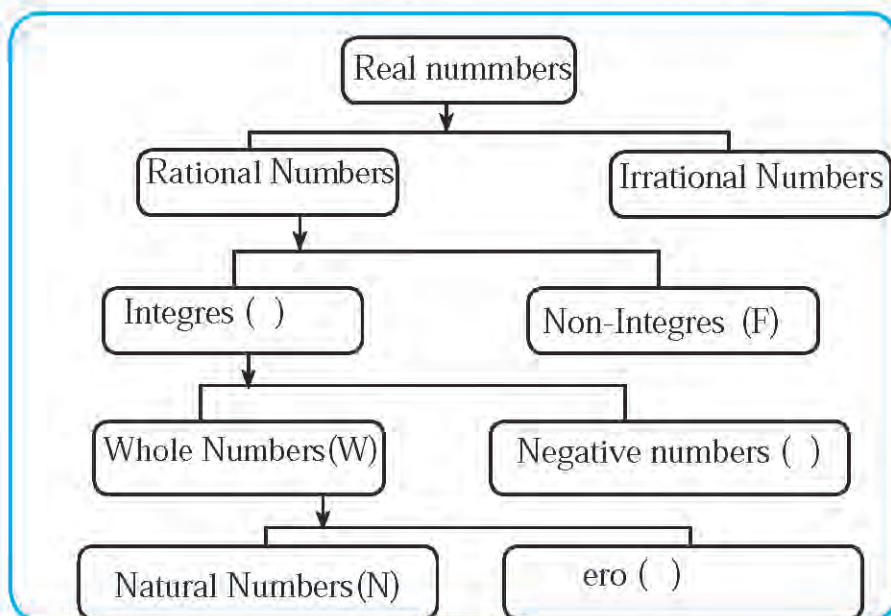
$$N \subseteq W \subseteq \mathbb{Z} \subseteq Q \subseteq R, Ir \subseteq R$$

The set of real numbers can be shown in the Venn diagram as follows.



$$N \subseteq W \subseteq \mathbb{Z} \subseteq Q \subseteq R, Ir \subseteq R$$

The set of real numbers can be presented in the chart as follows.



Activity 3

The integers can be shown on the number line. How can we show the number $\sqrt{2}$ on the number line?



Since $\sqrt{2}=1.4421\dots\dots$. How can the corresponding point be marked on the line? It is quite difficult to show on the number line. How is it possible?

Yes, it is really difficult. Let's go and ask to our teacher.



Respected madam ! how can the number $\sqrt{2}$ be shown on the number line?

Teacher: The value of $\sqrt{2}$ is $1.4421\dots\dots$. So, it is quite difficult to mark on the number line.

Student: Yes, madam.

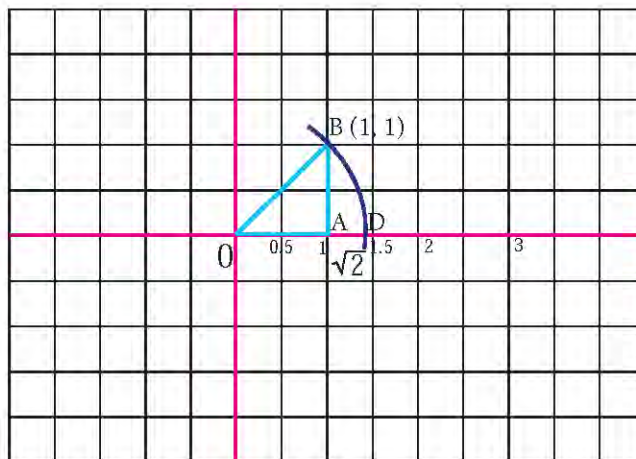
Teacher: You all know the Pythagoras theorem. According to the theorem, the relation of a , b and h in a right angled triangle is $a^2 + b^2 = h^2$. Can you tell the value of h , if $a = 1$ and $b = 1$?

Student: $h = \sqrt{(1^2 + 1^2)} = \sqrt{2}$

Teacher: Yes. Let's take a radius of $\sqrt{2}$ unit with the help pencil and compass and then mark on the line taking the same length from origin.

Student: How can it be done madam?

Teacher: Draw a number line on a graph paper and draw a right angled triangle OAB as shown in the figure. The length of OB is $\sqrt{2}$ unit. Take an arc of radius equal to $\sqrt{2}$ units from the origin and cut the number line XX' at the point D. Since, $OB = OD$. Hence, $OD = \sqrt{2}$.



Example 1

Identify the following numbers as rational or irrationals and also give reason.

(a) $\frac{5}{2}$

(b) $\sqrt{7}$

(c) $4.\overline{58}$

(d) 1.758621357

Solution:

- (a) $\frac{5}{2}$ is a rational number, because both the numbers in numerator and denominator are integers and also the integer in the denominator is non-zero.
- (b) $\sqrt{2} = 2.645751\dots\dots\dots$ is an irrational number as it is non – terminating and non-recurring decimal number.
- (c) $4.\overline{58}$ is a rational number as it is non-terminating recurring decimal number.
- (d) 1.758621357... an irrational number as it is non-terminating and non-recurring decimal number.

Example 2

Reduce the following decimal numbers to fractions:

(a) $0.\overline{3}$

(b) $0.\overline{41}$

Solution:

(a) $0.\overline{3}$

Let, $x = 0.\overline{3}$

$\therefore x = 0.333\dots$ (i)

Here, only single digit after decimal is repeated. So, multiplying equation (i) by 10,

$10x = 3.33\dots$ (ii)

Subtracting (ii) from (i),

$10x - x = 3.33\dots - 0.33\dots$

or, $9x = 3$

or, $x = \frac{3}{9} = \frac{1}{3}$

$\therefore 0.\overline{3} = \frac{1}{3}$

(b) $0.\overline{41}$

Let, $x = 0.\overline{41}$

$\therefore x = 0.4141\dots$ (i)

Here, two digits after decimal are repeated. So, multiplying equation (i) by 100,

$100x = 41.4141\dots$ (ii)

Subtracting (ii) from (i),

$100x - x = 41.4141\dots - 0.4141\dots$

or, $99x = 41$

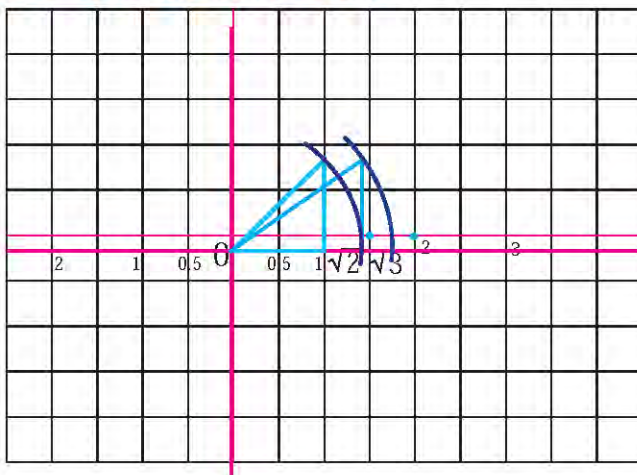
or, $x = \frac{41}{99}$

$\therefore 0.\overline{41} = \frac{41}{99}$

Example 3

Show the number $\sqrt{3}$ on the number line:

Solution:



In the figure, the length of each of the base and perpendicular of the right angled triangle is 1 unit. So, the length of hypotenuse is $\sqrt{2}$ units. Again, the hypotenuse of the right angled triangle with base $\sqrt{2}$ units and perpendicular 1 unit is $\sqrt{3}$ units. So, it should be taken an arc of radius $\sqrt{3}$ units from origin to mark the point on the number line.

Exercise 3.1

1. Write true or false to the following statements.

- All rational and irrational numbers are included in the set of real numbers.
- The set of all rational numbers and set of irrational numbers are disjoint.
- The set of whole numbers and set of integers are same.
- $Z \subset Q \subset R$
- $\sqrt{4}$ is an irrational number.
- $Z \supset N \supset W$
- All rational numbers are real numbers.

2. Identify the rational and irrational numbers from the following.

- | | | | | |
|--------------------|-------------------|-----------------|-------------------|---------------------|
| (a) $\frac{3}{4}$ | (b) $\sqrt{2}$ | (c) $\sqrt{5}$ | (d) $\frac{2}{5}$ | (e) $\frac{10}{20}$ |
| (f) 3.57 | (g) 3.5982 | (h) 15 | (i) 0.735. | |
| (j) $\sqrt{169}$ | (k) $\sqrt{3}$ | (l) $\sqrt{26}$ | (m) $2.\bar{5}$ | |
| (n) $\frac{35}{9}$ | (o) $\sqrt[3]{9}$ | | | |

3. Reduce the following decimal numbers to fractions.

- | | | | | |
|-----------------------|------------------------|-----------------------|------------------------|------------------------|
| (a) $0.\bar{5}$ | (b) $0.\bar{7}$ | (c) $0.\overline{24}$ | (d) $0.\overline{132}$ | (e) $0.\overline{27}$ |
| (f) $1.\overline{57}$ | (g) $0.\overline{365}$ | (h) $4.\overline{78}$ | (i) $0.\overline{445}$ | (j) $1.\overline{525}$ |

4. Show the given numbers on the number line.

- | | | | |
|----------------|----------------|----------------|----------------|
| (a) $\sqrt{5}$ | (b) $\sqrt{6}$ | (c) $\sqrt{7}$ | (d) $\sqrt{8}$ |
|----------------|----------------|----------------|----------------|

5. Define rational and irrational numbers with examples.

6. Show the relationship of rational and irrational numbers in venn diagram.

7. Write two differences between rational and irrational numbers.

Project work

1. Take a square shaped chart paper and prepare a graph assuming 2 cm length as 1 unit. Then, show the numbers $\sqrt{2}$ and $\sqrt{3}$ on the number line. Also, present it in the classroom.
2. Take any 5 rational numbers and reduce them to decimal form. Identify whether they are terminating or non terminating recurring decimals.

Answers

1. (a) True (b) True (c) False (d) True (e) False (f) False
2. Rational numbers: (a), (d), (e), (f), (h), (j), (m), (n)
Irrational numbers: (b), (c), (g), (i), (k), (l), (o)
3. (a) $\frac{5}{9}$ (b) $\frac{7}{9}$ (c) $\frac{24}{99}$ (d) $\frac{132}{999}$ (e) $\frac{27}{99}$ (f) $\frac{157}{99}$ (g) $\frac{365}{999}$
(h) $\frac{474}{99}$ (i) $\frac{445}{999}$ (j) $\frac{1524}{999}$

Show the answers of questions from number 4 to 7 to your teacher.

3.2 Scientific Notation of Numbers

Activity 4

The distance between the planets from the sun is given in the following table. How can we read the numbers showing the distance? Can they be written in short form? If so, how? Discuss these issues in your group.

The planet mercury is in the nearest distance from the sun. Its distance from the sun is 57910000 km. Which is read as Five crore seventy nine lakh and ten thousand kilometer according to our national system and Fifty seven million nine hundred ten thousand kilometer according to the international



Average Distance of the Planets from the Sun	
Planet	Average Distance (m)
Mercury	57,910,000
Venus	108,210,000
Earth	149,600,000
Mars	227,920,000
Jupiter	778,570,000
Saturn	1,433,530,000
Uranus	2,872,460,000
Neptune	4,495,060,000

system. The farthest planet from the sun is Neptune. Its distance from the sun is 4495060000 km. which is read as ‘Four Arab forty nine crore fifty thousand and sixty thousand kilometer’ according to Nepali system and ‘Four billion four hundred ninety five million sixty thousand kilometer’, according to international system. The distance between the earth and the sun is 149600000 km. which is read as ‘Fourteen crore ninety six lakh’ kilometer , according to Nepali system and one hundred forty nine million six hundred thousand kilometer’, according to the international system.

The distance of different planets from the sun is also written as the following.

The distance of the Mercury from the sun = 57,910,000 km = 5.791×10^7 km

The distance of the Neptune from the sun = 4,495,060,000 km = 4.49506×10^9 km

Also write the distance of other planets from the sun in the short notation as above.

Activity 5

The weights of the earth and hydrogen atom are given below. How can the weights be expressed as the powers of ten? Discuss in your group.

(a) The weight of the earth = 5,972,000,000,000,000,000,000 kg

(b) The weight of a hydrogen atom = 0.000,000,000,000,000,000,000,001,673 kg

Here, (a) the weight of the earth = 5,972,000,000,000,000,000,000,000 kg

$$= 5.972 \quad 1,000,000,000,000,000,000,000,000 \text{ kg}$$

$$= 5.972 \quad 10^{24} \text{ kg}$$

(b) The weight of hydrogen atom = 0.000,000,000,000,000,000,000,001,673 kg

$$= \frac{1673}{1,000,000,000,000,000,000,000,000,000}$$

$$= \frac{1.673 \quad 10^3}{10^{30}}$$

$$= 1.673 \quad 10^{3-30}$$

$$= 1.673 \quad 10^{-27} \text{ kg}$$

The scientific notation of the number is that method which expresses the number as the product of a decimal number containing a single digit before decimal and the number in the powers of 10.

For example, $1673 = 1.673 \quad 1000 = 1.673 \quad 10^3$

$$= \frac{157}{100000} = \frac{1.57 \quad 10^2}{10^5} = 1.57 \quad 10^{2-5} = 1.57 \quad 10^{-3}$$

Example 1

Write the following numbers in scientific notation.

- (a) 759 (b) 39000 (c) 0.00037 (d) 123.54 (e) 0.000213

Solution

$$(a) \quad 759 = 7.59 \quad 100 = 7.59 \quad 10^2$$

$$(b) \quad 39000 = 3.9 \quad 10000 = 3.9 \quad 10^4$$

$$(c) \quad 0.00037 = \frac{37}{100000} = \frac{3.7 \quad 10}{100000} = \frac{3.7 \quad 10^1}{10^5}$$

$$= 3.7 \times 10^{15}$$

$$= 3.7 \times 10^4 \quad \left[\because \frac{x^m}{x^n} = x^{m-n} \right]$$

$$(d) \ 123.54 = \frac{12354}{100} = \frac{1.2354 \times 10^4}{10^2}$$

$$= 1.2354 \times 10^{4-2} \quad \left[\because \frac{x^m}{x^n} = x^{m-n} \right]$$

$$= 1.2354 \times 10^2$$

$$(e) \ 0.000213 = \frac{213}{1000000} = \frac{2.13 \times 10^2}{10^6}$$

$$= 2.13 \times 10^{2-6} \quad \left[\because \frac{x^m}{x^n} = x^{m-n} \right]$$

$$= 2.13 \times 10^{-4}$$

Example 2

Write the following numbers given in scientific notation in decimal number system.

$$(a) \ 6.3 \times 10^3 \qquad (b) \ 4.579 \times 10^6$$

$$(c) \ 7.4 \times 10^{-5} \qquad (d) \ 3.579 \times 10^{-4}$$

Solution:

$$(a) \ 6.3 \times 10^3 = 6.3 \times 1000$$

$$= 6300.0$$

$$= 6300$$

The power of 10 is written in the expanded form, for example $10^3=1000$. Then, it is multiplied with second number.

$$(b) \ 4.579 \times 10^6 = 4.579 \times 1000000$$

$$= 4579000.000$$

$$= 4579000$$

$$\begin{aligned}
 \text{(c)} \quad 7.4 \times 10^{-5} &= \frac{7.4}{10^5} \\
 &= \frac{7.4}{1000000} \\
 &= 0.000074
 \end{aligned}$$

The negative power of 10 is expressed as the positive power of 10 writing in denominator, after then the power of 10 is written in the expanded form. The position of decimal in numerator is shifted to the left according to the number of zeros in the denominator.

$$\begin{aligned}
 \text{(d)} \quad 3.579 \times 10^{-4} &= \frac{3.579}{10^4} \\
 &= \frac{3.579}{100000} \\
 &= 0.0003579
 \end{aligned}$$

Exercise 3.2

1. Write the following decimal numbers in the scientific notation.

- | | | | |
|----------------|--------------|--------------|-------------|
| (a) 45 | (b) 3400 | (c) 0.000023 | (d) 101000 |
| (e) 0.010 | (f) 45.01 | (g) 7000000 | (h) 0.00671 |
| (i) 625.6 | (j) 0.07882 | (k) 118000 | (l) 87200 |
| (m) 0.00000272 | (n) 0.000037 | (o) 74171.7 | (p) 3456.78 |

2. Reduce the following scientific notation of numbers to the decimal numbers.

- | | | |
|---------------------------|---------------------------|---------------------------|
| (a) 2.30×10^4 | (b) 5.40×10^1 | (c) 1.76×10^0 |
| (d) 1.76×10^{-3} | (e) 7.4×10^{-5} | (f) 1.901×10^7 |
| (g) 1.525×10^6 | (h) 6.58157×10^7 | (i) 5.256×10^8 |
| (j) 5.23×10^7 | (k) 8.71×10^8 | (l) 7.75763×10^9 |

- The weight of a loaded truck is 12,000 kg . Express it in scientific notation.
- If the radius of an atom of the argon is 0.000,000,000,098 meter, write it in scientific notation.
- If 3×10^8 m s is the speed of light in air, what is this value in the decimal number?
- There are 6480000 seconds in a month of 30 days. Express this number in the scientific notation.

7. If the quantity of petrol stored by Nepal Oil Corporation, Kathmandu is 1.87×10^6 liters, how much kiloliters of petrol is there?
8. If speed of light is 3,00,000 m/s, express it in scientific notation.

Project work

Use various sources of information and find the distance of planets Mars, mercury and Venus from the Earth. Then, express them in scientific notation as well. Also, present your finding in the classroom.

Answer

1. (a) 4.5×10^1 (b) 3.4×10^3 (c) 2.3×10^{-5}
 (c) 1.01×10^5 (d) 1.0×10^1 (e) 4.501×10^1
 (f) 7.0×10^6 (g) 6.71×10^{-3} (h) 6.256×10^2
 (i) 7.882×10^{-2} (j) 1.18×10^5 (k) 8.72×10^4
 (l) 2.72×10^{-6} (m) 3.7×10^{-5} (n) 7.41717×10^4
 (o) 3.45678×10^3
2. (a) 23000 (b) 54 (c) 1.76
 (d) 0.00176 (e) 0.000074 (f) 0.0000001901
 (g) 1525000 (h) 65815700 (i) 525600000
 (j) 0.000000523 (k) 0.0000000871 (l) 0.00000000775763
3. 1.2×10^4 4. 9.8×10^{11} 5. 300000000 m/s
6. 6.48×10^6 7. 1870000 8. 3×10^5

3.3 Simplification of Numbers with Scientific Notations

Activity 6

Discuss in your group and solve the following problems.

- (a) A water tank is filled with water from the two tankers containing 1.2×10^4 liters and 7.0×10^3 liters of water respectively. How much quantity of water is there in the tank? Write in scientific notation.
- (b) There is 1.8×10^8 liters of water in a pond. If 1.6×10^5 liters of water are flowed outside, how much water is left? Express in scientific notation.
- (c) Lakhan has altogether 15 fish ponds each containing 1.8×10^8 liters of water. Find the total amount of water in the ponds writing in scientific notation.

Example 1

Simplify:

(a) $3.4 \times 10^2 + 4.57 \times 10^3$

(b) $4.54 \times 10^3 - 2.4 \times 10^3$

(c) $(2.00 \times 10^3) \times (4.12 \times 10^4)$

(d) $\frac{9.60 \times 10^7}{1.60 \times 10^4}$

Solution

(a) $3.4 \times 10^2 + 4.57 \times 10^3$
 $= 0.34 \times 10^3 + 4.57 \times 10^3$
 $= (0.34 + 4.57) \times 10^3$
 $= 4.91 \times 10^3$

(b) $4.54 \times 10^3 - 2.4 \times 10^3$
 $= (4.54 - 2.4) \times 10^3$
 $= 2.14 \times 10^3$

Alternative method

$$\begin{aligned} & 3.4 \times 10^2 + 4.57 \times 10^3 \\ &= 3.4 \times 100 + 4.57 \times 1000 \\ &= 340 + 4570 \\ &= 4910 = 4.91 \times 10^3 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad & (2.0 \times 10^3) (4.12 \times 10^4) \\
 &= (2.0 \times 10^3) (4.12 \times 10^4) \\
 &= 2.0 \times 4.12 \times 10^3 \times 10^4 \\
 &= 8.24 \times 10^7
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad & \frac{9.60 \times 10^7}{1.60 \times 10^4} \\
 &= \frac{9.60}{1.60} \times 10^{7-4} \\
 &= 6 \times 10^3
 \end{aligned}$$

Exercise 3.3

1. Simplify and write the answer in scientific notation:

$$\begin{aligned}
 \text{(a)} \quad & (1.2 \times 10^5) (5.35 \times 10^6) & \text{(b)} \quad & 6.91 \times 10^2 \times 2.4 \times 10^3 \\
 \text{(c)} \quad & 9.70 \times 10^6 \div 8.3 \times 10^5 & \text{(d)} \quad & 3.67 \times 10^2 \div 1.6 \times 10^1 \\
 \text{(e)} \quad & 8.41 \times 10^{-5} \div 7.00 \times 10^{-6} & \text{(f)} \quad & 1.33 \times 10^5 \div 4.9 \times 10^4
 \end{aligned}$$

2. Simplify and write the answer in scientific notation:

$$\begin{aligned}
 \text{(a)} \quad & (4.3 \times 10^8) (2.0 \times 10^6) & \text{(b)} \quad & (6.0 \times 10^3) (1.5 \times 10^2) \\
 \text{(c)} \quad & (1.5 \times 10^2) (8.0 \times 10^1) & \text{(d)} \quad & (5.23 \times 10^{11}) (3.0 \times 10^{10}) \\
 \text{(e)} \quad & \frac{1.20 \times 10^8}{3.0 \times 10^3} & \text{(f)} \quad & \frac{7.8 \times 10^{12}}{1.3 \times 10^{13}} & \text{(g)} \quad & \frac{8.4 \times 10^4}{1.2 \times 10^3} \\
 \text{(h)} \quad & \frac{5.6 \times 10^{18}}{1.4 \times 10^8} & \text{(i)} \quad & \frac{8.1 \times 10^9}{9.0 \times 10^8} & \text{(j)} \quad & \frac{3.25 \times 10^{10}}{1.625 \times 10^{15}}
 \end{aligned}$$

3. Simplify:

$$\begin{aligned}
 \text{(a)} \quad & \frac{(1.1 \times 10^3) (2.3 \times 10^3)}{1.7 \times 10^6} & \text{(b)} \quad & \frac{9.8 \times 10^8 \div 4.9 \times 10^8}{7.0 \times 10^7}
 \end{aligned}$$

$$(c) \frac{(2.1 \times 10^6) (4.0 \times 10^3)}{4.2 \times 10^4}$$

$$(d) \frac{6.48 \times 10^5}{(2.4 \times 10^4) (1.8 \times 10^2)}$$

4. There are 3.2×10^4 liters of water in a tank and 1.3×10^3 liters of water in a second tank. How much water is there in two tanks?
5. A rocket is projected to travel a distance of 2.7×10^9 km. How much distance is left after traveling the distance of 1.35×10^9 km?
6. How many tanks of capacity 1.6×10^3 liters are needed to store 9.6×10^6 liters of petrol?
7. There is 1.8×10^8 liters of water in a pond. If 1.6×10^5 liters of water are flowed outside, how much water is left? Express in scientific notation.

Project work

Write the weight of you and your friend in kilogram. Reduce the weights given in kilogram to milligram and express in scientific notation. Also present the sum and difference of your weights in scientific notation.

Answer

- | | | |
|-----------------------------|--------------------------|--------------------------|
| 1. (a) 5.47×10^6 | (b) 7.15×10^2 | (c) 10.53×10^6 |
| (d) 3.51×10^2 | (e) 7.71×10^5 | (f) 8.4×10^4 |
| 2. (a) 8.6×10^{14} | (b) 9.0×10^1 | (c) 1.20×10^2 |
| (d) 1.569×10^2 | (e) 0.4×10^5 | (f) 6.0×10^1 |
| (g) 7.0×10^7 | (h) 4.0×10^{10} | (i) 0.9×10^1 |
| (j) 2.0×10^5 | | |
| 3. (a) 2.0×10^9 | (b) 7.0×10^0 | (c) 2.0×10^7 |
| (d) 1.5×10^3 | 4. 3.33×10^4 | 5. 1.35×10^9 km |
| 6. 6000 tank | 7. 1.7984×10^8 | |

Lesson 4

Ratio and Proportion

4.0 Review

Divide the students into groups according to their needs and discuss the following questions and present the findings to the class.

- (a) What is the number of male and female students in your class?
- (b) Calculate the ratio of boys and girls.
- (c) Calculate the ratio of boys and total number of students.
- (d) Calculate the ratio of girls and total number of students.
- (e) Write the number of boys and total number of students in fraction.
- (f) Write the number of boys and total number of students in fraction.

4.1 Ratio

Activity 1

The cost of 1 dozen pencils is Rs. 120




Sovit

The cost of 6 pens is Rs. 150




Sovina

Observe the given figure and discuss the following questions:

- (a) Can we find the ratio of number of pencils and pens? If we can do, what is their ratio? Find it.
- (b) Find the cost of a pencil and a pen.
- (c) Find ratio of the cost of a pencil and a pen.

Here,

- (a) The ratio of number of pencil and pen

$$= \frac{12}{6} = 2:1$$

- (b) The cost of one = $\frac{120}{12}$

$$= \text{Rs. } 10$$

$$\text{The cost of one pencil} = \frac{150}{6} = \text{Rs. } 25$$

- (c) The ratio of the cost of one pencil and one pen = $\frac{10}{25} = 2:5$

Why is a dozen of pencils written as 12 while calculating ratio?



If the quantities to be compared a and b having the same unit, then the ratio of the a and b is $\frac{a}{b}$ or a:b and the ratio of b to a is written as $\frac{b}{a}$ or b:a Here a:b is read as a is to b and b:a is read as b is to a.

Example 1

Find the ratio of the following quantities.

- (a) 25 cm and 30 cm

- (b) 2 dozen and 20

- (c) 250 gm and 1 kg

Solution:

Here,

- (a) Ratio of 25 cm and 30 cm = $\frac{25}{30} = \frac{5}{6} = 5:6$

- (b) When 2 dozen are taken out = 2 12 = 24

[$\because$ Making both is same unit]

$$\text{Here, the ratio of 2 dozen and 20 eggs} = \frac{24}{20} = \frac{6}{5} = 6:5$$

[$\because$ Making both is same unit]

- (c) Here, converting 1 kg to gm,

$$1 \text{ kg} = 1000 \text{ gm}$$

Hence, the ratio of 250 gm and 1kg $= \frac{250}{1000} = \frac{1}{4} = 1:4$

Example 2

Mohammed and Abdul had invested in a factory in the ratio of 2:3. If Mohammad has invested Rs. 2,00,000, then how much had Abdul invested?

Solution:

Here, the ratio of investement of Mohammad and Abdul = 2:3

Mohammad's investment = Rs. 2,00,000

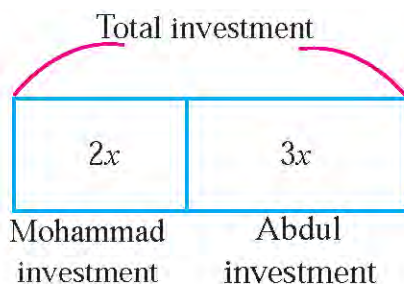
Abdul's investment = ?

$$\text{Now, } \frac{\text{Mohammad investment}}{\text{Abdul investment}} = \frac{2}{3}$$

$$\text{or, } \frac{2,00,000}{\text{Abdul investment}} = \frac{2}{3}$$

$$\therefore \text{ Abdul investment} = \frac{3 \times 2,00,000}{2}$$
$$= \text{Rs } 3,00,000$$

Hence, Abdul had invested Rs. 3,00,000



Alternative Method

Hence, Abdul had invested Rs. 3,00,000

According to the question

$$2x = 200000$$

$$\text{or, } x = 100000$$

$$\text{Now, Abdul investment} = 3x = 3 \times 1,00,000$$

Hence, Abdul had invested Rs. 3,00,000

Example 3

The ratio of the present age of Bishal and Dhaniya is 4:5. After 4 years, their ages ratio will be 5:6. What are their present ages? Find it.

Solution:

Let the present age of Bishal be $4x$ and present age of Dhaniya be $5x$

After 4 years,

The age of Bishal = $4x + 4$

The age of Dhaniya = $5x + 4$

By the question,

$$\frac{4x + 4}{5x + 4} = \frac{5}{6}$$

$$\text{or, } 24x + 24 = 25x + 20$$

$$\therefore x = 4$$

Hence, the present age of Bishal = $4x = 4 \times 4 = 16$ and

the present age of Dhaniya = $5x = 5 \times 4 = 20$

Example 4

The ratio of the yearly income of Dolma, Rambilash and Sonam = 3:4:5. If the sum of their yearly income is Rs. 24,00,000. Find each of their yearly income.

Solution:

Here,

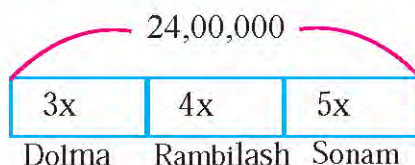
Total income = Rs. 24,00,000

The ratio of the yearly income of Dolma, Rambilash and Sonam = 3:4:5

Therefore, income of Dolma = $\frac{3}{12} \times 24,00,000 = 6,00,000$

Income of Dolma = $\frac{4}{12} \times 24,00,000 = 8,00,000$

Income of Sonam = $\frac{5}{12} \times 24,00,000 = 10,00,000$



Alternative Method

According to the question,

$$3x + 4x + 5x = 24,00,000$$

$$\text{or, } 12x = 24,00,000$$

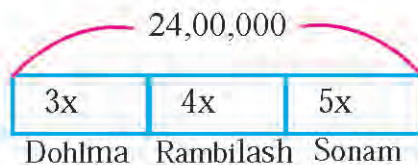
$$\therefore x = 2,00,000$$

Hence,

$$\text{Yearly income of Dolma} = 3x = 3 \times 2,00,000 = \text{Rs } 6,00,000,$$

$$\text{Yearly income of Rambilash} = 4x = 4 \times 2,00,000 = \text{Rs } 8,00,000$$

$$\text{Yearly income of Sonam} = 5x = 5 \times 2,00,000 = \text{Rs } 10,00,000$$



Exercise 4.1

1. Find the ratio of the following quantities.

- (a) 4 hrs and 6 hrs
- (b) 12 ft and 9 ft
- (c) 250 gram and 2 kg
- (d) 3 kg and 850 gram
- (e) 2 hrs and 45 minute
- (f) 40 minute and 1 hr
- (g) 2 l and 850 ml
- (h) Rs 5 and 90 paisa

2. (a) The ratio of the boys students and girls students in alika Secondary School is 5:8. If the number of girls in that school is 480, what is the number of boys? Find it.

- (b) The ratio of the number of teachers and students in Nepal Secondary School is 1:32. If there are 25 teachers, what is the number of students? Find it.

- (c) The ratio of the number of boys and girls of Mahendra Gram Secondary Shool is 330, what is the number of girls?

3. (a) Divide Rs. 840 to John and Joseph in the ratio of 3:4. How much will each get?

- (b) Two booksellers buy same type of 50 dozen exercise book from the wholesale shop. If they divided the exercise book in the ratio of 5:7, how many will each receive? Find it.

4. (a) A map is drawn in a scale of 1: 2,000. The distance between two

places is 8 cm. What is the real distance between the two places?

(b) A map is drawn in a scale of 1: 4,000. The distance between two places is 5 cm. What is the real distance between the two places?

5. (a) The ratio of present age of Rita and Nagma is 4:5. If the ratio of their ages was 3:4 two years ago, what are their present ages? Find it.

(b) Two brothers spent in the ratio of 4:5 on the occasion of birthday of friend. If they had added Rs. 10/10 of each expense then, the ratio of their expense is 5:6. Find each of their expenses.

(c) Two numbers are in the ratio of 1:3. If 5 is added in both numbers, the ratio becomes 1:2. Find the numbers.

(d) The ratio of the present age of Ali and Amir is 3:4. If their age ratio 3 years ago was 2:3, find their present ages.

6. (a) The age of the girls are 8, 9 and 10 years. If Rs. 216 is divided in the ratio of their ages, how many rupees will each get?

(b) Bipin, Amrit and Aashish carried out a business with investments in the ratio of 2: 5: 6. After 1 year if they earned Rs. 65, 00, 000, how much amount each of them would have earned?

7. (a) If B collects twice as much as A and C collects triples as much as B, Rs 98, 460 is collected in total. How much amount has, each of them collected?

(b) If Krishna collects twice as much as Ram and Hari collects triples as much as Krishna so that Rs 16,200 is collected in total. How much amounts, each of them has collected?

8. (a) The ratio of the angles of triangle is 1:1:2. Find the angles of triangle.

(b) The ratio of the angles of triangle is 2:3:4. Find the angles of triangle.

Project work

Write down the ages of any 5 family members in your neighborhood. Tabulate the obtained information on the basis of age, up to 16 years, 17 years to 40 years and above 40 years. Present the ratio of the age of three groups. Discuss the obtained results in the classroom.

Answer

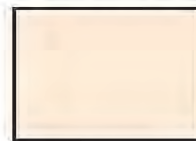
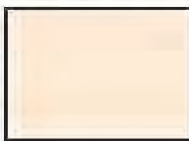
1. (a) 2:3 (b) 4:3 (c) 1:8 (d) 60:17
(e) 8:3 (f) 2:3 (g) 40:17 (h) 50:9
2. (a) 300 (b) 800 (c) 550
3. (a) 360,480 (b) 250,350
4. (a) 160 m (b) 200 m
5. (a) 8, 10 (b) 40, 50 (c) 5, 15 (d) 9:12
6. (a) Rs 64, Rs 72 and Rs 80 (b) Rs 10,00,000, Rs 25,00,000 and Rs 30,00,000
7. (a) A has Rs 10940, B has Rs 21880, C has Rs 65640
(b) Ram has Rs 1800, Krishna has Rs 3600, Hari has Rs 10800
8. (a) 45° , 45° , 90° (b) 40° , 60° , 80°

4.2 Proportion

Activity 2

Provide a photocopy paper for each pair of students and ask them to divide as per the instruction given below. Divide into two pairs and take one piece of photocopy paper each. The first one divides the paper into three equal parts and applies colour to one part, while the second one divides the paper into six equal parts and applies the colour into two parts. Present both (first one and second one) colour and uncoloured parts in proportion. Discuss the following topics in groups and present the conclusion to the class.

- (a) What is the relationship between first and second ratio?
- (b) Can we say these two ratios are proportion?



If the lowest terms of two ratios are equal, then such ratios are called proportional. For example: If the ratio of two quantities in the same unit a and b and the ratio of c and d are equal, then a, b, c and d are proportional. It is written as $a:b::c:d$. Here a and d are called extreme and b and c are called means. The product of extremes and the means are separately equal.

Example 1

5, 8 10, a are in proportion, find the value of a.

Solution

5, 8 10, a are in proportion

$$\frac{5}{8} = \frac{10}{a}$$

$$\text{or, } 5a = 80$$

$$\therefore a = 16$$

Example 2

What number must be added to the numbers 4 and 5 so that their ratio becomes 5:6?

Solution

Let the added number with 4 and 5 is y.

By the question

$$\frac{4+y}{5+y} = \frac{5}{6}$$

$$\text{or, } 24 + 6y = 25 + 5y$$

$$\therefore y = 1$$

Hence, 1 is added to the numbers 4 and 5 so that the ratio becomes 5:6.

Example 3

12 bananas are available in Rs. 150 and how many bananas are available in Rs. 900?

Solution

Let we can buy x bananas in Rs. 900

We know that

Ratio of value = Ratio of bananas

Now,

$$\frac{150}{900} = \frac{12}{x}$$

$$\text{or, } 150x = 900 \times 12$$

$$\therefore x = \frac{900 \times 12}{150} = 72$$

Price	Banana
Rs 150	12
Rs 900	x

Hence, 72 bananas are available in Rs. 900

Example 4

A tea seller used to mix milk and water in the ratio of 3: 5 while making milk tea. If 300 ml of milk was mixed in making two glasses of tea, how many ml of water was mixed? Find out.

Alternative method,

Let, the tea seller mixed the x ml water.

$$3x = 300$$

Now,

$$x = \frac{300}{3} = 100$$

$$\frac{300}{x} = \frac{3}{5}$$

$$5x = 5 \times 100 = 500$$

$$\text{or, } 3x = 300 \quad 5$$

$$\therefore x = \frac{300 \times 5}{3} = 500 \text{ ml}$$



Hence, the tea seller mixed 500 ml of water to make two glasses of tea.

Exercise 4.2

1. Check and write whether the following numbers are proportional.

- (a) 5, 8, 10, 15
- (b) 3, 5, 6, 10
- (c) 1kg, 4kg, 6kg, 10kg
- (d) 5 cm, 8 cm, 10 cm, 16 cm
- (e) 5m, 3m, 25m, 15m
- (f) 3ft, 8ft, 12ft, 22ft

2. Find the unknown terms if the following numbers are proportional.

- (a) a, 3, 3, 9
- (b) 3, x, 6, 8
- (c) 2, 5, 8, d
- (d) x, 2, 6, 4
- (e) 16, 4, 4, y
- (f) 7, 9, z, 18

3. Find the value of x.

- (a) $x:5 = 10:25$
- (b) $3:7 = 21:x$
- (c) $10:x = 2:11$
- (d) $25:15 = x:3$

4. (a) What should be added to the numbers 12 and 21 to make the ratio 5:8?

- (b) What should be added to the numbers 15 and 25 to make the ratio 2:3?
- (c) What should be subtracted from the numbers 24 and 30 to make the ratio 3:4?

5. (a) If 12 notebooks are available in Rs. 600, how many notebooks are available in Rs. 900?

- (b) If 5 pens are available in Rs. 50, how many pens are available in Rs. 240?

6. (a) The ratio of the students using pencils and pens in Rastriya Basic School is 10: 11. If there are 110 students using pencils, find the number of students using pens.

- (b) The ratio of the students who have lunch at school and bring food from home in Mahendra Secondary School is 3: 2. If the number of students who have lunch at school is 321, find the number of students who bring their lunch from home.

- (c) Reena and Meena are two sisters. Reena is in class 10 and Meena is in class 8. The ratio of their expenditure to buy books is 3: 2. If Meena paid Rs. 824, how much did Reena pay?
- (d) The ratio of the milk and sugar in a type of sweet is 5:2. If the milk is 750 gm, what is the part of sugar?
- (e) The ratio of gravity of moon and earth is 1:6. Find the weight of an object on moon whose weight is 90N on earth
- 7 (a) Kopila obtained marks in Nepali, English, Maths and Science in proportion. If she obtained 25, 30, 75 and x respectively in the given subjects, what is the value of x ?
- (b) The amount Abdul spends on lunch in Baisakh, Jestha, Ashad and Shrawn is in proportion. If she spends 150, 200, x , and 240 respectively in those months, what will be the value of x ?

Answer

- | | | | | | |
|------------|---------|----------|------------|----------|--------|
| 1. (a) No | (b) Yes | (c) No | (d) Yes | (e) Yes | (f) No |
| 2. (a) 1 | (b) 4 | (c) 20 | (d) 3 | (e) 1 | (f) 14 |
| 3. (a) 2 | (b) 49 | (c) 55 | (d) 5 | | |
| 4. (a) 3 | (b) 5 | (c) 6 | 5. (a) 18 | (b) 24 | |
| 6. (a) 121 | (b) 214 | (c) 1236 | (d) 300 gm | (e) 15 N | |
| 7. (a) 90 | (b) 180 | | | | |

5.0 Review)

Study the given conversation and discuss on the following questions:

Dina is a girl studying in class 8. One day she went to the market with her mother. In one shop, she saw the clothes on sale as shown in the figure. She was curious to know about this and told her mother and went near the shop. Coincidentally at that time, she saw one of her neighbor brothers in the store, putting other clothes in the place where it was written as sale. She went near to him and started the conversation. Read this conversation.

Dina: Namasker, brother. Why are you here?

Brother: h Namaskar Dina sister. I am working here as a shopkeeper.

Dina: h, yes brother. Why is this cloth kept in sale?

Brother: h, this.....(slowly smiles), the clothes which have only one measurement or they are unseasonal are kept in sale with the aim of selling in cheap price.

Dina: (ust showing the pants, written Rs. 500). Is this its selling price, brother?

Brother: Yes, we have left only 3 pieces of pants of the same measurement (size). Actually, its cost price was Rs. 700 (in small voice).

Dina: Brother, what was (ust showing the sweater, written Rs. 1200) this cost price? Can you say?

At the same time Dina's mother came near to them.

Mother: (ust showing the shop of Utensil which is little bit far from there)
Let's go there.

Both of them move from there saying bye bye to the neighbor brother.



- What was the cost price and selling price of the pants?
- Which one is more in cost price and selling price of sweater?
- What is the profit or loss of the shopkeeper after selling the pants? Also find in percent.
- What is the profit or loss of the shopkeeper after selling the sweater? Also find in percent.

5.1 Discount

Activity 1

Study the following condition and discuss the given question in group and find the conclusion:

Sandeep is a class 8 student of Sarswati Secondary School. He went to a shop to buy a jacket for winter. The price of jacket was kept as shown as in figure. After buying the jacket, the shopkeeper gave a bill as given in the right side.



Rs. 2200

**10%
DISCOUNT**

Acharya Store Tadi, Chitwan

Name: sandip Bhandari

S.N.	Particular	Price	Qty.	Price (Rs.)
1.	jacket	2200	1	2200
Discount 10				220
Total amount				1980
In words: one thousand nine hundred eighty only.				

Binaya
saler

- What is the initial price of jacket?
- What is the discount percent while buying jacket?
- What is the discount amount while buying jacket?
- Compare the initial price of jacket and the price he paid while buying?

Here,

The initial price or marked price of jacket is Rs. 2200. When 10% discount was given in the marked price, Sandeep gets Rs. 220 as discount amount. It means, he was buying the jacket Rs.220 less than the initial price.

Points to remember

- (a) The price businessmen fix and tell to the customer is called marked price.
- (b) If an item is sold after bought in marked price, the reduced price is called discount amount.
- (c) If the goods are buying with some discount amount on the marked price, then the price after the discount is called selling price.
- (d) Selling price (SP) = Marked price (MP) - Discount amount (D)
- (e) Discount amount = discount percent of MP = $MP \times d$

Example 1

Rajukanta bought a story book of cost Rs. 600 at a discount of 15%. how much amount does he have to pay to the shopkeeper?

Solution

Here, the marked price of story book (MP) = Rs. 600

Discount (D) = 15

Selling price (SP) = ?

We know that,

Discount amount = 15% of Rs. 600

Selling Price (SP) = MP - Discount amount

$$= 600 - 90$$

$$= \text{Rs } 510$$

Alternative method

Let, marked price (MP) = $100x$

Discount (D) = $15x$

Selling price (SP) = $85x$

By the question,

MP = Rs. 600

or, $100x = 600$

$\therefore x = 6$

$85x = 85 \times 6$

= Rs 510



Hence, he has to pay Rs. 510 for the story books.

Example 2

If Nabin buys a mobile at 8% discount and has to pay Rs. 22,080 to the shopkeeper, find the marked price of the mobile.

Solution

Here, Discount of mobile = 8%

Selling price (SP) = Rs. 22,080

Marked price (MP) = ?

Let, marked price (MP) = x

We know that,

SP = MP - discount % of MP

or, $22080 = x - x \times 8\%$

or, $22080 = x - \frac{8}{100}x$

or, $22080 = \frac{100x - 8x}{100}$

or, $22080 \times 100 = 92x$

or, $x = \frac{22080 \times 100}{92}$

$$\therefore x = 24000$$

Hence, the marked price of mobile is Rs. 24,000.

Alternative method

Here,

Let, marked price (MP) = $100x$

Discount (D) = $8x$

Selling price (SP) = MP - discount amount

$$= 100x - 8x$$

$$= 92x$$

By the question,

$$SP = \text{Rs. } 22,080$$

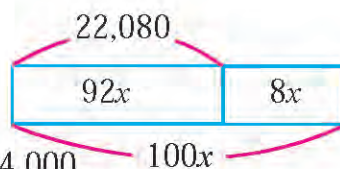
$$\text{or, } 92x = 22,080$$

$$\therefore x = 240$$

$$MP = 100x$$

$$= 100 \times 240$$

$$= \text{Rs } 24,000$$



Hence, the marked price of mobile is Rs.24,000.

Example 3

Ajuman buys a computer at Rs. 36,000. The marked price of computer is 25% above the cost price. If he sells the computer at 25% discount then,

- What is the marked price of computer?
- What is the discount amount?
- What is the selling price of computer?
- What is his profit or loss percent from that computer? Find it.

Solution

Here, cost price of computer (CP) = Rs. 36,000

By the question,

(a) Marked price (MP) = ?

$$\begin{aligned}\text{Marked price (MP)} &= \text{CP} + 25\% \text{ of CP} \\ &= 36000 + 36000 \times \frac{25}{100} \\ &= 36000 + 9000 \\ &= 45000\end{aligned}$$

(b) Discount = ?

We know that,

$$\begin{aligned}\text{Discount} &= \text{discount \% of MP} \\ &= 45,000 \times \frac{25}{100} = \text{Rs } 11,250\end{aligned}$$

(c) Selling price (SP) = ?

By formula,

$$\begin{aligned}\text{SP} &= \text{MP} - \text{Discount} \\ &= 45000 - 11250 \\ &= \text{Rs } 33,750\end{aligned}$$

(d) Since, the selling price of computer is less than cost price so there is loss.

$$\begin{aligned}\text{Loss} &= \text{CP} - \text{SP} \\ &= 36,000 - 33,750 \\ &= \text{Rs } 2,250\end{aligned}$$

$$\begin{aligned}\text{Loss percent} &= \frac{\text{loss}}{\text{CP}} \times 100 \\ &= \frac{2250}{36000} \times 100 \\ &= 6.25\end{aligned}$$

Exercise 5.1

1. Find the selling price of the following items.

Items	Marked Price	Discount
Books	Rs 480	Rs 48
Shoes	Rs 1250	Rs 250
Calculator	Rs 750	Rs 50
Shirt	Rs 1500	Rs 300
Mobile	Rs 24500	Rs 480

2. Find the selling price on the basis of the given table.

Items	Marked Price	Discount
Watch	Rs 2200	15
Bag	Rs 1750	12
Cupboard	Rs 30000	25
Computer	Rs 40000	10
TV	Rs 54500	20

3. Find the marked price from the following table.

Items	Selling Price	Discount
Table	Rs 2150	14
Sari	Rs 1360	20
Coat	Rs 4100	18
Computer	Rs 28800	10
Laptop	Rs 80750	5

4. On the basis of the given marked price and selling price, find the discount percent.

Items	Marked Price	Selling Price
Iron	Rs 1800	1530
Telephone set	Rs 2500	2200
Motorcycle	Rs 245000	208250
Watch	Rs 3000	2700
Printer	Rs 30000	26100

5. (a) The marked price of a book is Rs. 250. What is the price of book after 12 discount?
 (b) The marked price of a jacket is Rs. 2,250. What is the price of jacket if the shopkeeper sells it after 8 discount?
6. (a) If the price of a colour television after 10 discount is Rs. 13950. What is the marked price of television? Find it.
 (b) If the price of a sweater after 14 discount is Rs. 1075. What is the marked price of sweater? Find it.
7. A shopkeeper bought a torchlight in Rs. 1400. The marked price of the torch is 40 above the cost price. If the shopkeeper sold the torch after 20 discount then,
 (a) What is the marked price of the torch light?
 (b) What is the discount amount for the customer?
 (c) What is the selling price of the torch?
 (d) What is the profit of the shopkeeper after selling the torch, find it.
8. The cost price of a mobile is Rs. 21,000. The marked price of mobile is 30 above the cost price. If the shopkeeper sold the mobile after 20 discount then,
 (a) What is the marked price of mobile?
 (b) What is the discount amount for the customer?
 (c) What is the selling price of the mobile?
 (d) What is the profit of the shopkeeper after selling the mobile? Find it.
9. Ramhari bought a computer in Rs. 45,000. The marked price of computer is 30 above the cost price. If Ramhari sold the computer after 30 discount then,
 (a) What is the marked price of computer?
 (b) What is the discount amount?
 (c) What is the selling price of computer?
 (d) What is his profit or loss with that computer? Find it.
10. Rahaman marks the price of a suitcase as Rs. 6,000. If 15 discount is allowed in the suitcase, he got Rs. 500 profit, then,
 (a) What is the discount amount?

- (b) What is the selling price of computer?
- (c) What was the cost price of suitcase for Rahaman?
- (d) What was his profit percent from the suitcase? Find it.,

Project Work

Prepare a list of five items which are used daily in your home. On the basis of the prepared list, go to the nearest shop to find their marked price, selling price and discount amount. Ask to the responsible person of the shop, what was the cost price of those items? After the collection of information, what was their profit and loss? Also find its percentage. Prepare a report, and present it in your class.

Answer

1. Rs 432, Rs 1000, Rs 700, Rs 1200, Rs 24020
2. Rs 1870, Rs 1540, Rs 22500, Rs 36000, Rs 43600
3. Rs 2500, Rs 1700, Rs 5000, Rs 32000, Rs 85000
4. 15 , 12 , 15 , 10 , 13
5. (a) Rs 220 (b) Rs 2070 6. (a) Rs 15500 (b) Rs 1250
7. (a) Rs 1960 (b) Rs 392 (c) Rs 1568 (d) Rs 168
8. (a) Rs 27300 (b) Rs 5460 (c) Rs 21840 (d) 4
9. (a) Rs 58500 (b) Rs 17550 (c) Rs 40950 (d) 9
10. (a) Rs 900 (b) Rs 5100 (c) Rs 4600 (d) 10.87 (Approx)

6.0 Review

Fill in the tables and discuss on the basis of given information.

- (i) A bus is running with 50 km hr. constant speed. Calculate the value of unknow and fill out the box.

Time(hour)	1	2	3	4	
Distance(m)	50				250

- (ii) It takes 16 days to complete a work for 10 people. If the work is done in same rate ,then find the days on the basis of given number of people and number of people on the basis of days in table below.

Days	16	8		2	1
No. of people	10		40		

- How the distance changes as per change in time?
- What is the relationship between the distance travelled by the bus and time?
- What happened to the number of the people required to complete the work while decreasing the time? How short the required number of people changes as per changes in finishing time?
- What is the relationship between the days to complete the work and number of workers?

Here, in (i) as the bus travels more hours, the distance travelled by it has also increased at same ratio. Here, it takes more hours to travel more distance. So, there is a direct relationship between distance travelled and time taken by the bus.

n the other hand, in (ii) when the days to complete the work are decreasing the ratio of number of people to complete work is increasing. Similarly, if the number of people is to be decreased then, the days required to complete the work is to be increased. So, there is indirect inverse relationship between number of people and number of days required to complete the work.

Among two quantities, when one quantity is increases the other also increases by the same factor. Conversely, when one quantity is decreases the other quantity also decreases by the same factor, it is called direct variation. In the same way, if there is increase in one quantity, then there is decrease in other quantity in the same ratio and conversly, then it is called indirect variation.

6.1 Application of Direct Variation

Activity 1

The pencil bought by Bhargab and Bhumika and its price is given in the following figure.

The cost of 10 pencils = Rs. 50



Bhargab

The cost of 8 pencils = Rs 120



Bhumika

- Whose price per pencil is more, Bhargab's or Bhumika's?
- What is the ratio of number of pencils with Bhargab to the number of pencil with Bhumika?
- What is the ratio of the price of 10 pencils paid by Bhargav to the price of 8 pencils paid by Bhumika?
- Is the ratio of number of pencil and price of pencils equal?
- If Bhumika buys 12 pencils, how much would she pay for it?

Here, if the price per pencil is Rs.15 for both Bharga and Bhumika, the ratio of number of pencils with them is 5:4 and its price ratio is also 5:4. The number of pencils and price of pencils are direct variation. If Bhumika had bought 12 pencils,

$$\frac{8}{12} = \frac{120}{x}$$

$$\text{or, } 8x = 12 \times 120$$

$$\text{or, } x = \frac{1440}{8}$$

$$= \text{Rs. } 180$$

Therefore, the price of 12 pencils is Rs. 180

Numbers of pencils	The price of pencils
8	120
12	x (suppose)

Example 1

If the price of 10 kg apples is Rs.950, then what will be the price of 3 kg apples at the same rate?

Solution

Quantity(kg)	Price (Rs.)
10	950
3	x (suppose)

Being direct variation between quantity of apple and price

$$\frac{10}{3} = \frac{950}{x}$$

$$\text{or, } 10 \times x = 3 \times 950$$

$$\text{or, } x = \frac{3 \times 950}{10} = \text{Rs. } 285$$

Therefore, the price of 3 kg apple costs Rs. 285

Activity 2

Study the following condition and discuss it.

If 10 people earn Rs. 160000 in 20 days, then,

- How much will 1 person earn in a day?
- How much will 15 people earn in 10 days at same rate?
- How many days should 10 people work to earn Rs. 96000 at the same rate?

Here,

(a) In 20 days 10 people can earn Rs. 160000

In 1 days 10 people can earn Rs. $\frac{1,60,000}{10}$

In 1 days 1 people can earn Rs. $\frac{1,60,000}{(10 \div 20)} = \text{Rs } 800$

Here,

people	Day	Income
10 	20 	1,60,000 
1 	1 	x (suppose) 

Next Method (a)

$$\frac{1}{10} \cdot \frac{1}{20} = \frac{x}{1,60,000}$$

$$\text{or, } x = \frac{1}{10} \cdot \frac{1}{20} \cdot 1,60,000$$

$$\text{or, } x = 800$$



The number of people and income or the working days and the income are in direct variation. That's why, all the proportion are taken as same way or arrow sign are facing towards same direction.

Therefore, in 1 day a man can earn Rs.800

b) In 20 days ,10 men can earn Rs 1,60,000

In 20 days, 1 man can earn Rs. $\frac{1,60,000}{10}$

In 1 day 1 man can earn Rs. $\frac{1,60,000}{(10 \div 20)} = \text{Rs } 800$

In 1 day 15 men can earn Rs 800 15

In 10 day 15 men can earn Rs (800 15 10)

$$= \text{Rs } 1,20,000$$

Here,

People	Day	Income (Rs)
10	20	1,60,000
15	10	x (Suppose)

$$\text{Now, } \frac{x}{1,60,000} = \frac{15}{10} \times \frac{10}{20}$$

$$\text{or } x = \frac{15}{10} \times \frac{10}{20} \times 1,60,000$$

$$\text{or } x = 1,20,000$$

Therefore, in 10 days 15 people can earn Rs 1,20,000

c) 10 men can earn Rs 1,60,000 in 20 days

10 people can earn Rs 1 in $\frac{20}{1,60,000}$ days

Therefore, 10 people can earn Rs 96,000 in $\frac{20}{1,60,000} \times 96,000$
 $= 12$ days

Alternative Method

Here,

People	Day	Income (Rs)
10	20	1,60,000
10	x (suppose)	96,000

$$\text{now } \frac{x}{20} = \frac{96,000}{1,60,000} \times \frac{10}{10}$$

$$\text{or, } x = \frac{96,000}{1,60,000} \times \frac{10}{10} \times 20$$

$$\text{or, } x = 12$$

Therefore, 10 people can earn Rs. 96000 in 12 days.

When the number of men increase, then their income also increases and as the number of men decreases, then their income also decreases. Therefore, there is direct relationship between men and income. Likewise, if people work for more days, then they earn more income and if men work for fewer days, then they earn less income. Hence, there is direct relationship between working days and income.

Example 2

30 men can construct 600 m long wall in 20 days. Calculate how long wall will be constructed by 15 workers in 18 days.

Solution,

Here,

In 20 days, 30 workers can construct 600m long wall

In 1 day, 30 workers can construct $\frac{600}{20}$ long wall (worker and day : direct variation)

In 1 day 1 worker can construct $\frac{600}{20 \times 30}$ long wall (worker and day : direct variation)

In 18 days, 1 workers can construct $\frac{600}{20 \times 30} \times 18$ long wall (worker and day : direct variation)

In 18 days 15 workers can construct $\frac{600}{20 \times 30} \times 18 \times 15$ long wall (work and worker : direct variation)

= 270 long wall

Alternative method

Here

Worker	Day	Length of wall (m)
30 ↑	20 ↑	600 ↑
15 ↑	18 ↑	x (Let) ↑

$$\frac{x}{600} = \frac{15}{30} \times \frac{18}{20}$$

$$\text{or, } x = \frac{15}{30} \times \frac{18}{20} \times 600$$

$$\therefore x = 270$$

Therefore, in 18 days 15 workers can construct 270 m long wall.

6.2 Application of indirect variation

Activity 3

Discuss the given problem and solve it.

10 people of a community constructed a shelter with bamboo in 14 days. Likewise, 7 people of a community take a task to complete the construction of shelter with the same measurement.

- If the people work 8 hours per day to construct the first shelter, how long will it take for 7 men to complete the construction of the second shelter in 16 days?
- How long does it take to construct a second shelter if 7 people work for 8 hours per day?

Here,

In 14 days, 10 people constructed a shelter working 8 hours a day.

In 14 days, 1 person constructed a shelter working 8×10 hours a day

(People and working hour indirect variation)

In 1 day, 1 person should work $8 \times 10 \times 14$ hours to construct a shelter.

In 1 day, 7 people should work $\frac{(8 \times 10 \times 14)}{7}$ hours to construct a shelter.

In 16 days, 1 person should work $\frac{(8 \times 10 \times 14)}{7} = 10$ hours per day to construct the shelter. (Day and working hour per day: indirect variation)

Therefore, in 16 days, 7 people should work 10 hours per day to construct the bamboo shelter.

Alternative Method

now,

men	days	hour per day
10 	14 	8 
7 	16 	x 

$$\frac{x}{8} = \frac{14}{16} \times \frac{10}{7}$$

$$\text{or, } x = \frac{14}{16} \times \frac{10}{7} \times 8$$

$$\therefore x = 10$$

Therefore,



Per day working hour and working day have indirect relationship. So, their proportion is taken inversely or the arrow sign is shown in opposite direction.

7 people should 10 hours per day to construct bamboo shelter in 16 days.

(b) Here,

10 people have to work 8 hour per day to construct bamboo shelter in 14 days.

1 people has to work 8 hours per day to construct bamboo shelter in 14x10 days

(people and day: indirect variation)

7 people have to work 8 hours per day to construct bamboo shelter in

$$\frac{(10 \times 14)}{7} = 20 \text{ days}$$

Alternative way

Here,

People	Day	Hours per day
10 ↓	14 ↑	8 ↓
7 ↓	x (Suppose) ↑	8 ↓

Now,

$$\frac{x}{14} = \frac{10}{7} \times \frac{8}{8}$$

$$\text{or, } x = \frac{14 \times 10}{7}$$

$$\therefore x = 20$$

Therefore, 7 people work for 8 hours per day to construct bamboo shelter in 20 days.

When number of people increase, then working hours per day decreases and vice versa. Likewise, when working hours per day increases, then working day decreases and vice versa. Similarly, when number of workers increase, then working days decrease and vice versa. Likewise per days working hours and men or per day working hours, people and working days have indirect variation.

Example 3

20 people require 24 days to do a work. How long will it take for 15 men to do same work?

Solution,

here,

No of workers	Days
20	24
15	x (let)

Number of workers and number of days required to complete are indirect variation.

$$\frac{20}{15} = \frac{x}{24}$$

$$\text{or, } x \cdot 15 = 20 \cdot 24$$

$$\text{or, } x = \frac{20 \cdot 24}{15} = 32$$

Hence, 15 people required 32 days to do the work.

Example 4

If 32 people take 24 days to paint 6 houses ,then

- (a) How long will it take to paint 6 houses by 1 man?
- (b) If the work is to be completed in 8 days, how many peoples should be added?

32 men take 24 days to paint 6 houses.

$\therefore$ 1 man takes 24×32 days to paint 6 houses.

$$= 768 \text{ days}$$

Alternative method

Now,

People	Hours	Working days
32	6	24
1	6	x (let)

$$\frac{x}{24} = \frac{32}{1} \times \frac{6}{6}$$

$$\text{or, } x = 32 \times 24$$

$$\therefore x = 768$$

Therefore,

1 man requires 768 days to paint 6 houses.

(b) Here,

In 24 days, 32 workers required to paint 6 houses

In 1 day 32×24 workers required to paint 6 houses

In 8 days $\frac{32 \times 24}{8}$ workers required to paint 6 houses
 $= 96$ workers are required

Therefore, number of additional workers = $96 - 32 = 64$ people

Next method

Here,

Working days	No. of houses	Men
24 ↓	6 ↓	32 ↑
8 ↓	6 ↓	x (let) ↑

now,

$$\frac{x}{32} = \frac{24}{8} \times \frac{6}{6}$$

$$\text{or, } x = 32 \times 3$$

$$x = 96$$

Therefore, number of additional worker = $96 - 32$
 $= 64$ men

Exercise 6

1. Write D for the direct variation and I for the indirect variation against the following statements.

- a) The relationship between the length of the diameter of circle and the measurement of its circumference.
 - b) The relationship between the numbers of students in the hostel and the number of the provision last for.
 - c) The relationship between the principal and interest earned on the fixed time and the fixed rate of interest.
 - d) The relationship between the speed and distance travelled by the vehicle.
 - e) The relationship between the speed of vehicle and time to cover a fixed distance.
2. If the price of 4 dozen pens is Rs.576, then how many pens can be bought with Rs 228?
 3. If an athlete can complete 18 km distance in 45 minutes, then estimate the the time to cover distance of 30 km by the athlete.
 4. The loaded truck can cover a distance in 6 hours at the speed of 48 km per hours. If the speed of the truck decreases to 36 km per hour, then, how long will it take to cover that distance?
 5. If 20 workers can complete one task in 15 days. How many workers are to be added to complete the same work in 12 days?
 6. It takes 14 days to complete the work by 12 worker,. If 21 workers are added, then, how many days will it take to complete the work?
 7. In a barrack, There is a provision of food for 200 soldiers for 30 days. How many soldiers should be transfered to other place in order to last the food for 40 days?
 8. A motorcycle takes 6 hours to cover a certain distance at the speed of 30 km per hours. What should be its speed to cover same distance in 5 hours?
 9. The total price of 3 chairs and 4 tables is Rs.7, 540. If the price of one chair is Rs. 220, then find the price of one table.
 10. The total price of 5 cows and 2 oxen is Rs.1, 35,000. If the price of one ox is Rs.17500, then, find the price of one cow.
 11. If 4 men can earn Rs.28, 000 in 10 days, then

- a) How much will a man earn in a day?
 - b) How much will 3 men earn in 15 days?
 - c) By what percent will it be less or more between the earning of 4 men in 10 days and earning of 3 men in 15 days?
12. If 48 men require 700 kg of rice for 30 days, then
 - a) How much rice is needed for 40 men for 36 days?
 - b) What is the total price of rice for 40 men for 36 days at the rate of Rs.60 per kg?
 13. If 15 men can construct a 50 m long wall in 8 days, then find the length of the wall that can be constructed by 16 men in 12 days working at the same pace.
 14. If 20 men can construct a 80m long wall in 8 days, then how many men are required to construct a 60m long wall in 24 days?
 15. Bhargav deposited Rs.6,000 for 3 years in Nepal Bank Limited. If he receives Rs.1800 as interest at the end of 3 years, at the same calculate the interest of Rs.10, 000 for 4 years at same rate of interest?
 16. 6 men and 8 boys can harvest the crops of 30 ropanis in 4 days. If the capacity to harvest of 4 boys is equal to 2 men, then, at the same rate how much ropanis can 14 men and 8 boys harvest in 8 days?
 17. If 10 men can construct a road in 2 days working 7 hours a day, then, working at the same pace, calculate the number of hours per day 5 men should work to construct a road in 14 days.?
 18. If 18 men can construct a 54m long wall in 10 days, then how many men of same capacity is required to construct a 66m long wall in 22 days?
 19. If 20 men construct a park having 40m long and 20m width in 25 days then, how long will it take for 50 men to construct a park of 50m long and 40m width?
 20. If 25 men earn Rs.5, 00,000 in 30 days, then
 - a) How much does a man earn in a day?
 - b) How many men will earn 5, 00,000 in 10 days?
 - c) How many days are required for 25 men to earn 1, 00,000?
 - d) What is the earning of 5 men in 40 days?

Project Work

Write down the 3/3 examples related to the direct and indirect variation used in our daily life and present in the classroom.

Answer

1. Show to your teacher
2. 19
3. 75 Minute
4. 8 Hour
5. 5 workers
6. 8 Day
7. 50 workers
8. 6 km hr
9. Rs1720
10. Rs 20,000
11. (a) Rs. 700 (b) Rs 31,500 (c) Greater than 12.5
12. (a) 700 kg (b) Rs 42,000
13. 80 m.
14. 5 men
15. ₹.4,000
16. 108 ropani
17. 2 hour
18. 10 men
19. 25 day
20. (a) Rs 666.67 (b) 75 men (c) 6 day
- (d) Rs.1,33,333.33

7.0 Review

ari needed some money to start a goat farming business, so he decided to take a loan of Rs. 5000 from the savings group in his village. The savings group also agreed to give him Rs. 100 for a period of one year at a rate of Rs.10 per year. Based on this, we can discuss the following questions.

- What is the interest of Rs. 100 for 1 year?
- What is the interest of Rs. 500 for 1 year?
- What is the interest of Rs. 1000 for 1 year?
- What is the interest of Rs. 5000 for 1 year?
- What is the interest of Rs. 5000 for 3 years?
- At the end of 3 years, how much will he pay the loan amount and including interest?

7.1 Simple Interest

Activity 1

Bhumika has deposited Rs.5,000 in a saving account at Nepal Bank. After 2 years, the bank returned Rs.5800. Based on this information, discuss on the following questions.

- What is the amount deposited in the bank by Bhumika called?
- What is the additional amount given by the bank called?
- How long was it deposited for?
- What do you call the total amount of Rs. 5800, when Rs.800 is added to Rs. 5000?
- If the interest on Rs.5000 is Rs. 800 for 2 years, how much will be the interest for Rs. 100 in 1 year?

The amount of Rs. 5000 deposited in the bank is called sum or principal (P).
 The additional amount of 800 given by the bank is called interest earned (I).
 Bhumika deposited the principal for a period of 2 years, denoted as time (T).
 The total amount of Rs. 5800 by the bank is known as the total amount (A).
 Here, the interest of Rs.5000 for 2 years is Rs. 800.

The interest of Rs. 1 for 2 years is Rs $\frac{800}{5000}$

The interest of Rs. 1 for 1 years is Rs = $\frac{800}{5000 \times 2}$

The interest of Rs.100 for 1 years is Rs. $\frac{800 \times 100}{5000 \times 2} = \text{Rs } 8$

Here,

$R=8$,So, 8 interest rate per annum means that interest of Rs.100 for 1 year Rs.8.

Activity 2

Study and discuss on the following situation.

- (a) Sandesh's mom decided to open a grocery store. For that, She has taken loan of Rs. 50000 from a commercial bank for 3 years at 12 per annum interest rate. How much interest will she pay to the bank at the end of 3 years?

Here, according to the meaning of interest rate of 12 .

the Interest of Rs.100 for 1 year is Rs.12

the Interest of Rs.1 for 1 year is Rs. $\frac{12}{100}$

the Interest of Rs.50000 for 1 year is Rs. $\frac{50,000 \times 12}{100}$

the Interest of Rs.50000 for 3 year is Rs. $\frac{50,000 \times 3 \times 12}{100}$

$$= \text{Rs. } 18,000$$

So, Sandesh's mom has to pay Rs. 18000 interest in 3 years.

- (b) If a loan of Rs. P is taken at an interest rate of R . per annum for T years, How much interest is payable?

According to interest rate of R . per annum .

Interest of Rs. 100 for 1 year is Rs. R

Interest of Rs. 1 for 1 year is Rs. $\frac{R}{100}$

Interest of Rs. 100 for T year is Rs. $\frac{T \cdot R}{100}$

Interest of Rs. P for T year is Rs. $\frac{P \cdot T \cdot R}{100}$

$$\therefore \text{Interest (I)} = \frac{P \cdot T \cdot R}{100}$$

We borrow loan from different institutions for purchasing some goods or operating business or fulfilling our needs if we have not enough amounts with us. These institutions take some additional amount after fixed time for lending the loan. This additional amount is called interest. If we deposit some amount in a bank, then we also receive additional amount. This additional amount is called interest.

Terms related to simple interest

- Sum or principal (P): The amount borrowed or lend or sum of money deposited in the bank is called principle.
- Interest (I): The additional money deposited in principal after fixed time is called interest.
- Interest rate(R) : Interest rate is the interest of Rs.100 for one year.
- Time(T) : A person borrows or gives a loan for a fixed period of time. The fixed time period is called time (T)
- Amount(A): The sum amount comes after adding interest.

Example 1

Somaya borrowed loan of Rs 20000 at the rate of 10 % per year, for vegetable cultivation from a cooperative. How much interest should she pay for 2 years. Find out by unitary method.

Solution

Here, according to meaning of 10 % rate of interest per year.

The interest of Rs. 100 for 1 year = 10

$$\text{The interest of Rs. 1 for 1 year} = \frac{10}{100}$$

$$\text{The interest of Rs. 20000 for 1 year} = \frac{10}{100} \times 20,000$$

$$\begin{aligned}\text{The interest of Rs. 20000 for 2 year} &= \frac{10}{100} \times 20,000 \times 2 \\ &= \text{Rs. 4,000}\end{aligned}$$

Therefore, Somaya s paid Rs. 4000 as interest for 2 years.

Example 2

Santaman took a loan of Rs. 5000 at the rate of 10 % per year from a bank for starting animal husbandry. How much interest should pay for the loan for 3 years and 6 months?

Solution

Here,

Principal (P) = Rs. 5000

Rate of interest (R) = 10

$$\text{Time (T)} = 3 \text{ years } 6 \text{ month} = \left(3 + \frac{6}{12} \right) \text{ year} = 3.5 \text{ year}$$

Interest(I) = ?

Here, we know that,

$$\begin{aligned}\text{Interest (I)} &= \frac{P \times T \times R}{100} \\ &= \frac{(5,000 \times 3.5 \times 10)}{100} \\ &= \text{Rs. 1,750}\end{aligned}$$

Therefore, Santaman should paid Rs. 1750 as interest for 3 years and 6 months.

Example 3

Manjura has to pay Rs. 2400 as interest at 10 per annum for 3 years. What is the principal?

Solution

Here,

Rate of interest (R) = 10

Time (T) = 3 year

Interest(I) = Rs. 2400

Suppose, the amount borrowed by Manjura as loan be = Rs. P

Here, we know that,

$$\text{Interest(I)} = \frac{P \times T \times R}{100}$$

$$\text{or, } 2400 = \frac{P \times 3 \times 10}{100}$$

$$\text{or, } 30 \times P = 2400 \times 100$$

$$\text{or, } P = \frac{2400 \times 100}{30}$$

$$\therefore P = \text{Rs. 8,000}$$

Hence, the sum of money that Manjura had borrowed is Rs. 8000.

Exercise 7.1

1. Write very short answer to following questions.

- a) What is interest?
- b) What is Principal?
- c) Write 30 months in years.

2. Find the simple interest by using unitary method?

- (a) Principal = Rs. 500 Rate of interest (R) = 10 Time = 3 years
- (b) Principal = Rs. 5,200 Rate of interest (R) = 12 Time = 5 years
- (c) Principal = Rs. 3,300 Rate of interest (R) = 7.5 Time = 3 years
6 months
- (d) Principal = Rs. 1,200 Rate of interest (R) = 3 Time = 3 years
3 months

3. Find the time T

- (a) Principal = Rs. 1,260 Rate of interest (R) = 5 Interest (I) = Rs. 378
- (b) Principal = Rs. 1,250 Rate of interest (R) = 13 Interest (I) = Rs. 650
- (c) Principal = Rs. 4,500 Rate of interest (R) = 4 Interest (I) = Rs. 900

4. Find Interest rate R

- (a) Principal = Rs. 7,200. Time (T) = 5 years Interest = Rs. 1,080
- (b) Principal = Rs. 6,000. Time (T) = 3 years 6 months Interest = Rs. 1,155
- (c) Principal = Rs. 2,160. Time (T) = 4 years Interest = Rs. 648

5. Find the principal P

- (a) Rate of interest (R) = 9 Time (T) = 9 year Interest = Rs. 810
- (b) Rate of interest (R) = 4.8 Time (T) = 1 month Interest = Rs. 3996
- (c) Rate of interest (R) = $6\frac{2}{3}$ Time (T) = 5 year Interest = Rs. 400

6. If Ganga deposited a sum of Rs. 3500 in Nepal Bank Limited at 7 rate of interest for 4 years. Calculate the interest she will get from the bank?

7. If Sandesh borrowed a loan of Rs.18000 from a bank at 6.6% rate of interest per year, how much interest should he pay after 30 months?
8. Bhargav received Rs.550 interest of certain sum of money after 4 years. If he got the interest at 5.5% rate of interest on that sum, then, find the sum deposited by him?
9. If a bank gave Rs. 1254 to Sarisma as the interest after 3 years for the deposit of Rs. 7600, then find the rate of interest.
10. How many years will Rs. 900 need to be deposited at the interest rate of 12% to get the amount equal to the interest of Rs. 1080 in 4 years at the interest rate of 10.

Answer

1. Show to your teacher
2. (a) Rs.150 (b) Rs.3,120
(c) Rs.866.25 (d) Rs.117
3. (e) 6 year (f) 4 year (g) 5 year
4. (a) 3 (b) 5.5 (c) 7.5
5. (a) Rs.1,000 (b) Rs.9,990 (c) Rs.1,200
6. Rs.980 7. Rs.2,970
8. Rs.2,500 9. 5.5 10. Rs.432, 4 year

7.2 Amount

Activity 3

Studying following condition and discuss

- a) Lakpa deposited Rs 10000 as a saving from his monthly income in Nepal Bank at the rate of 8% per year, then
 - i) How much interest does he receive?
 - ii) How much does he receive as the total of the principal and interest?

Here,

Principal (P) = Rs. 10,000

Rate of interest (R) = 8

Time (T) = 3 years

(a) The interest he will receive (I) = ?.

$$(I) = \frac{(P \cdot T \cdot R)}{100} = \frac{10000 \cdot 3 \cdot 8}{100}$$
$$= \text{Rs. } 2400$$

(b) The total amount with principal and interest = Principal + interest

$$= 10000 + 2400$$
$$= \text{Rs. } 12400$$

Here, the sum of amount of loan and interest is Rs. 12400.

The additional amount with interest on principal after certain time is called amount. It is denoted by A.

Therefore, Amount = Principal + Interest

If the interest of sum of Rs P for T years at rate of interest is Rs.1, then what is the amount.

Here,

Amount = principal + interest

$$A = P + I \quad \dots (i)$$

We know that,

$$I = \frac{PTR}{100} \quad \dots (ii)$$

Combining equation (i) and (ii)

$$A = P + \frac{PTR}{100}$$

$$\therefore A = P \left(1 + \frac{TR}{100} \right)$$

$$= P \left(\frac{100 + TR}{100} \right)$$

Example 1

Find the interest earned by depositing Rs 5000, in a bank for 2 years at 8 interest rate and the total amount after 2 years.

Solution

Here,

Principal (P) = Rs. 5,000

Rate of interest (R) = 8

Time (T) = 2 years

Amount (A) = P + I = ?

$$\begin{aligned}\text{Interest (I)} &= \frac{PTR}{100} \\ &= \frac{5000 \times 2 \times 8}{100} \\ &= \text{Rs. } 800\end{aligned}$$

$$\begin{aligned}\text{Now, Amount (A)} &= P + I \\ &= 5,000 + 800 \\ &= \text{Rs. } 5,800\end{aligned}$$

So, Interest = Rs. 800 and Amount = Rs. 5,800

Example 2

Rajkishor has deposited Rs 8000, which is his earning from the shop, in a bank for 3 years to get 12 annual interest rate. Find the total amount he will get back after 3 years.

Solution

Here,

Rate of Interest (R) = 12

Principal (P) = Rs. 8,000

Time (T) = 3 years

Amount (A) = ?

We know that,

$$\begin{aligned}\text{Amount (A)} &= P \left(\frac{100 + TR}{100} \right) \\ &= 8,000 \left(\frac{100 + 3 \times 12}{100} \right) \\ &= 8,000 \left(\frac{136}{100} \right) \\ &= \text{Rs. } 10,880\end{aligned}$$

Alternative method

$$\begin{aligned}\text{Interest (I)} &= \frac{P \times T \times R}{100} \\ &= \frac{8000 \times 3 \times 12}{100} \\ &= \text{Rs. } 2880\end{aligned}$$

$$\begin{aligned}\text{Amount (A)} &= 8000 + 2880 \\ &= \text{Rs. } 10,880\end{aligned}$$

So, after 3 years, Rajkishor, will be refunded Rs.10880.

Example 3

How much should be deposited at 7.5% interest to get lump sum of Rs 2375 in 40 months?

Solution

$$\text{Here, Time (T)} = 40 \text{ Month} = \frac{40}{12} \text{ year} = \frac{10}{3} \text{ year}$$

$$\text{Amount (A)} = \text{Rs. } 2,375$$

$$\text{Rate of interest (R)} = 7.5$$

$$\text{Principal (P)} = ?$$

We know that,

$$\text{Amount (A)} = P \left(\frac{100 + TR}{100} \right)$$

$$\text{or, } 2375 = P \left(\frac{100 + \frac{10}{3} \times 7.5}{100} \right)$$

$$\text{or, } P = \left(\frac{100 \times 2,375}{100 + \frac{10}{3} \times 7.5} \right) = \frac{2,37,500}{125}$$

$$\therefore P = \text{Rs. } 1,900$$

$$\text{So, Principal (P)} = \text{Rs. } 1,900$$

Exercise 7.2

1 Find the amount A:

- (a) Principal = ₹. 50,000 Time = 7 year Rate of interest (R) = 3
- (b) Principal = ₹. 2,160 Time = 4 year Rate of interest (R) = 3.5
- (c) Principal = ₹. 55,500 Time = 2 year Rate of interest (R) = 7.5
- (d) Principal = ₹. 5,24,000 Time = 3 Month Rate of interest (R) = 11

2 Aruna took Rs 200000 at 12% interest rate from a bank for cow husbandry. If she paid the amount including interest after 30 months, then how much did she pay in total?

3 What will be the amount if Rs 35000 is deposited at 3% interest rate for 54 months?

4 How much should be deposited in the bank at 10% interest rate to get Rs. 610 as interest for 4 months?

5 How much sum should be deposited to get total amount Rs 251225 at 5% interest rate for 4 years 6 months?

6 What amount will be refunded when Rs 7500 is deposited at 5.5% interest rate per year for 42 months?

7 Adira saves Rs 40000 in bank at 5% interest rate per year and she pays 5% of interest as income tax, then how much will she receive after 4 years? Find it.

8 If the bank levied 5% tax in the interest of sum Rs 75000 at 5.6% interest rate for 6 months, then what amount will be received after 6 months?

9 If 5% of interest on Rs 10800 at 10% interest rate for 4 years is to be paid as tax, then what will be the total amount after 4 years?

10 Hari borrowed a loan of Rs 300000 at 12% interest rate for 2 years and 6 months for foreign employment.

- a Find the simple interest Hari has to pay.
- b Find the total amount to clear the loan.

Project work

Visit your nearest financial institution or bank or search the information of interest rate of different account and find out the type of account to deposit our sum from where we get the highest interest. Find it and present in classroom.

Answer

1. (a) Rs.60,500 (b) Rs.2462.40 (c) Rs.63,825 (d)Rs.5,38,410
2. Rs.2,60,000 3. Rs.39725 4. Rs.18,300
5. Rs.205081.63 6. Rs.8,943.75
7. Rs.47,600 8. Rs.76,995 9. Rs.14,904
10. (a) Rs.90,000 (b) Rs.3,90,000

Mixed Exercise

1. Two students Rish and Purna studying at grade 8 have got new rulers. Their friend Sonam asked how much price they paid it. The prices paid by them were as follows:
Rish : Rs. 10100₂
Purna: Rs. 100₅
(a) Sonam could not understand the numbers they told. Give your idea to express the numbers in the proper form so that everyone could understand the prices easily.
(b) If Sonam also needs to buy a ruler, which shop is better for him?
2. The height of Mt. Everest is 8,848.86 m. express it in scientific notation.
3. According to the data of World Bank, the population of Pakistan is 200,960,000 and Sri Lanka is 2.167×10^7 . Which country has larger population and by how much?
4. The total number of pages in Pratima's favorite novel is 200.
(a) Write the number in scientific notation.

- (b) Express the number in quinary number system.
- (c) Express the number in binary number system.
- (d) If she has read its 150 pages, find the ratio of the number of completed pages and the remaining pages.
5. The monthly salary of Manisha is Rs. 43689.
- (a) Write the amount in scientific notation.
- (b) Express the number in quinary number system.
- (c) Find her expenditure and saving, if their ratio is 2 : 1.
6. There are two water tanks. The first tank contains 3.4×10^2 liters and second contains 4.57×10^3 liters of water.
- (a) How much water is there altogether? Express in scientific notation.
- (b) How much water should be added to the first tank to make equal as the second tank?
- (c) If the cost of 1 liter water is 10 paisa, find the total cost of water contained in the second tank.
7. The length of Mahendra Highway from Akadvitta in the east to Mahendranagar in the west is 1027.67 kilometers.
- (a) Which of the followings is in the scientific notation of given length?
- (i) 1.02767×10^4 (ii) 10.2767×10^4
- (iii) 102.767×10^4 (iv) 0.102767×10^4
- (b) Write the number 1028 in quinary number system.
- (c) If the symbol ■ denotes 1 and □ denotes 0, Color the following blocks to represent 35 in binary system.
-
8. Rita and Gita are two sisters, and their ages are 8 years and 12 years respectively. One day, their mother gives Rs. 500 and asked to divide it in the ratio of their ages.
- (a) How much money each of them receives?
- (b) How much is the percentage Gita receives more than that of Rita?
9. Ram and Laxman are two brothers and their ages are 20 years and 15 years respectively. They can spend the total of Rs. 10500 monthly in such a way that their individual expenditures are fixed in the ratio of their ages.

- (a) How much money each of them can spend?
 - (b) How much is the percentage Laxman can spend less than that of Ram?
10. The ratio of amount of pure milk and water in the mixture of 50 liters of milk is 7:3.
- (a) Find the amount of pure milk and water in the mixture.
 - (b) If the cost of 1 liter mixture of milk is Rs. 100, how much liters of milk can be bought in Rs. 900?
 - (c) A shopkeeper bought the total of 50 liters milk in Rs. 5000. He mixed 5 liters of water and sold the milk allowing 5 % discount on the price that he purchased. How much profit or loss will the shopkeeper have finally?
 - (d) How much price should the shopkeeper sell the total amount of milk without mixing water to make a profit of 10%?
11. Rohan has deposited the amount of his money in two banks A and B in the ratio of 3 : 2. The amount of money deposited in the bank A is Rs. 60,000.
- (a) What is the amount that he deposited in the bank B?
 - (b) How much interest and amount that Rohan will receive from the bank after 2 years , if the rate of interest is 5 % per annum?
 - (c) How long should he deposit the sum in the bank B at the same rate of interest in order to earn the interest same as that from bank A?
12. Ramesh deposits a sum of Rs. 80,000 for 4 years at the rate of interest 10 % per annum.
- (a) Write the formula to find out the simple interest.
 - (b) How much interest does Ramesh earn in 2 years?
 - (c) If Ramesh withdraws Rs. 30,000 from his saving after 2 years, what is the total amount that he will receive after 4 years?
 - (d) How long does it take the sum of money becomes three times?
 - (e) Ramesh spends Rs.30,000 for his children s education and health in the ratio of 3 : 2. Find the amount of expenditure in the children's education and health.

13. The ratio of cost of a table and 4 chairs shown in the figure is 3 : 4.

- If the total cost of a table and 4 chairs is Rs. 16,000 , find the cost of a table.
- How much does a chair cost?
- What price should the set of table and chairs be sold after allowing 10 % discount.



14. Shivani was suffering from anemia as the amount of hemoglobin in her blood is lower than normal. Doctor prescribed her to take a tablet of medicine two times a day for 15 days. The cost of 10 tablets in a pharmacy is Rs. 17 .

- How much does the medicine for 15 days cost?
- Which of the following price is taken to calculate discount?
 - Buying price
 - Selling price
 - Marked price
 - All of above
- If Shivai got 10 % discount on the price of medicine, how much actual discount did she get?
- Shivani did not recovered completely in 15 days, so, her doctor advised her to take one tablet of medicines each day for one week more. Can she buy the additional medicines for Rs.10?

15. A supermarket has announced a discount offer for the sales of electronics items on the occasion of Dahain and Tihar festival. While Sita visited the market to buy a laptop, she knew that a mobile set of price Rs. 20000 can be purchased at Rs. 17,000 after discount. If the price of laptop was marked as Rs. 90,000,



- Find the rate of discount offered by the supermarket.
 - How much discount does Sita get on the purchase of laptop?
 - If Sita sold the same laptop to her friend Ramita making a profit of 10 % , how much did Ramita pay for it?
16. Three friends Ram, Shyam and Hari have jointly invested Rs. 60,00,000

to establish a company. The ratio of share of their individual capital is 3 : 4 : 5. After one year the company went to loss and they sold it on the price less by 12,00,000.

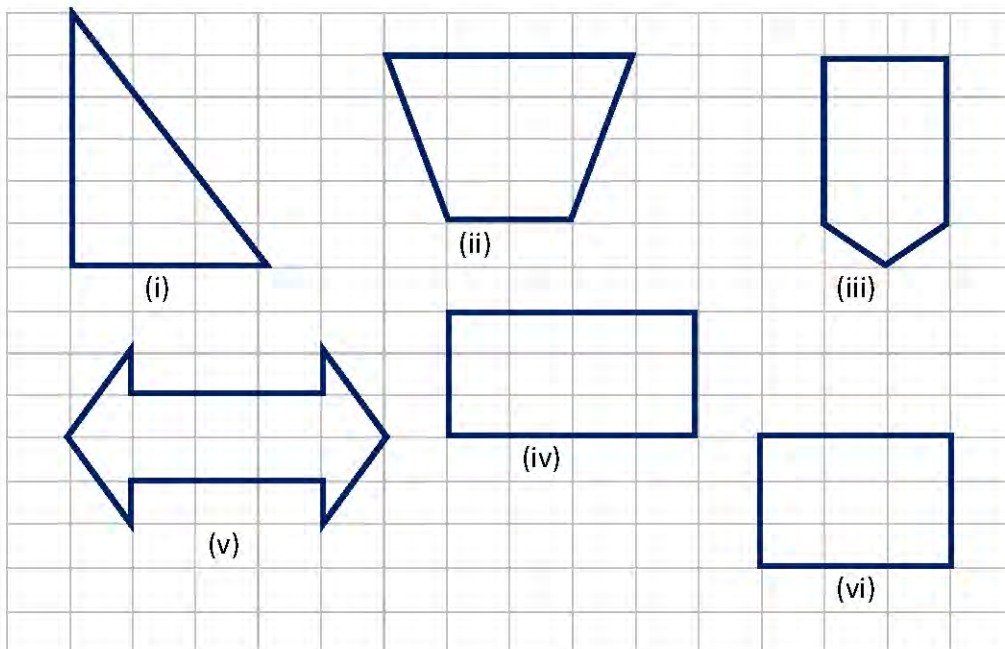
- How much capital did they invest individually?
- How much loss did Ram bear after one year?
- If Hari deposited the sum of money so obtained in a bank for 2 years at the rate of interest 10%, how much amount will he receive from the bank after 2 years?

Answer

- Both the prices can be reduced to the base 10 number system. Arish had paid Rs. 20 and Purna had paid Rs. 25.
 - It is better to buy from the shop which Arish had bought. Because the price is cheaper by Rs. 5.
- 8.84886×10^3
- Population of Pakistan larger by 1.7929×10^8
 - 1300_5
 - 11001000_2
 - $3 : 1$
- 4.3689×10^4
 - 2344224_5
 - Saving Rs. 29126, Expenditure Rs. 14563
- 4.91×10^3
 - 4230 litres.
 - Rs. 457
- i
 - 13103_5
-
- Rita: Rs. 200, Gita : Rs. 300
 - Gita receives more than Rita by 50%.
- Ram : Rs.6000, Laxman : Rs.4500
 - Laxman spends 25% less
- 35 literes and 15 liters.
 - 9 liters
 - Profit Rs. 225
 - Rs.110
- Rs. 40,000
 - Rs. 6,000, Rs. 66,000
 - 3 years
- Rs. 16000
 - Rs.26,000
 - 20 years
 - Rs.18,000
- Rs. 6000
 - Rs. 2500
 - Rs. 14,400
- Show to the teacher
 - 15
 - Rs. 13,500
- Rs. 86,445
- 15
 - Rs. 3,00,000
 - Rs. 24,00,000

8.0 Review

What are the areas of the shapes given in the following square paper? Discuss in pairs and present the conclusion in the classroom.



Area is the surface covered by an object on a plane surface .

8.1 Area of Triangles and Quadrilaterals

8.1.1 Area of Triangles

i Area of Right-angled Triangle

Activity 1

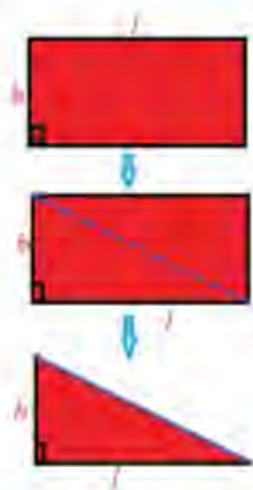
Take a rectangular paper of length (l) and breadth (b). So, area of rectangle is $l \times b$ square unit.

Draw a diagonal in the rectangle as shown in the second figure. Fold the paper and cut the folded part. Now, the rectangle is divided into two right angled triangles.

Now, keep the cut part over the initial triangle. It exactly fits with the initial triangle. Therefore,

Area of rectangle = $2 \times$ Area of right angled triangles

$\therefore$ Area of right-angled triangle = $\frac{1}{2} \times$ Area of rectangle.



So, if b and p be the base and perpendicular of a right angled triangle then,

Area of right angled triangle (A) = $\frac{1}{2} \times b \times p$

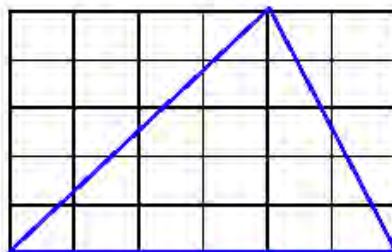
Note: Since, perpendicular and the base of a right-angled isosceles triangles are equal, the area of the

right-angled isosceles triangle is (A) = $\frac{p^2}{2}$ or $\frac{b^2}{2}$

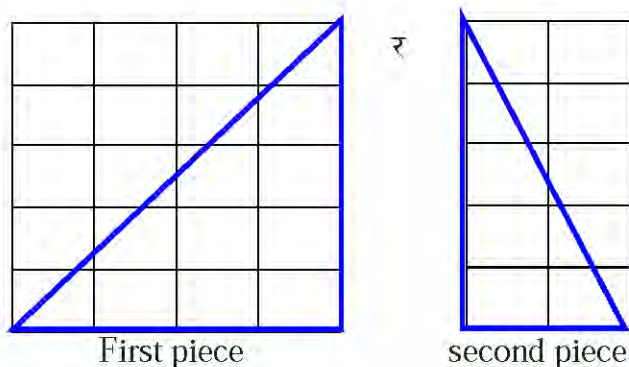


Activity 2

Take one rectangular squared paper in pairs and make one triangle of equal size as shown in the figure. Now, what is the area of that triangle? Discuss in pairs.



Perform the following activities to find the area of a triangle. Divide the triangle you made into two right angled triangles as shown in the figure. Now, consider the hypotenuse of the right angled triangle as the diagonal and fold both the pieces of rectangular paper. Discuss what you found.



Now, calculate the area of both the triangles separately.
From the first piece,

$$\begin{aligned}\text{Area of a triangle} &= \frac{1}{2} \text{ Area of rectangular paper (after cutting)} \\ &= \frac{1}{2} \quad 4 \quad 5 = 10 \text{ square units}\end{aligned}$$

Similarly, from the second piece

$$\begin{aligned}\text{Area of a triangle} &= \frac{1}{2} \text{ Area of rectangular paper (after cutting)} \\ &= \frac{1}{2} \quad 2 \quad 5 = 5 \text{ square units}\end{aligned}$$

So, total area of triangle = $10 + 5 = 15$ square units

Now, find the area of rectangular paper (before cutting) made by your friend, and compare with yours.

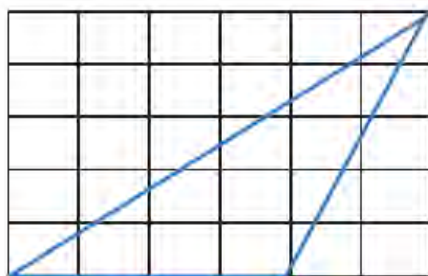
Here,

Area of rectangular paper (before cutting) = base $\times$ height = $6 \times 5 = 30$ square unit

Comparing the area of triangular and rectangular paper,

$$\text{Area of triangle} = \frac{1}{2} \text{ Area of rectangle}$$

Now, what is the area of the triangle given in the figure below? Discuss with your friends and find out.



Activity 3

Take a rectangular paper of certain size. Let, the base be (b) and height be (h). Then,

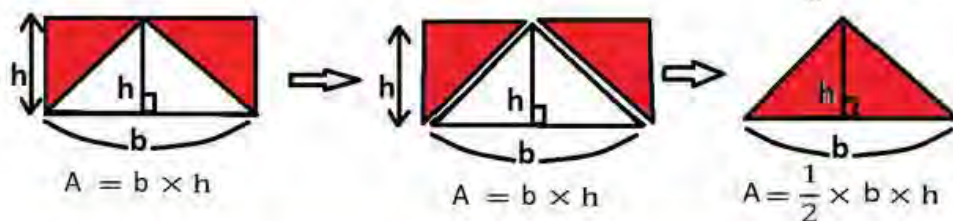
$$\begin{aligned}\text{Area of rectangle} &= \text{base} \times \text{height} \\ &= b \times h \text{ square units}\end{aligned}$$

Now, construct a triangle inside of the rectangle as shown in the first figure and draw the perpendicular from the vertex to the base. Fold the paper from the vertices of the triangle as shown in the figure and cut the folded part. Now place the cut part over the previous triangle. It exactly fits with the previous triangle.

Therefore,

$$\text{Area of triangle} = \frac{1}{2} \text{Area of rectangle} = \frac{1}{2} b \times h$$

$$\begin{aligned}\text{The area of a triangle with base length (b) and height (h)} &= \frac{1}{2} \text{base} \times \text{height} \\ &= \frac{1}{2} b \times h\end{aligned}$$



The area of a triangle with base length (b) and height (h) = $\frac{1}{2}$ base $\times$ height

$$= \frac{1}{2} b \times h$$

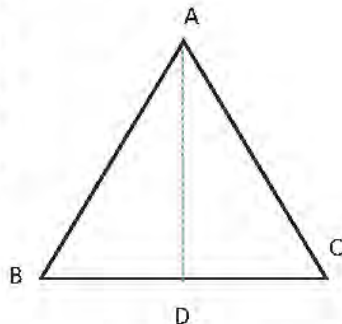
ii Area of Equilateral Triangle

Activity 4

Take a photocopy paper and make an equilateral triangle of certain length on it. Now, cut the triangle you have made with the help of scissors. Fold the triangle and make it half from any one of the vertex of triangle. What will be the relationship between the two angles formed at the vertex and the base of the triangle? Discuss with the friends.

Here,

The adjoining figure is an equilateral triangle. Suppose the length of the side be a . Then, draw a perpendicular AD on BC . Now, the base BC is bisected by AD . The bisector of the vertical angle of an equilateral triangle bisects the base side.



$$\therefore BD = DC = \frac{a}{2}$$

According to the Pythagoras theorem,

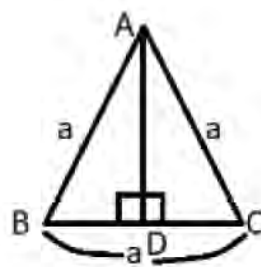
$$AD^2 = AC^2 - CD^2$$

$$\text{or, } AD = \sqrt{AC^2 - CD^2}$$

$$= \sqrt{a^2 - \left(\frac{a}{2}\right)^2}$$

$$= \sqrt{\frac{4a^2 - a^2}{4}}$$

$$= \frac{\sqrt{3}}{2} a$$



$$\text{Now, area of triangle (A)} = \frac{1}{2} \text{ base} \times \text{height}$$

$$= \frac{1}{2} a \cdot \frac{\sqrt{3}}{2} a = \frac{\sqrt{3}}{4} a^2$$

$$\text{Area of equilateral triangle (A)} = \frac{\sqrt{3}}{4} a^2$$

ii Area of Isosceles Triangle

Activity 5

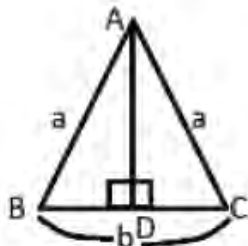
The given figure is an isosceles triangle. Let, $AB = AC = a$ and $BC = b$. Let us draw AD perpendicular to BC from A which base BC . The perpendicular drawn from the vertex to the base of an isosceles triangle bisects the base]

$$\therefore BD = DC = \frac{b}{2}$$

According to Pythagoras theorem,

$$AD^2 = AC^2 - CD^2$$

$$\begin{aligned}\text{or, } AD &= \sqrt{AC^2 - CD^2} = \sqrt{a^2 - \left(\frac{b}{2}\right)^2} \\ &= \sqrt{\frac{4a^2 - b^2}{4}} = \frac{\sqrt{4a^2 - b^2}}{2}\end{aligned}$$



$$\text{Now, area of triangle (A)} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times b \times \frac{\sqrt{4a^2 - b^2}}{2} = \frac{b\sqrt{4a^2 - b^2}}{4}$$

$$\text{Thus, Area of isosceles triangle (A)} = \frac{b\sqrt{4a^2 - b^2}}{4}$$

8.1.2 Area of different Quadrilaterals

i Area of a Parallelogram

Activity 6

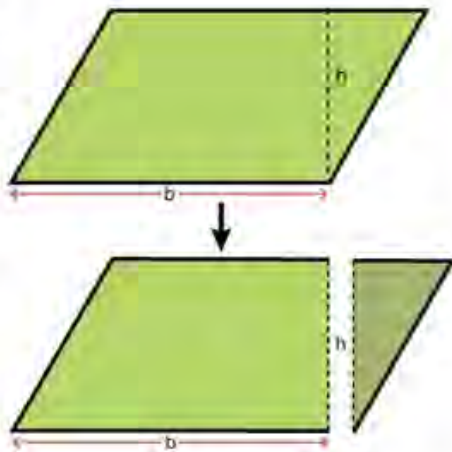
Use cardboard and construct a parallelogram with base (b) and height (h) as shown in the figure.

As shown in the figure, fold the cardboard perpendicularly from the bottom right vertex and cut folded part.

Now, bring the cut part (triangle) to the left and join as shown the given figure.

What shape do you get?

Here, the rectangle has length (b) and breadth (h).



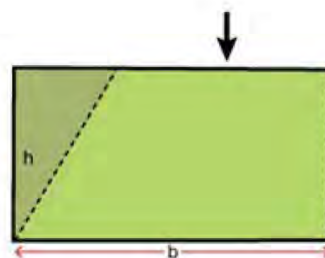
Hence the

Area of rectangle = length $\times$ breadth = $b \times h$

Here,

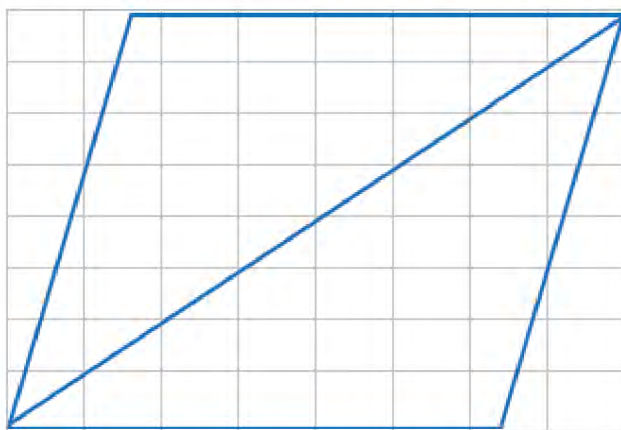
Area of rectangle = area of parallelogram.

Area of parallelogram = base $\times$ height = $b \times h$



Activity 7

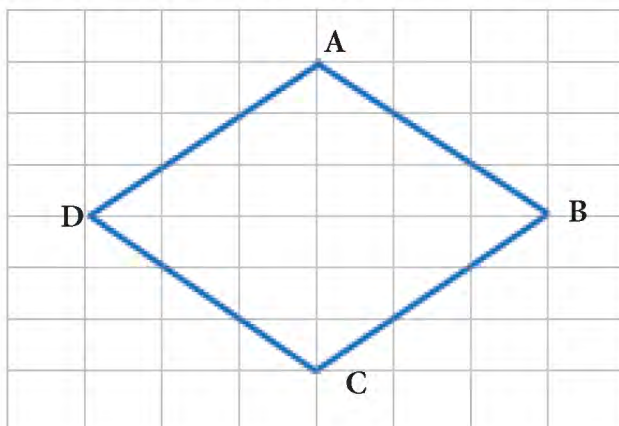
Take a square paper. Draw a parallelogram on the squared paper as shown in figure. Now, what is the area of the parallelogram you have made? Discuss in pairs.



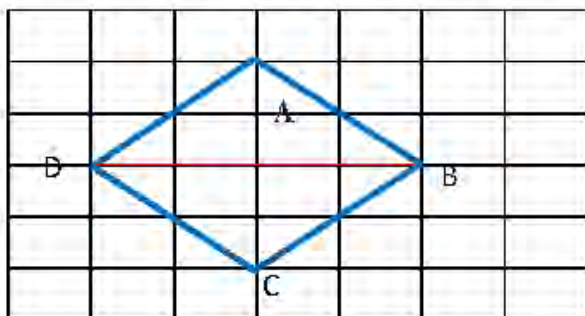
ii Area of a Rhombus

Activity 8

Take a rectangular squared paper and make a rhombus as shown in the figure. Now, what is the area of the rhombus you made? Discuss in pairs.



Here, Draw a diagonal BD. Now, the rhombus you made is divided into two triangles.



Hence, the area of rhombus = Area of $\triangle ABD$ + Area of $\triangle CBD$

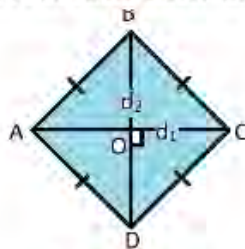
$$\begin{aligned}
 &= \frac{1}{2} 4 \times 2 + \frac{1}{2} 4 \times 2 \\
 &= \frac{1}{2} 4 (2 + 2) \\
 &= \frac{1}{2} 4 \times 4 \\
 &= \frac{1}{2} BD \times AC
 \end{aligned}$$

Here,

In the adjoining figure, diagonals AC (d_1) and BD (d_2) of rhombus ABCD are drawn.

Since, the diagonals of a rhombus intersect each other at perpendicular.

So, $A \perp BD$ and $C \perp BD$.



Now, area of rhombus ABCD (A) = Area of $\triangle ABD$ + area of $\triangle CBD$

$$\begin{aligned}
 &= \frac{1}{2} A \times BD + \frac{1}{2} C \times BD \\
 &= \frac{1}{2} BD (A + C) \\
 &= \frac{1}{2} BD \times AC = \frac{1}{2} d_1 d_2
 \end{aligned}$$

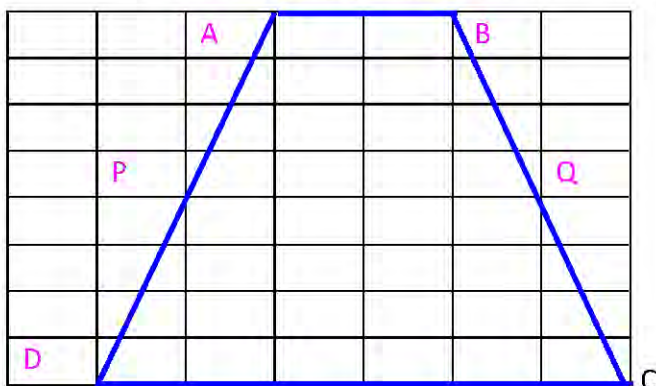
So,

$$\text{Area of rhombus} = \frac{1}{2} d_1 d_2$$

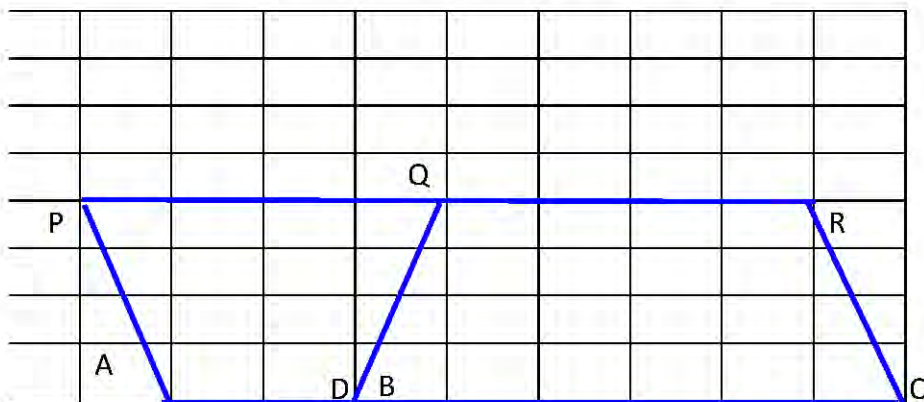
iii Area of a Trapezium

Activity 9

Take a graph paper and make a trapezium of any length as shown in the figure. Now, what is the area of the trapezium you made? Discuss in pairs.



Now, draw a line joining the midpoints of legs AD and BC and make a parallelogram from the trapezium you made as shown in the figure.



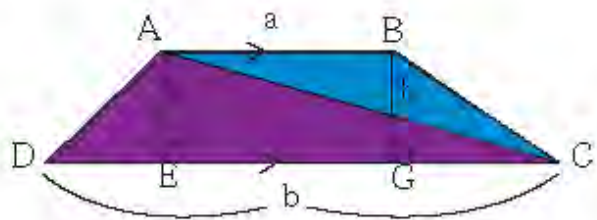
Here,

$$\begin{aligned}
 \text{Area of trapezium } ABCD &= \text{Area of parallelogram } ACRP \\
 &= \text{base} \times \text{height} \\
 &= (2 + 6) \times 4 \\
 &= (AB + DC) \times \text{height}
 \end{aligned}$$

Hence,

$$\text{Area of trapezium} = \frac{1}{2} \text{ Sum of parallel side} \times \text{height}$$

In the adjoining figure, the side AB (a) and CD (b) of trapezium ABCD are parallel to each other. $AE = BG = h$ is the height of the trapezium and AC is the diagonal.



Now,

Area of trapezium (A) = Area of $\triangle ADC$ + Area of $\triangle ABC$

$$\begin{aligned}
 &= \frac{1}{2} CD \times AE + \frac{1}{2} AB \times BG \\
 &= \frac{1}{2} b \times h + \frac{1}{2} a \times h \\
 &= \frac{1}{2} h \times (a + b)
 \end{aligned}$$

Hence,

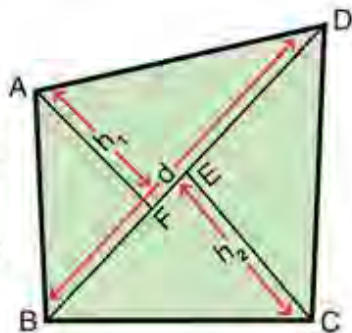
$$\begin{aligned}
 \text{Area of trapezium} &= \frac{1}{2} h \times (a + b) \\
 &= \frac{1}{2} \text{ height} \times \text{sum of parallel sides}
 \end{aligned}$$

iv Area of Quadrilateral

In the figure, ABCD is a quadrilateral. If the diagonal of a quadrilateral is BD (d), perpendiculars are drawn from vertices A and C to the diagonal BD. Therefore,

Height of $\triangle ABD$ (AF) = h_1 and height of $\triangle CBD$ height (CE) = h_2

Now,



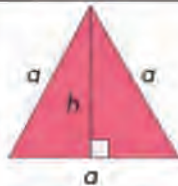
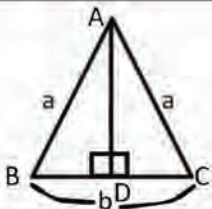
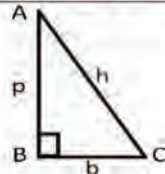
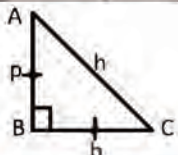
Area of quadrilateral ABCD (A) = Area of $\triangle ABD$ + Area of $\triangle CBD$

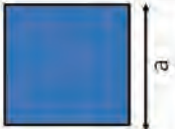
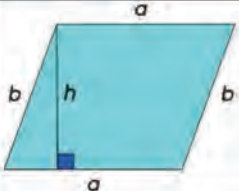
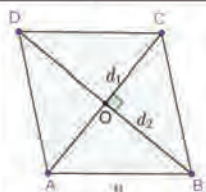
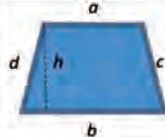
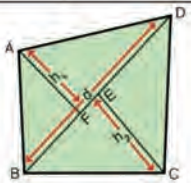
$$\begin{aligned}
 &= \frac{1}{2} BD \times AF + \frac{1}{2} BD \times CE \\
 &= \frac{1}{2} BD \times (AF + CE) \\
 &= \frac{1}{2} \times d \times (h_1 + h_2)
 \end{aligned}$$

So,

$$\begin{aligned}\text{Area of quadrilateral (A)} &= \frac{1}{2} \times d \times (h_1 + h_2) \\ &= \frac{1}{2} \times \text{diagonal} \times \text{Sum of the perpendiculars drawn into the diagonal.}\end{aligned}$$

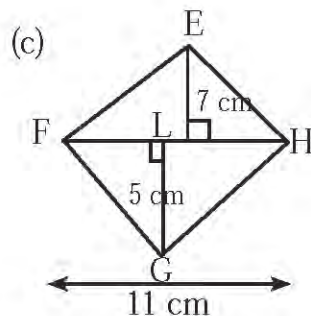
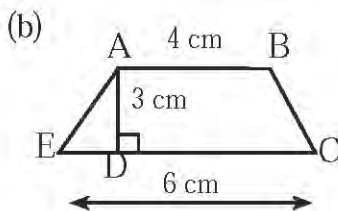
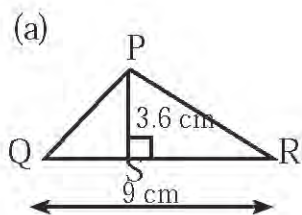
The perimeter and area of the above polygons can be presented in the following table:

Name of polygon	Figure	Perimeter	Area
Triangle		$P = a + b + c$	$A = \frac{1}{2} \times b \times h$
Equilateral triangle		$P = 3a$	$A = \frac{\sqrt{3}a^2}{4}$
Isosceles triangle		$P = 2a + b$	$A = \frac{b\sqrt{4a^2 - b^2}}{4}$
Right angled triangle		$P = p + b + h$	$A = \frac{1}{2} \times p \times b$
Right angled Isosceles triangle		$p = b$ then $P = 2p + h$ or $= 2b + h$	$A = \frac{p^2}{2} \text{ or } \frac{b^2}{2}$
Ractangle		$P = 2(l + b)$	$A = l \times b$

Square		$P = 4a$	$A = a^2$ or diagonal d then, $A = \frac{d^2}{2}$
Parallelogram		$P = 2(a + b)$	$A = b \times h$
Rhombus		$P = 4a$	$A = \frac{1}{2} \times d_1 \times d_2$
Kite		$P = 2(S_1 + S_2)$	$A = \frac{1}{2} \times d_1 \times d_2$
Trapezium		$P = a + b + c + d$	$A = \frac{1}{2} \times h \times (a + b)$
Quadrilateral		$P = AB + BC + CD + AD$	$A = \frac{1}{2} \times d \times (h_1 + h_2)$

Example 1

Find the area of the given geometrical shapes. (The length is not exact.)



Solution

(a) Here, In $\triangle PQR$

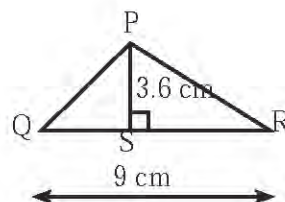
Base (b) = $QR = 9\text{ cm}$

Height (h) = $PS = 3.6\text{ cm}$

Area(A) = ?

We know that,

$$\begin{aligned}\text{Area of triangle (A)} &= \frac{1}{2} \times b \times h \\ &= \frac{1}{2} \times 9 \times 3.6 \\ &= 16.2\text{ cm}^2\end{aligned}$$



(b) Here, in trapezium ABCE,

Parallel sides are

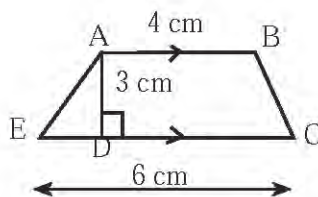
$a = AB = 4\text{ cm}$, $b = EC = 6\text{ cm}$

Height (h) = $AD = 3\text{ cm}$

Area (A) = ?

We know that,

$$\begin{aligned}\text{Area of trapezium (A)} &= \frac{1}{2} \times h \times (a + b) \\ &= \frac{1}{2} \times 3 \times (4 + 6) \\ &= \frac{1}{2} \times 3 \times 10 \\ &= 15\text{ cm}^2\end{aligned}$$



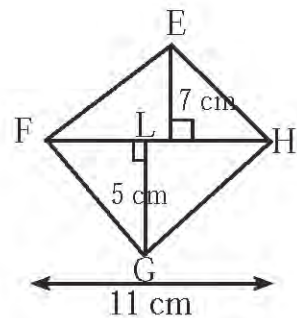
- (c) Here, in quadrilateral EFGH,
height (h_1) = 7 cm, height (h_2) = 5 cm

Diagonal (d) = 11 cm

Area (A) = ?

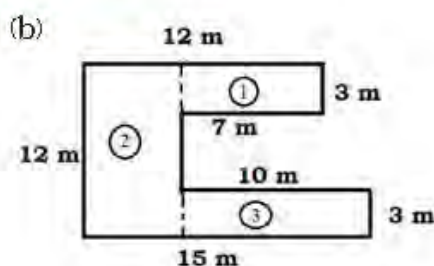
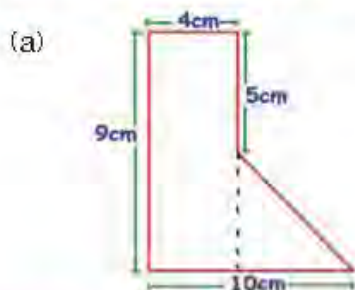
We know that,

$$\begin{aligned}\text{Area of quadrilateral (A)} &= \frac{1}{2} \times d \times (h_1 + h_2) \\ &= \frac{1}{2} \times 11 \times (7 + 5) \\ &= \frac{1}{2} \times 11 \times 12 \\ &= 66 \text{ cm}^2\end{aligned}$$



Example 2

Find the area of the given geometrical shapes.



Solution

(a) Given geometrical shape is made up of combination of rectangle and triangle.

So, area of shape = Area of rectangle + Area of triangle

Here,

Length of rectangle (l) = 9 cm

Breadth of rectangle (b) = 4 cm

Area of rectangle = $l \times b = 9 \times 4 = 36 \text{ cm}^2$

Base of right-angled triangle (b) = $10 - 4 = 6 \text{ cm}$

Perpendicular of right angle triangle (p) = $9 - 5 = 4 \text{ cm}$

Therefore, area of right-angled triangle (A_2) = $\frac{1}{2} \times p \times b = \frac{1}{2} \times 4 \times 6 = 12 \text{ cm}^2$

Here, Area of given shape = Area of rectangle + Area of right angle triangle
 $= 36 + 12$
 $= 48 \text{ cm}^2$

(b) Given geometrical shape is made up of three rectangles.

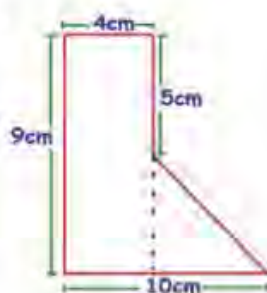
Hence, area of given shape = Sum of area of all three rectangle

Here, length of first rectangle (l_1) = 7 m,

Breadth (b_1) = 3 m

Therefore, area of first rectangle (A_1) = $l_1 \times b_1 = 7 \times 3 = 21 \text{ m}^2$

Length of second rectangle (l_2) = 12 m,



Breadth (b_2) = $15 - 10 = 5$ m,

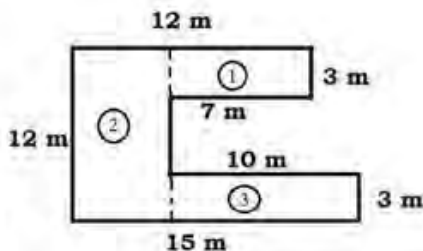
$$\begin{aligned}\text{Area of second rectangle} &= (A_2) = l_2 \times b_2 \\ &= 12 \times 5 \\ &= 60 \text{ m}^2\end{aligned}$$

Length of third rectangle (l_3) = 10 m,

Breadth (b_3) = 3 m,

$$\begin{aligned}\text{Area of third Rectangle } (A_3) &= l_3 \times b_3 = 10 \\ &\times 3 = 30 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{Now, Area of given shape} &= \text{Sum of area of all three rectangle} \\ &= 21 + 60 + 30 \\ &= 111 \text{ m}^2\end{aligned}$$



Example 3

Find the area of shaded region from the adjoining figure.

Solution

Here, ABCD is a parallelogram

Base of parallelogram (b) = 9 cm

Height (h) = 7 cm

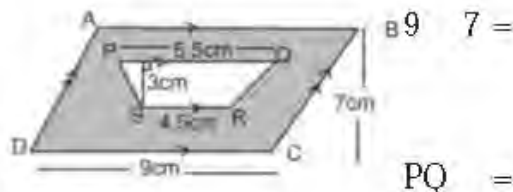
$$\begin{aligned}\text{Area of parallelogram } (A_1) &= b \times h = \\ &9 \times 7 = 63 \text{ cm}^2\end{aligned}$$

Again, in trapezium PQRS,

Parallel sides $a = SR = 4.5$ cm, $b = PQ = 5.5$ cm

Height (h) = 3 cm

$$\begin{aligned}\therefore \text{Area of trapezium } (A_2) &= \frac{1}{2} \times h \times (a + b) \\ &= \frac{1}{2} \times 3 \times (4.5 + 5.5) \\ &= \frac{1}{2} \times 3 \times 10 \\ &= 15 \text{ cm}^2\end{aligned}$$



$$\begin{aligned}
 \text{Now, area of shaded part (A)} &= A_1 - A_2 \\
 &= 63 - 15 \\
 &= 48 \text{ cm}^2
 \end{aligned}$$

Example 4

If the area of triangle is $16\sqrt{3} \text{ cm}^2$ then find the value of x .

Solution

Here, ABC is an equilateral triangle.

Side of equilateral triangle (a) = x = ?

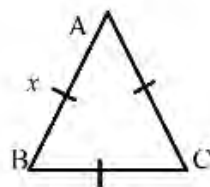
We know that,

$$\text{Area of equilateral triangle (A)} = \frac{\sqrt{3}}{4} a^2$$

$$\text{or, } 16\sqrt{3} = \frac{\sqrt{3}}{4} x^2$$

$$\text{or, } x^2 = 64$$

$$\therefore x = 8 \text{ cm}$$



Example 5

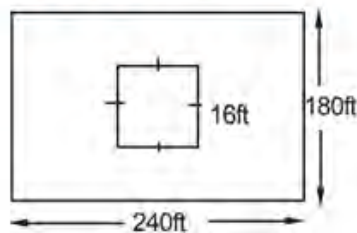
A rectangular field has 240 ft length and 180 ft breadth and there is a square shape pond of length 16 ft in the middle of the field. Find the area of field excluding the pond.

Solution

Length of rectangular field (l) = 240 ft

Breadth of rectangular field (b) = 180 ft

Therefore,



$$\text{Area of rectangular field (A}_1\text{)} = l \times b = 240 \times 180 = 43200 \text{ ft}^2$$

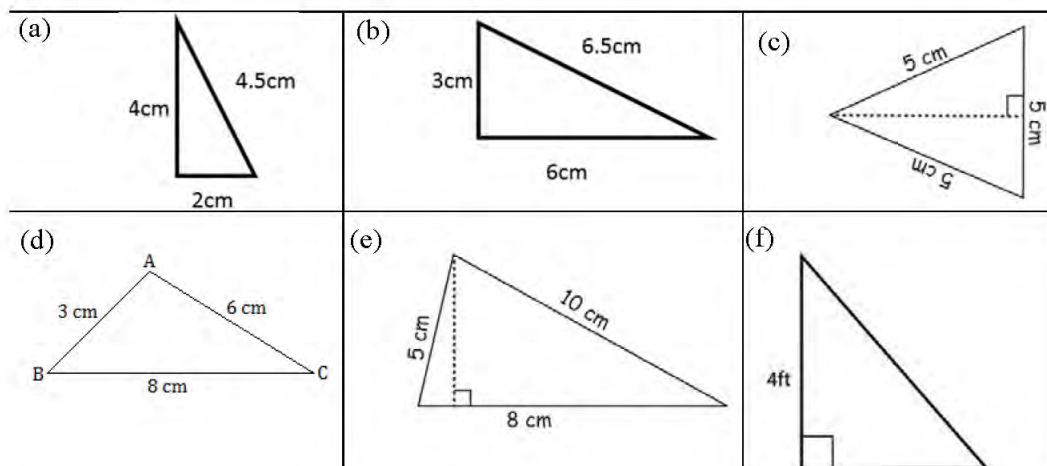
$$\text{Again, area of square shape pond } l^2 = (16)^2 = 256 \text{ ft}^2$$

Now,

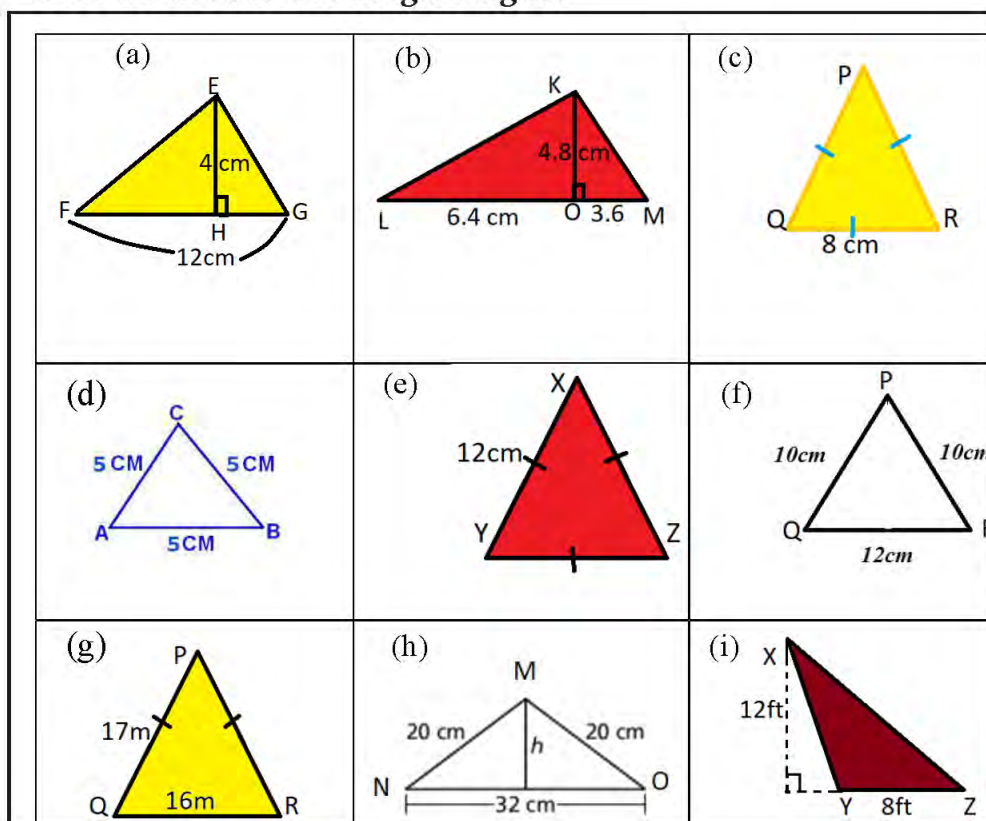
$$\begin{aligned}
 \text{Area of field excluding the pond (A)} &= \text{Area of field} - \text{Area of pond} \\
 &= 43200 - 256 \\
 &= 42944 \text{ ft}^2
 \end{aligned}$$

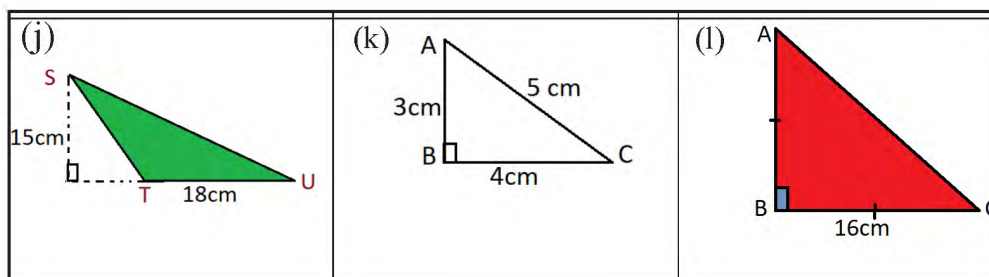
Exercise 8.1

1. Find the perimeter of following triangles:

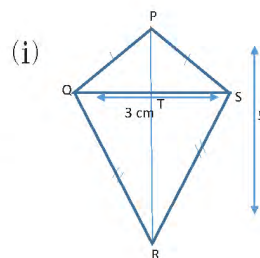
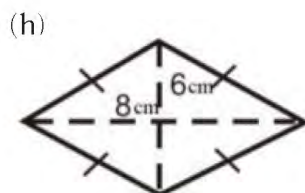
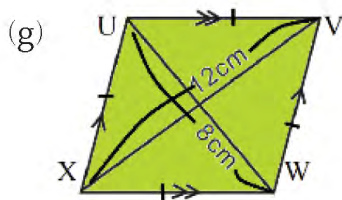
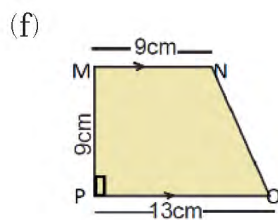
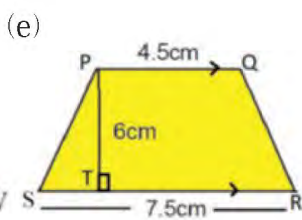
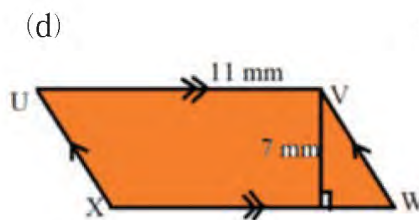
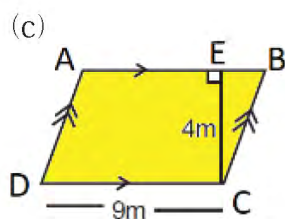
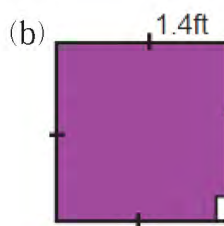
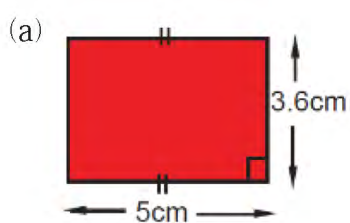


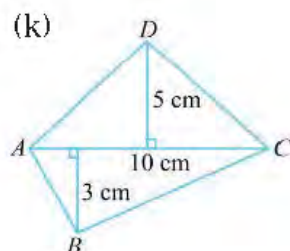
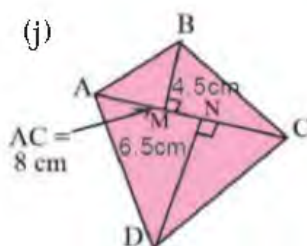
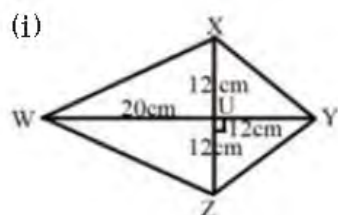
2. Find the area of following triangles:



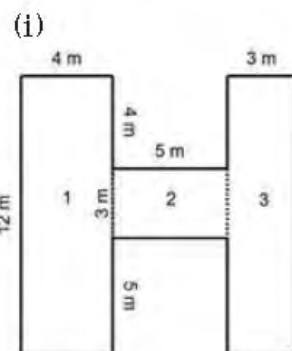
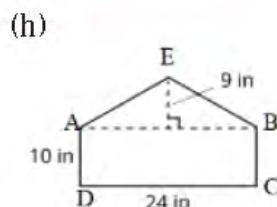
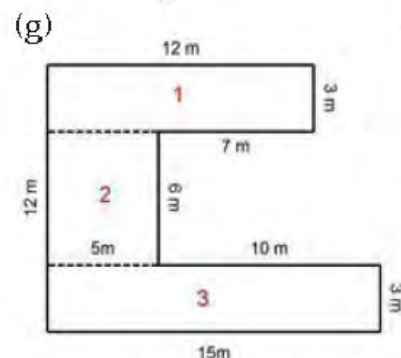
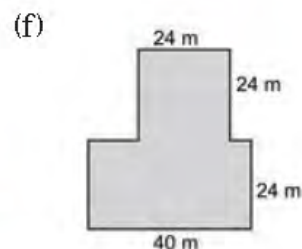
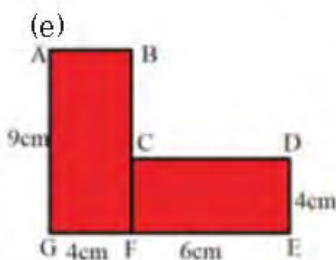
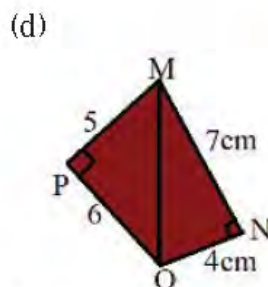
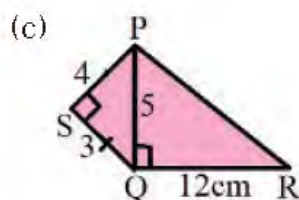
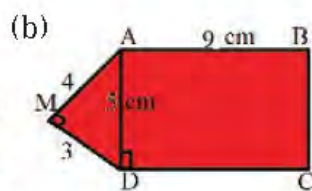
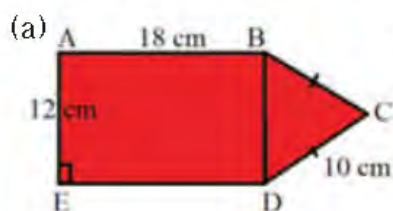


3. Find the area of given quadrilaterals.

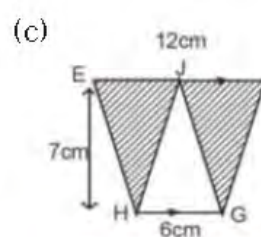
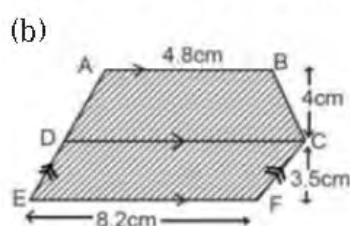
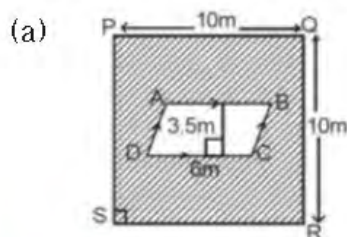




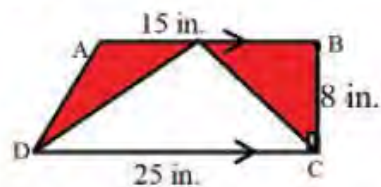
4. Find the area of following geometrical shape:



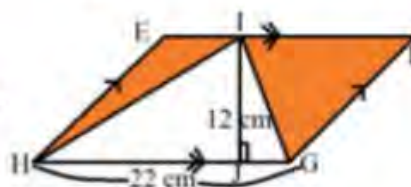
5. Find the area of the shaded part from the given figure:



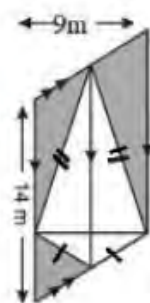
(d)



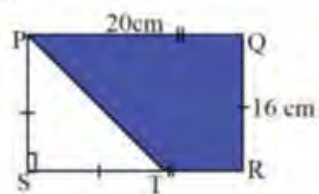
(e)



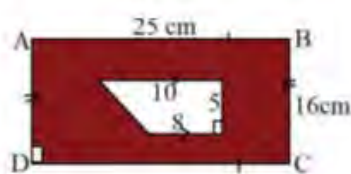
(f)



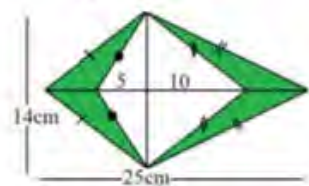
(g)



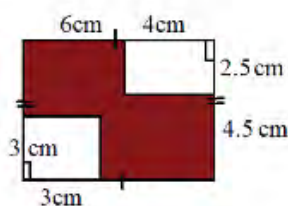
(h)



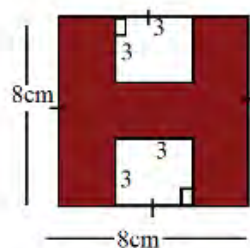
(i)



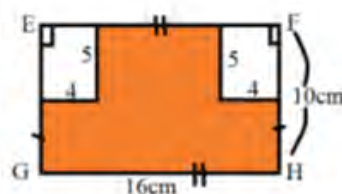
(j)



(k)

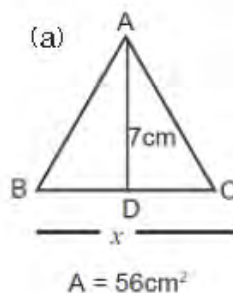


(l)

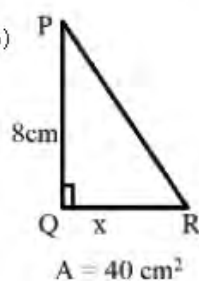


6. Find the value of x from the given figures:

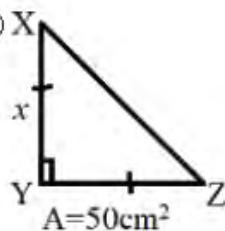
(a)

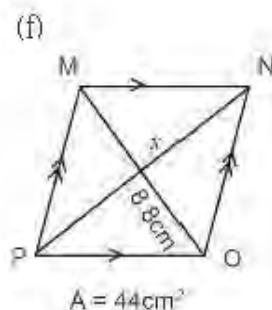
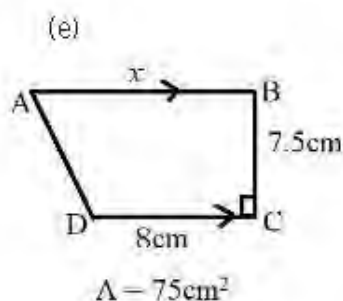
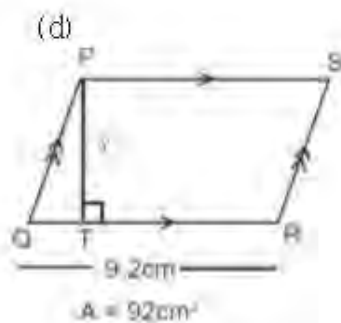


(b)



(c)



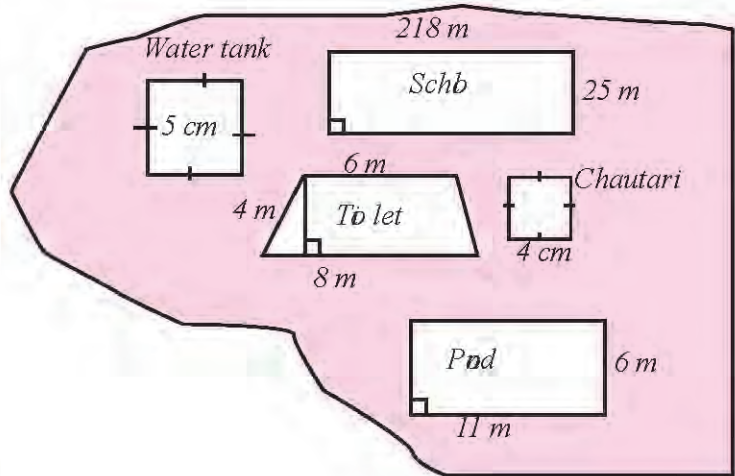


7. (a) A rectangular park of length 120m and breadth 110m has a volleyball court of length 18m and breadth 9m. Find the area of **the park excluding the volleyball court.**
- (b) A square fish pond of length 50 m is constructed in the middle of a rectangular field of length 125 m and breadth 100 m. Find the area **of the field excluding the fish pond.**
8. (a) How many boards of area 1 sq. ft. required to be past on a wall having 9 ft length and 7 ft wide without overlapping?
- (b) In a wall of 25 m length and 3 m wide, how many 1 sq. m. ply can be pasted without over placing?
9. (a) If the perimeter of a square field is 200 feet,
 - (i) Find the length of the field.
 - (ii) Find the area of the field.
- (b) If the length of the wall surrounding a square ground is 80 m,
 - (i) Find the length of ground.
 - (ii) Find the area of the ground.
- c) If the length of rectangular room is 15 ft and its perimeter is 54 ft,
 - (i) What is breadth of the room?
 - (ii) Find the area of room?
- (d) The length of a rectangular room is twice the breadth. If the perimeter is 60 ft,
 - (i) What is the length and breadth of the room?

(ii) How many square feet of carpet is required to cover the floor of room?

10. In the adjoining figure, different important places of a village are given. Find the area of the following place.

- (a) Public toilets
- (b) Pond
- (c) School
- (d) Chautari
- (e) Water tank



Project work

Take an A4 size paper. Construct a rhombus with a length 5 cm in the middle of the paper. Cut out the rhombus with the help of scissors. Now, find the area of the remaining part of and present in the classroom.

Answer

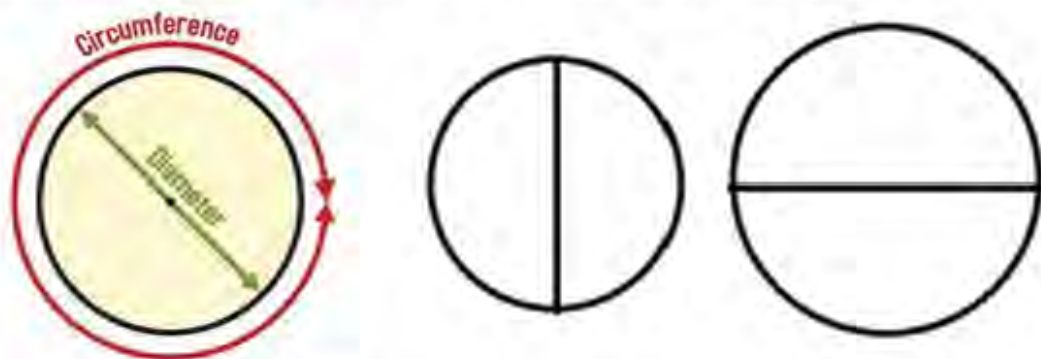
1. (a) 10.5 cm (b) 15.5 cm (c) 15 cm (d) 17cm (e) 23cm
(f) 12 ft
2. (a) 24 cm^2 (b) 24 cm^2 (c) $16\sqrt{3} \text{ cm}^2$ (d) $\frac{25\sqrt{3}}{4} \text{ cm}^2$
(e) $36\sqrt{3} \text{ cm}^2$ (f) 48 cm^2 (g) 120 m^2 (h) 192 cm^2 (i) 48 sq.ft
(j) 135 cm^2 (k) 6 cm^2 (l) 128 cm^2
3. (a) 18 cm^2 (b) 1.96 sq.ft (c) 36 m^2 (d) 77 mm^2 (e) 36 cm^2
(f) 99 cm^2 (g) 48 cm^2 (h) 24 cm^2 (i) 7.5 cm^2 (j) 384 cm^2
(k) 44 cm^2 (l) 40 cm^2

4. (a) 264 cm^2 (b) 51 cm^2 (c) 36 cm^2 (d) 29 cm^2 (e) 60 cm^2
(f) 1536 m^2 (g) 111 m^2 (h) 348 sq.inch (i) 99 m^2
5. (a) 79 m^2 (b) 54.7 cm^2 (c) 42 cm^2 (d) 60 sq.inch (e) 132 cm^2
(f) 63 m^2 (g) 192 m^2 (h) 355 cm^2 (i) 70 cm^2 (j) 51 cm^2
(k) 46 cm^2 (l) 120 cm^2
6. (a) 16 cm (b) 10 cm (c) 10 cm (d) 10 cm (e) 12 cm (f) 10 cm
7. (a) 13038 m^2 (b) 10000 m^2 8.(a) 63 b) 75
9. (a) (i) 50 ft (ii) 2500 sq.ft (b) (i) 20 m (ii) 400 m^2
(c) (i) 12 ft (ii) 180 sq.ft (d) (i) $20 \text{ ft}, 10 \text{ ft}$ (ii) 200 sq.ft
10. (a) 28 m^2 (b) 66 m^2 (c) 5450 m^2 (d) 16 m^2 (e) 25 m^2

8.2 Area of Circle

Activity 10

Make circle of different measurement and do the following activities:



- (a) Measure the length of radius and diameter.
- (b) Cut out the circles with scissors and measure its perimeter or circumference with the help of a ruler.
- (c) Find the ratio of circumference to diameter and what it is denoted for. Discuss with friends.

The total length of the circular boundary of any circle is called the circumference. The ratio of the circle's circumference to its diameter is denoted by π (Pie), so $\pi = \frac{C}{d}$, $d = 2r$ or $C = \pi d = 2\pi r$. Usually, approximate value of π is $\frac{22}{7}$ i.e. 3.142. Therefore, when solving the problem, the value of π is kept $\frac{22}{7}$.

Activity 11

Do the following activities.

1. Take a cardboard and make a circle. Its radius is denoted by r .
 2. As shown in the figure (a), divide it into equal parts, so that the areas are as small as possible. (If the circle is big, it can be separated in more than 32, 64 parts)
 3. Now, colour each part of circle as shown in figure and cut with scissors and separate it.
 4. Arrange those triangular pieces as shown in figure (b) and make a shape of parallelogram from it.
- (One of the sides can be made into 2 equal pieces to make a rectangle)

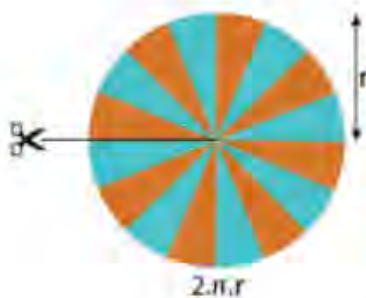


Figure (A)

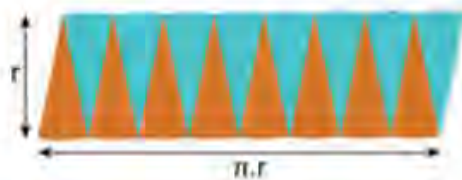


Figure (B)

5. The area of the circle is equal to the area of the parallelogram so formed. Now, Radius of circle (r) then
Perimeter of circle (P) = $2\pi r$
The perimeter of circle is $2\pi r$ having its radius r .
Therefore, the height of the parallelogram (h) = r

$$\text{Base of parallelogram (b)} = \frac{2\pi r}{2} = \pi r$$

$$\therefore \text{Area of parallelogram (A)} = b \times h = \pi r \times r = \pi r^2$$

Therefore, area of circle = πr^2

Half of diameter is radius so, $= \frac{d}{2}$

$$\text{Area of circle (A)} = \pi \left(\frac{d}{2} \right)^2 = \frac{1}{4} \pi d^2$$

Example 1

Find the area of circle from given measurement. ($\pi = 3.14$)

(a) radius = 7cm (b) Diameter = 34 ft (c) Circumference 62.8 inch

Solution,

(a) Here,

Radius of circle (r) = 7cm

Area of circle (A) = ?

We know that,

$$\text{Area of circle (A)} = \pi r^2 = 3.14 \times 7 \times 7 = 153.86 \text{ cm}^2$$

Alternative method,

Diameter of circle (d) = 34ft

Area of circle (A) = ?

We know that,

$$\text{Area of circle (A)} = \frac{1}{4} \pi d^2 = \frac{3.14 \times 34 \times 34}{4} = 907.46 \text{ sq.ft}$$

(c) Here, circumference of circle (c) = 62.8 inch

Area of circle (A) = ?

We know that,

Circumference of circle (C) = $2\pi r$

$$\text{or, } 62.8 = 2 \times 3.14 \times r$$

$$\therefore r = \frac{62.8}{6.28} = 10 \text{ inch}$$

Again,

$$\text{Area of circle (A)} = \pi r^2 = 3.14 \times 10 \times 10 = 314 \text{ sq. inch}$$

$$\therefore \text{Area of circle (A)} = 314 \text{ sq. inch}$$

Example 2

Find the radius of circle where area is 5544 cm^2 . ($\pi = \frac{22}{7}$)

Solution

$$\text{Here, area of circle (A)} = 5544 \text{ cm}^2$$

$$\text{Radius of circle (r)} = ?$$

We know that,

$$\text{Area of circle (A)} = \pi r^2$$

$$\text{or, } 5544 = \frac{22}{7} r^2$$

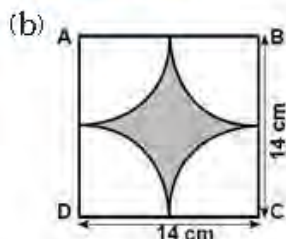
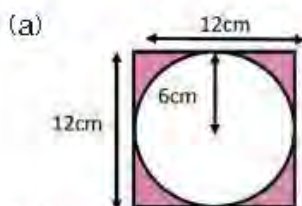
$$\text{or, } r^2 = \frac{5544 \times 7}{22} = 1764$$

$$\therefore r = 42 \text{ cm}$$

$$\text{Therefore, radius of circle (r)} = 42 \text{ cm}$$

Example 3

Find the area of shaded region from given figure.



Solution

(a) Here, the complete part of given figure is a square. The non-shaded part is a circle.

$$\text{Side of square (a)} = 12 \text{ cm}$$

$$\therefore \text{Area of circle (A)} = \pi r^2 = 3.14 \times 12 \times 12 = 471.6 \text{ cm}^2$$

$$\text{And radius of circle (r)} = 6 \text{ cm}$$

Therefore, area of circle $(A_2) = \pi r^2 = 3.14 \times 6 \times 6 = 113.04 \text{ cm}^2$

$$\begin{aligned}\text{Area of shaded region } (A) &= A_1 - A_2 \\ &= 144 - 113.04 \\ &= 30.96 \text{ cm}^2\end{aligned}$$

- b) Here, in figure, whole part ABCD is a square and non shaded part is circle.

Square of side $(a) = 14 \text{ cm}$

$$\therefore \text{Area of square } (A_1) = a^2 = 14 \times 14 = 196 \text{ cm}^2$$

Again, the 4 non-shaded parts of figure are the one fourth of a circle.

$$\text{And radius of circle } (r) = \frac{14}{2} \text{ cm} = 7 \text{ cm}$$

$$\therefore \text{Area of one fourth circle} = \frac{1}{4} \pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 7 \times 7 = \frac{154}{4} \text{ cm}^2$$

$$\text{Therefore, Area of 4 one fourth circle } (A_2) = 4 \times \frac{154}{4} = 154 \text{ cm}^2$$

$$\begin{aligned}\text{Now, area of shaded part } (A) &= A_1 - A_2 \\ &= 196 - 154 \\ &= 42 \text{ cm}^2\end{aligned}$$

Example 4

If the circumference of circular pond is 125.6 m, then, find the radius and area of that pond. ($\pi = 3.14$)

Solution

Here, circumference of circular pond $(C) = 125.6 \text{ m}$

Radius $r) = ?$

Area $(A_1) = ?$

Now, circumference $(C) = 2\pi r$

$$\text{or, } 125.6 \text{ m} = 2 \times 3.14 \times r$$

$$\text{or, } \frac{125.6}{6.28} \text{ m} = r$$

$\therefore$ Therefore, radius of circle $(r) = 20 \text{ m}$

$$\begin{aligned}\text{Again, Area of circle } (A) &= \pi r^2 \\ &= 3.14 \times 20 \times 20 \\ &= 1256 \text{ m}^2\end{aligned}$$

Hence, radius of circular pond is 20m and Area is 1256 m².

Example 5

The area of cylindrical well cover is 15400 cm^2 . ($\pi = \frac{22}{7}$)

- (i) Find the radius of cover of well?
- (ii) How long steel is required to surround the area of cover of well?
- (iii) If cost of steel per meter. is Rs 250, find total cost to surround the well cover with steel?

Solution

Here, cylindrical well has circular shape.

- (i) Radius(r) = ?

We know that,

Area (A)

$$(A) = \pi r^2$$

$$\text{or, } 15400 = \frac{22}{7} r^2$$

$$\text{or, } \frac{15400 \times 7}{22} = r^2$$

$$\text{or, } 4900 = r^2$$

$$\therefore r = 70 \text{ cm}$$

$\therefore$ Therefore, radius of circle (r) = 70cm

- (b) The circular surrounding of a well cover is circumference.

$$\text{Now, circumference (C)} = 2\pi r = 2 \times \frac{22}{7} \times 70 = 440 \text{ cm} = 4.4 \text{ m}$$

$\therefore$ Therefore, the fence of steel surrounds the well cover is 4.4m.

- (c) Here,

$$\text{Price of 1m steel} = \text{Rs.}250$$

$$\text{Price of 4.4m steel} = \text{Rs.}4.4 \times 250$$

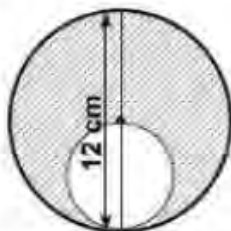
$$= \text{Rs.}1100$$

Hence, The cost of covering lid with steels Rs. 1100.

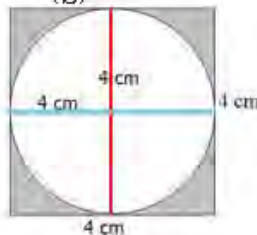
Exercise 8.2

- Find the area of a circle with the following measurements: ($\pi = 3.14$)
 - radius = 3 cm
 - radius = 8 ft
 - Radius = 16 m
 - Diameter = 5 cm
 - diameter = 12 inch
 - diameter = 18 m
 - diameter = 20 km
 - diameter = 15 mm
 - diameter = 22 cm
- Find the area of a circle with the following circumference: ($\pi = 3.14$)
 - 34.54 cm
 - 65.94 m
 - 18.84 inch
 - 113.04 m
 - 376.80 ft
 - 157 m
- Find radius of a circle having the following area: ($\pi = 22/7$)
 - 154 cm²
 - 346.5 ft²
 - 616 m²
 - 1386 m²
 - 38.5 km²
 - 3850 ft²
- Find the area of the shaded part of the figures below: ($\pi = 3.14$)

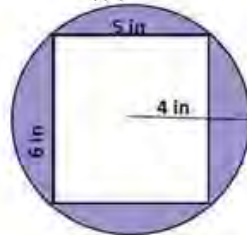
(a)



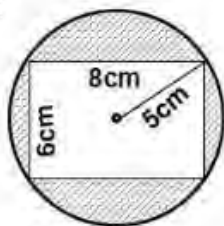
(b)



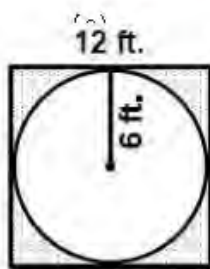
(c)



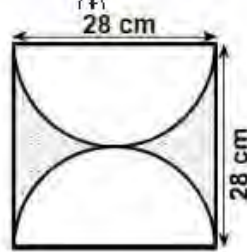
(d)



(e)



(f)



- If the radius of a circular room is 14 m then, how much area does it cover? ($\pi = 22/7$)
 - If the radius of a circular fun park is 21 m, then, how much area does it cover? ($\pi = 22/7$)
 - A cow is tied with a rope of 7 ft. length. Find the grazing area of that cow. ($\pi = 22/7$)
- If a diameter of the base of a cylindrical bowl is 9 cm, find the area of the base of the bowl. ($\pi = 3.14$)

- (b) If the diameter of the base of a cylindrical pipe is 30 cm, find the area of the base of pipe. ($\pi = 3.14$)
7. (a) If the area of base of cylindrical tank is 154 sq. ft. Find the radius and the circumference of it. ($\pi = 22/7$)
- (b) A circular playground of area 153.86 m^2 is plastered. What is the diameter of the plastered part of the playground? Also find its circumference. ($\pi = 3.14$)
8. (a) Sharmila has drawn a circle of radius 5 cm. Similarly, Prakash also has drawn a circle of radius 7 cm. Now, whose area of circle is more and by how much? ($\pi = 3.14$)
- (b) Salman dug a well of 14 m diameter. Similarly, Pramila also dug another well with a diameter of 18 m. Now whose well occupies more land and how much? ($\pi = 3.14$)
9. The area of a circular pond is 616 m^2 . ($\pi = 22/7$)
- (a) What is the radius of the pond?
- (b) How long wire is required to fence around the pond?
- (c) If the costs to fence the pond around is Rs.250. What is the total cost of fencing surround the pond?
10. If a runner complete a 3520 m run on a circular track by 4 rotations,
- (a) What is the length of the circular track?
- (b) What is its diameter?
- (c) What area does the track cover?
- (d) What is the total cost for fencing with wire around the track, if the cost of wire per meter is Rs 600?

Project work

- (a) Name any 3 circular objects around you. Now measure their diameter and find out their circumference and area. Also present the result in class.
- (b) Take three pieces of thread 18 cm long. Now make a square, a rectangular and a circle from that thread respectively. Now find their area and discuss which figure has the least and the most area.

Answer

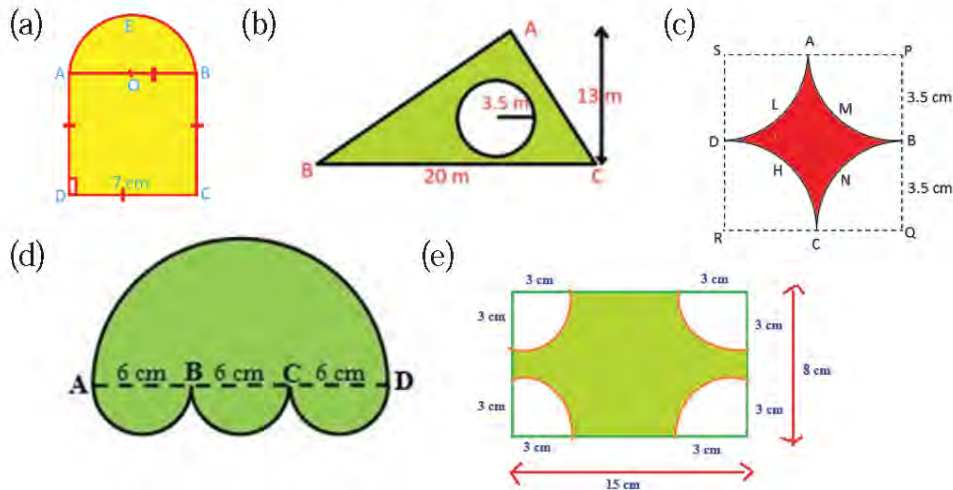
1. (a) 28.26 cm^2 (b) 200.96 sq.ft (c) 803.84 m^2 (d) 19.625 cm^2
(e) 113.04 sq.inch (f) 254.34 m^2 (g) 314 km^2
(h) 176.625 mm^2 (i) 379.94 cm^2
2. (a) 94.985 cm^2 (b) 346.185 m^2 (c) 28.26 sq.inch
(d) 1017.36 m^2 (e) 11304 sq.ft (f) 1886.5 m^2
3. (a) 7 cm (b) 10.5 ft (c) 14 m (d) 21 m
(e) 3.5 km (f) 35 ft
4. (a) 84.78 cm^2 (b) 3.44 cm^2 (c) 20.24 sq.in (d) 30.5 cm^2
(e) 30.96 sq.ft (f) 168.56 cm^2
5. (a) 154 m^2 (b) 1386 m^2 (c) 154 sq.ft
6. (a) 63.585 cm^2 (b) 706.5 cm^2
7. (a) 7 ft 44ft (b) 14m 43.96 m
8. (a) prakash 75.36 cm^2 (b) Pramila 401.92 m^2
9. (a) 14 m (b) 88 m (c) ₹=22,000
10. (a) 880m (b) 280 m (c) 61600 m^2 (d) ₹.5,28,000

Mixed Exercise

1. Find the area of the following shapes with given measurements.

- A triangle having base = 5 cm and height = 10 cm
- A parallelogram having base = 12 cm and height = 8 cm
- A square having a diagonal = 6 ft.
- A rhombus having diagonals 5.4 cm and 6.8 cm long.
- A kite having diagonals 9 cm and 5 cm long.
- A circle with diameter = 7 m [$\pi = \frac{22}{7}$]
- A circle with radius = 6 cm [$\pi = 03.14$]

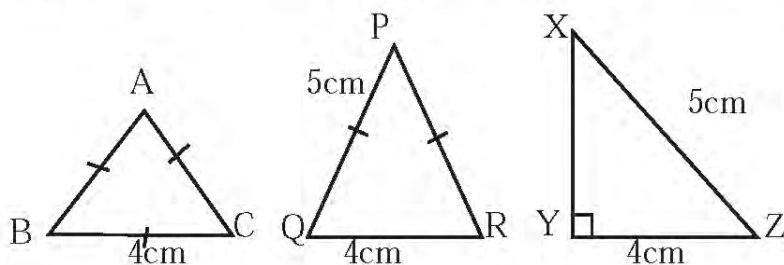
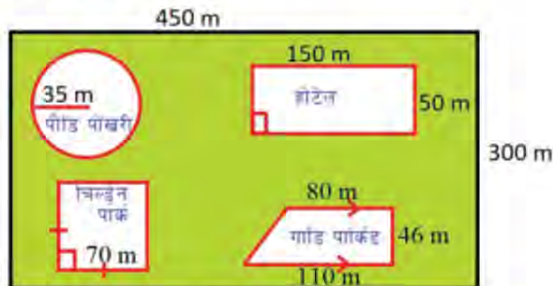
2. Find the area of shaded part of the following figures. [$\pi = 3.14$]



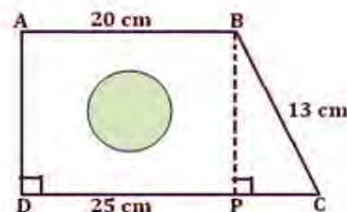
3. There is a circular carpet of diameter 7 ft. placed in the middle of the floor in a square shaped room of length 10 ft.

- What is the area of the room?
 - What is the area of the carpet? [$\pi = \frac{22}{7}$]
 - Find the area of room excluding the carpet.
4. A building of length 40 meters and breadth 30 meters is constructed in a rectangular field of length 200 meters and breadth 120 meters.
- What is the total area of land?

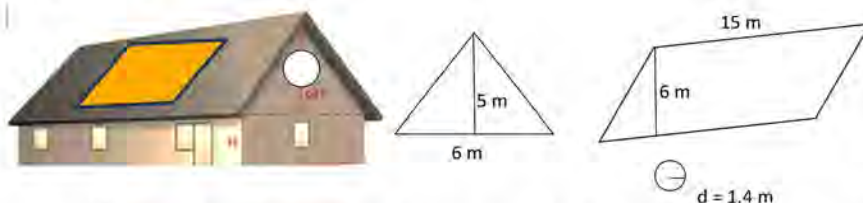
- (b) What is the area of building?
- (c) What is the area of land excluding the building?
5. The adjoining figure shows the major parts of a hotel in its premises. Find the area of following parts from the figure.
- Area of the whole premises
 - Swimming pool
 - Children park
 - Vehicle parking
 - Land occupied by hotel building
 - If the remaining open space is formed a garden, then find its area.
6. Find the area of $\triangle ABC$, $\triangle PQR$ and $\triangle XYZ$ given in the figure.



7. There is a circular carpet of diameter 3.5 meters placed in the middle of the floor of rectangular shaped room of length 6 meters and breadth 4 meters.
- What is the area of the room?
 - What is the area of the carpet?
 - Find the area of room excluding the carpet.
 - What is the total cost of carpeting the room at the rate of Rs. 1000 per squared meter.
8. In the adjoining figure, ABCD is a trapezium and BPC is a right angled triangle. Where $AB = 20$ cm, $DC = 25$ cm and $BC = 13$ cm.

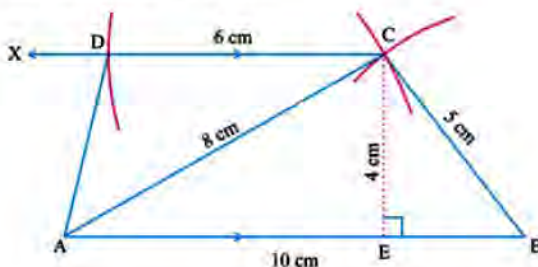


- (a) Find the length of BP.
 - (b) Find the area of trapezium ABCD.
 - (c) How much times is the area of trapezium ABCD more than that of triangle BPC?
9. It can be seen a triangular, a parallelogram and a circular shapes on a house given in the following figure and also their models are given separately on the right side.

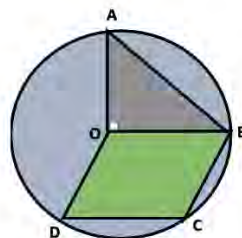


- (a) Write the relationship between the diameter and radius of a circle.
 - (b) What is the area of triangular shape at the front side of the house?
 - (c) How much does it cost to color the triangular shaped part at the front side excluding the circular part, if the rate of coloring is Rs. 50 per squared meter?
 - (d) If the number of tiles required for tiling 1 squared meter area is 10, find the total numbers of tiles required for tiling the two parallelogram shaped roofs.
10. The construction of a trapezium is shown in the figure, where $CD=6$ cm, $AE = 10$ cm, $AC = 8$ cm, $BC = 5$ cm and $EC = 4$ cm.

- (a) What is the length of side AB?
- (b) Find the area of trapezium.
- (c) Which of the triangles BEC and AEC has greater area and by how much?



11. Rabina has drawn a right angled triangle and a rhombus in her circular garden of radius 8 meters as shown in the figure.



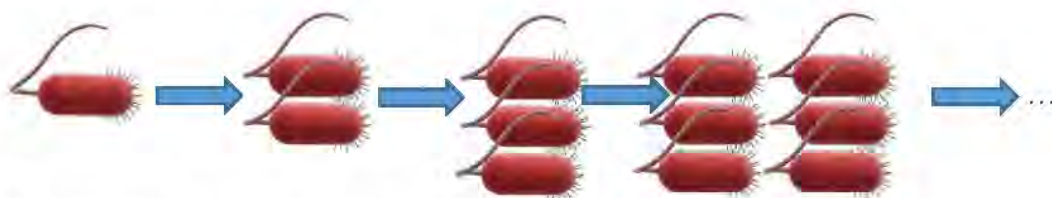
- Write the formula to find the area of circle with radius r meter.
- Find the area of right angled triangle AOB .
- Find the area of rhombus BCD , if the length of its longer diagonal is 12 meters. Which of the rhombus BCD and triangle AOB has greater area and by how much?
- Compare the area occupied by the triangular and rhombus shaped part and the area of remaining part of the circular garden. Which has greater area and by how much?

Answer

- (a) 25 cm^2 (b) 96 cm^2 (c) 18 sq.ft (d) 18.36 cm^2 (e) 22.5 cm^2
(f) 38.5 m^2 (g) 113.04 cm^2
- (a) 68.25 cm^2 (b) 91.5 m^2 (c) 10.5 cm^2 (d) 169.56 m^2 (e) 91.74 cm^2
- (a) 100 ft^2 (c) 38.5 ft^2 (d) 61.5 ft^2
- (a) 24000 m^2 (b) 1200 m^2 (c) 22800 m^2
- (a) 135000 m^2 (b) 3850 m^2 (c) 4900 m^2 (d) 4370 m^2
(e) 7500 m^2 (f) 114380 m^2
- (a) $4\sqrt{3} \text{ cm}^2$ (b) $2\sqrt{21} \text{ cm}^2$ (c) 6 cm^2
- (a) 24 m^2 (b) 9.625 m^2 (c) 14.375 m^2 (d) Rs. 9625
- (a) 12 m (b) 270 cm^2 (c) 9 times
- (a) $d = 2r$ (b) 15 m^2 (c) Rs. 1423 (d) 1800
- (a) 13 cm (b) 38 cm^2 (c) Show to your teacher.
- (a) $\pi r^2 \text{ m}^2$ (b) 32 m^2 (c) Show to your teacher.
(d) 121.14 m^2

9.0 Review

If the growth of bacteria is found in such a way that its number increases twice in every twenty minutes, what will be the number of bacteria in one hundred twenty minutes? Discuss in your group.



- (a) Number of bacteria at the beginning = 1
- (b) Number of bacteria in first twenty seconds = $2 = 2^1$
- (c) Number of bacteria in second twenty seconds = $2 \times 2 = 2^2 = 4$
- (d) Number of bacteria in third twenty seconds = $2 \times 2 \times 2 = 2^3 = 8$
- (e) Number of bacteria in fourth twenty seconds = $2 \times 2 \times 2 \times 2 = 2^4 = 16$
- (f) Number of bacteria in fifth twenty seconds = $2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$
- (g) Number of bacteria in sixth twenty seconds = $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6 = 64$

The product of a number with itself in a given number of times can be represented using indices.

In a^n , a is called the base and n is called the index.

Base $\rightarrow 2^6 \leftarrow$ Index

9.1 Laws of Indices

Activity 1

Study the following pattern and discuss to find the conclusion.

(a) $x^m \times x^n = x^{m+n}$

$$\text{Here, } x^2 \times x^3 = (x \times x) \times (x \times x \times x) = x \times x \times x \times x \times x = x^5 = x^{2+3}$$

$$x^2 \times x^4 = (x \times x) \times (x \times x \times x \times x) = x \times x \times x \times x \times x \times x = x^6 = x^{2+4}$$

$$x^3 \times x^1 = (x \times x \times x) \times (x) = x \times x \times x \times x = x^4 = x^{3+1}$$

$$\therefore x^m \times x^n = x^{m+n}$$

When the indices of same base are multiplied, the base remains the same and the indices are added. $x^m \times x^n = x^{m+n}$

$$(b) \quad x^m \div x^n = x^{m-n}$$

Here,

$$x^2 \div x^1 = \frac{x^2}{x^1} = \frac{x \times x}{x} = x = x^1 = x^{2-1}$$

$$x^3 \div x^1 = \frac{x^3}{x^1} = \frac{x \times x \times x}{x} = x \times x = x^2 = x^{3-1}$$

$$x^5 \div x^2 = \frac{x^5}{x^2} = \frac{x \times x \times x \times x \times x}{x \times x} = x \times x \times x = x^3 = x^{5-2}$$

$$\therefore x^m \div x^n = x^{m-n}$$

When the indices of same base are divided, the base remains the same and the index is obtained by subtracting the index of denominator from the index of numerator $x^m \div x^n = x^{m-n}$

$$(c) \quad x^0 = 1, \quad x \neq 0$$

We know that, $x^m \div x^n = x^{m-n}$, ($x \neq 0$)

$$x^m \div x^m = x^{m-m}$$

$$\text{or, } 1 = x^0$$

$$\therefore x^0 = 1$$

If the index of any number except zero is zero, then its value is 1.

$$x^0 = 1, \quad (x \neq 0)$$

$$(d) \quad (x \times y)^m = x^m y^m$$

Here, $(x \times y)^m = xy \times xy \times xy \times xy \times xy$ to m terms

$= (x \times x \times x \times x \times x$ to m terms) $\times$ ($y \times y \times y \times y \times y$ to m terms)

$$= x^m y^m$$

Similarly, $\left(\frac{x}{y}\right)^m = \frac{x}{y} \cdot \frac{x}{y} \cdot \frac{x}{y} \cdot \frac{x}{y} \cdot \frac{x}{y} \dots$ to m terms

$$= \frac{x^m}{y^m}$$

$$\therefore \left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$$

If the base contains product or quotient of two numbers with the same index, then the index can be assigned separately to each number of the

base. $(xy)^m = x^m y^m$ & $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$

(e) $x^{-m} = \frac{1}{x^m}$

Here, $x^m \cdot x^{-m} = x^{m-m} = x^0 = 1$

or, $\frac{x^m \cdot x^{-m}}{x^m} = \frac{1}{x^m}$

or, $x^{m-m-m} = \frac{1}{x^m}$

$$\therefore x^{-m} = \frac{1}{x^m}$$

If the index of the base in numerator is negative, then it can be shifted to denominator raising the positive index to the base. $x^{-m} = \frac{1}{x^m}$

Similarly, If the index of the base in denominator is negative, then it can be shifted to numerator raising the positive index to the base. $\frac{1}{x^{-m}} = x^m$

(f) $(x^m)^n = x^{mn}$

Here,

$$(x^2)^3 = x^2 \cdot x^2 \cdot x^2 = (x \cdot x) \cdot (x \cdot x) \cdot (x \cdot x) = x^6 = x^{2 \cdot 3}$$

$$(x^3)^4 = x^3 \cdot x^3 \cdot x^3 \cdot x^3 = (x \cdot x \cdot x) \cdot (x \cdot x \cdot x) \cdot (x \cdot x \cdot x) \cdot (x \cdot x \cdot x)$$

$$= x^{12} = x^{3 \times 4}$$

$$(x^4)^5 = x^4 \cdot x^4 \cdot x^4 \cdot x^4 \cdot x^4$$

$$= (x \cdot x \cdot x \cdot x) \cdot (x \cdot x \cdot x \cdot x) \cdot (x \cdot x \cdot x \cdot x) \cdot (x \cdot x \cdot x \cdot x)$$

$$= x^{20} = x^{4 \times 5}$$

$$\therefore (x^m)^n = x^{mn}$$

Alternative method,

$$(x^m \cdot x^m \cdot x^m \cdot \dots \cdot x^m \text{ } n \text{ times}) = x^{mn}$$

$$\therefore (x^m)^n = x^{mn}$$

If the second index is raised to the expression which has already one index, then the two indices are multiplied on the same base.

$$(x^m)^n = x^{mn}$$

Example 1

Simplify the followings using the law of indices.

(a) $2^3 \cdot 2^3$

(b) $x^3 \cdot x^2$

(c) $p^4 \cdot p^3 \cdot p^5$

Solution:

(a) Here, $2^3 \cdot 2^3$

$$= 2^{3+3}$$

$$= 2^6 = 64$$

(b) $x^3 \cdot x^2$

$$= x^{3+2}$$

$$= x^5$$

(c) $p^4 \cdot p^3 \cdot p^5$

$$= p^{4+3+5}$$

$$= p^{12}$$

Example 2

Simplify using law of indices.

(a) $x^5 \cdot x^2$

(b) $8x^5 \cdot 2x^2$

(c) $x^{3n-2} \cdot x^{2n-5}$

Solution:

(a) Here, $x^5 \cdot x^2$

$$= x^{5+2}$$

$$= x^7$$

(b) $8x^5 \cdot 2x^2$

$$= 4x^{5+2}$$

$$= 4x^7$$

(c) $x^{3n-2} \cdot x^{2n-5}$

$$= \frac{x^{3n-2}}{x^{2n-5}}$$

$$= x^{(3n-2)-(2n-5)}$$

$$= x^{3n-2-2n+5} = x^{n+3}$$

Example 3

Simplify using the law of indices.

$$(a) \frac{x^{4n-2}}{x^{2(2n-1)}}$$

$$(b) \frac{a^{n-2}}{a^{n-2}}$$

Solution:

$$(a) \frac{x^{4n-2}}{x^{2(2n-1)}}$$

$$= x^{(4n-2)-(4n-2)}$$

$$= x^{4n-2-4n+2}$$

$$= x^0$$

$$= 1$$

$$(b) \frac{a^{n-2}}{a^{n-2}}$$

$$= a^{(n-2)-(n-2)}$$

$$= a^{n-2-n+2}$$

$$= a^0$$

$$= \frac{1}{a^0}$$

Example 4

Simplify:

$$(a) \left(\frac{x^2 y^2}{x^3 y} \right)^3$$

$$(b) (x^3 y^2)^3 (2 x^2 y^3)^4$$

Solution

$$\text{Here, (a) } \left(\frac{x^2 y^2}{x^3 y} \right)^3$$

$$= \frac{(x^2 y^2)^3}{(x^3 y)^3}$$

$$= \frac{x^6 y^6}{x^9 y^3} = \frac{y^{6-3}}{x^{9-6}} = \frac{y^3}{x^3}$$

$$\begin{aligned} (b) (x^3 y^2)^3 (2 x^2 y^3)^4 &= (x^9 y^6) (16 x^8 y^{12}) \\ &= 16 x^{9+8} y^{6+12} \\ &= 16 x^{17} y^{18} \end{aligned}$$

Example 5

Simplify:

$$(a) (x)^{(a-b)} \times (x)^{(b-c)} \times (x)^{(c-a)}$$

$$(b) a^y \times a^x \times a^{(y-x)}$$

Solution

$$\begin{aligned} \text{(a)} \quad (x)^{(a-b)} \times (x)^{(b-c)} \times (x)^{(c-a)} \\ = x^{a-b+b-c+c-a} \\ = x^0 \\ = 1 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad a^y \times a^x \times a^{(y-x)} \\ = a^{y+x-y-x} \\ = a^0 \\ = 1 \end{aligned}$$

Example 6

If $a = 1$, $b = 2$ and $c = 3$, find the value of $a^b \cdot b^c \cdot c^a$.

Solution

$$\begin{aligned} \text{Here, } a^b \cdot b^c \cdot c^a \\ = 1^2 \cdot 2^3 \cdot 3^1 = 1 \cdot 8 \cdot 3 = 24 \end{aligned}$$

Exercise 9

1. Simplify using the law of indices.

$$\begin{aligned} \text{(a)} \quad 3^4 \cdot 3^3 & \quad \text{(b)} \quad x^3 \cdot x^5 & \quad \text{(c)} \quad ab^4 \cdot b^3 \\ \text{(d)} \quad (a^2 b) \cdot (ab^3) & \quad \text{(e)} \quad 3x^4 \cdot 2x^3 & \quad \text{(f)} \quad (2 \cdot x^4) \cdot (3x^3) \\ \text{(g)} \quad (ab) \cdot (a^3 b^3) & \quad (a^2 b) \end{aligned}$$

2. Simplify using the law of indices.

$$\begin{aligned} \text{(a)} \quad 4^4 \cdot 4^2 & \quad \text{(b)} \quad x^8 \cdot x^5 & \quad \text{(c)} \quad a^4 b^4 \cdot a^3 b^3 \\ \text{(d)} \quad (x^6 y^3) \cdot (x^3 y^3) & \quad \text{(e)} \quad 8x^4 \cdot 2x^3 & \quad \text{(f)} \quad 16x^4 \cdot 8x^3 \end{aligned}$$

3. Simplify using the law of indices.

$$\begin{aligned} \text{(a)} \quad (3a)^0 & \quad \text{(b)} \quad (2b)^3 & \quad \text{(c)} \quad (3 \cdot x)^4 & \quad \text{(d)} \quad (4 \cdot ab^2)^3 \\ \text{(e)} \quad (3a^3 b^2)^2 & \quad \text{(f)} \quad \left(\frac{x^2}{y^2}\right)^2 & \quad \text{(g)} \quad \frac{(3xy)^2}{3xy} & \quad \text{(h)} \quad \frac{a^{4n-2}}{a^{2(2n-1)}} \end{aligned}$$

4. Simplify using the law of indices.

$$\begin{aligned} \text{(a)} \quad \frac{2^2 \cdot 4^2}{8^2} & \quad \text{(b)} \quad \frac{5^3 \cdot 125^3}{25^3} & \quad \text{(c)} \quad \frac{4^4 \cdot 5^5}{25^3 \cdot 16^2} \end{aligned}$$

5. Fill the box with appropriate number.

(a) $4^{\square} = 8^2$ (b) $(3x^{\square})^2 = 9x^6$ (c) $(4x)^{\square} = 1$

6. Prove that

(a) $\frac{x^{m+n-2} \cdot x^{m+n-2}}{x^{2(m+n-1)}} = x^2$ (b) $\frac{x^{p+q-1} \cdot x^{q+r-1} \cdot x^{r+p-1}}{x^3} = 1$

(c) $(x^a \cdot b)^{a-b} \cdot (x^b \cdot c)^{b-c} \cdot (x^c \cdot a)^{c-a} = 1$

7. If $a = 2$, $b = 3$, $c = 1$, $m = 4$ and $n = 5$, find the value of

(a) $\frac{a^m \cdot b^n \cdot c^{ab}}{m^a \cdot n^b \cdot (b^a)^c}$

(b) $(a + b + c)^{m+n} \cdot (m + n)^{a+b+c}$

Project work

A children club decided to implement the following strategy in order to spread public awareness information in a community:

One person will share the information to 5 persons on the first day and each of the 5 persons will share it to other 5 persons the next day. If this continues, how many persons will get the information in the period of 5 days?

Answer

1. (a) 3^7 (b) x^8 (c) ab^7 (d) $a^3 b^4$
(e) $6x^7$ (f) $6x^7$ (g) $a^6 b^5$
2. (a) 4^2 (b) x^3 (c) ab (d) x^3 (e) $4x$ (f) $2x$
3. (a) 1 (b) $8b^3$ (c) $81x^4$ (d) $64a^3 b^6$ (e) $9a^6 b^4$
(f) $\frac{x^4}{y^4}$ (g) $3xy$ (h) 1
4. (a) 1 (b) 5^6 (c) $\frac{1}{5}$
5. (a) 4 (b) 3 (c) 0
6. Show to your teacher 7. (a) $\left(\frac{3}{5}\right)^3$ (b) $\left(\frac{8}{3}\right)^3$

10.0 Review

Discuss in your group to find the answer to the following questions:



- What is the area of rectangular field, if its length is $(2x + y)$ m and breadth is $(x + 2y)$ m?
- What will be the perimeter of the field?
- If there is a garden having length $(x + 2y)$ m and breadth y m inside the field, by how much the area of garden is less than the area of field?
- Find the area of field and garden, if $x = 10$ and $y = 2$

10.1 Factorisation

Activity 1

The product of two algebraic expressions is given in the following examples. Study in each of your pair.

- $x(x + 2) = x^2 + 2x$
- $(x + 2)(x + 3) = x^2 + 5x + 6$
- $(x + 2)(x + 1) = x^2 + 3x + 2$

Discuss the following questions on the basis of the above examples.

- Do the expressions x and $(x + 2)$ divide $x^2 + 2x$ exactly? If so, what is called the relation of x and $(x + 2)$ to $(x^2 + 2x)$?
- Do the expressions $(x + 2)$ and $(x + 3)$ divide $x^2 + 5x + 6$ exactly? If so, what is called the relation of $(x + 2)$ and $(x + 3)$ to $(x^2 + 5x + 6)$?

- (c) Do the expressions $(x - 2)$ and $(x - 1)$ divide $x^2 - 2x - 2$ exactly? If so, what is called the relation of $(x - 2)$ and $(x - 1)$ to $(x^2 - x - 2)$?

The algebraic expressions which divide the given algebraic expression exactly are called factors of the given expression. If an algebraic expression is expressed as the product of its factors, then it is called factorization.

10.1.1 Factorization by taking common factor and grouping the terms

Activity 2

Observe the following algebraic models where the areas of cards are given separately. The green and red colors of cards indicate positive and negative respectively. Discuss the following questions in your group in order to find the conclusion.

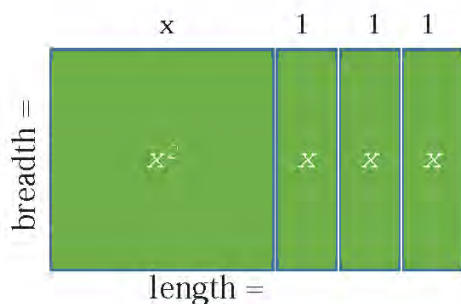


Figure 1

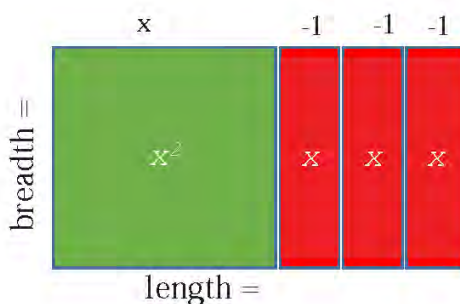


Figure 2

- What is the total area of cards given in figure no. 1?
- What is the total area of cards given in figure no. 2?
- What are the length and breadth of the rectangle formed by combining all the cards in figure no. 1?
- What are the length and breadth of the rectangle formed by combining all the cards in figure no. 2?

There is one card of positive x^2 and 3 cards of positive x in the first figure. So, the total area of the cards is $x^2 + 3x$. On the other hand, there is one card of positive x^2 and 3 cards of negative x in the second figure. So, the total area of cards is $x^2 - 3x$.

In the figure no. 1, the rectangle formed by combining all the cards has length $x + 3$ and breadth x .

So, $x^2 + 3x = x(x + 3)$. According to this, $x^2 + 3x$ can be expressed as the product of a common factor x and the remaining factor $(x + 3)$.

Similarly, in the figure no. 2, the rectangle formed by combining all the cards has length $x - 3$ and breadth x . So, $x^2 - 3x = x(x - 3)$. According to this, $x^2 - 3x$ can be expressed as the product of a common factor x and the remaining factor $(x - 3)$.

If there is no common factor to all the terms of a polynomial expression, the terms can be grouped so that a common factor can be taken from each group. For example:

$$\begin{aligned} & 2xy - 3 - 6x - y \\ &= 2xy - 6x - y - 3 \quad (\text{Grouping the terms}) \\ &= 2x(y - 3) - 1(y - 3) \\ &= (2x - 1)(y - 3) \end{aligned}$$

Example 1

Factorize the following expression.

(a) $2x^2 - 4x$

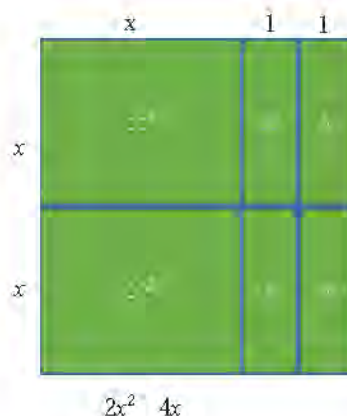
Solution

$$\begin{aligned} & \text{Here, } 2x^2 - 4x \\ &= 2x(x - 2) \quad (2x \text{ is common to both}) \\ &= 2x(x - 2) \end{aligned}$$

(b) $a^2 - 15b - 5a - 3ab$

Solution

$$\text{Here, } a^2 - 15b - 5a - 3ab$$



(Arranging the terms having a common factor)

$$= a^2 - 5a + 3ab - 15b$$

$= a(a - 5) + 3b(a - 5)$ [Taking a as the common factor from first two terms and $3b$ as the common factor from last two terms]

$$= (a - 5)(a + 3b)$$

Exercise 10.1.1

1. Factorise the following algebraic expressions:

(a) $6x^2 - 3$

(b) $x^2 - 4x$

(c) $12a^2 - 3b$

(d) $12p^2 - 6q^2$

(e) $14xy - 7y$

(f) $x^2 - x^3$

(g) $12x^2 - xy - xz$

(h) $x^3 - x^2 - x$

(i) $2x^2 - 2x^3 - 8x^4$

2. Factorise the following by grouping the terms:

(a) $ax^2 - bx - ay^2 + by$

(b) $2ab + a^2b - 2b - ab$

(c) $x^2y - xy - 2x^2y - 2xy$

(d) $x^2 - 3x - xy - 3y$

(e) $2ab - 3a - 2b^2 - 3b$

(f) $a^2 - b^2 - a^2 - ab$

(g) $2a^2 - 5a - 6a - 15$

(h) $2xa - x^2a + 2a - ax$

(i) $x^2y - 4xy - xy^2 - 4y^2$

(j) $3x(x - y) - 3y(x - y)$

(k) $2x^2 - 3ax - 2ax - 3a^2$

Answer

1. (a) $3(2x^2 - 1)$ (b) $x(x - 4)$ (c) $3(4a^2 - b)$ (d) $6(2p^2 - q^2)$
 (e) $7y(2x - 1)$ (f) $x(1 - x^2)$ (g) $x(12x - y - z)$ (h) $x(x^2 + x + 1)$
 (i) $2x^2(1 - x - 4x^2)$

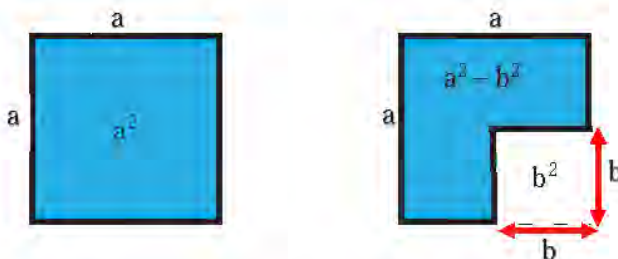
2. (a) $(a + b)(x + y)$ (b) $b(2 - a)(2a - 1)$ (c) $3xy(x - 1)$
 (d) $(x - 3)(x - y)$ (e) $(a - b)(2b - 3)$ (f) $(a - b)(a - 1)$
 (g) $(a - 3)(2a - 5)$ (h) $a(x - 1)(2 - x)$ (i) $y(x + 4)(x - y)$
 (j) $3(x - y)(x - y)$ (k) $(x - a)(2x - 3a)$

10.1.2 Factorization of the expression of the form $(a^2 - b^2)$

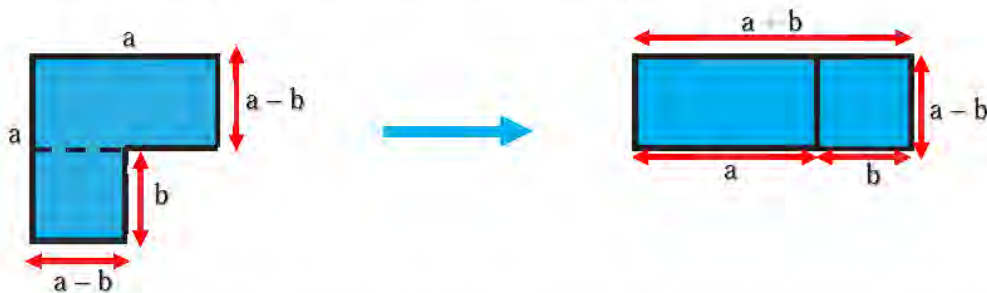
Activity 2

Perform the following activity in each of your pair:

- Take a square shaped paper of length a unit .Its area is a^2
- Cut a square shaped part of length b units and remove it from the original as shown in the figure..



- While removing the part of area b^2 from the whole part of area a^2 , the area of remaining part is $a^2 - b^2$. Again, modify the remaining part to form a rectangular shape as shown in the figure.



What will be the length and breadth of rectangular shape so formed? Discuss in your group and find the answer.

Here, Area of the rectangular surface = $a^2 - b^2$

Length of the rectangular surface = $(a + b)$

Breadth of the rectangular surface = $(a - b)$

Therefore,

Area of the rectangular surface = length $\times$ breadth

$$a^2 - b^2 = (a - b)(a + b)$$

Example 1

Factorise:

(a) $a^2 - 16$ (b) $25x^2 - 36y^2$ (c) $p^2 - \frac{1}{49q^2}$

Solution:

$$\begin{aligned} \text{Here, } a^2 - 16 \\ &= a^2 - (4)^2 \\ &= (a - 4)(a + 4) \quad [\because a^2 - b^2 = (a - b)(a + b)] \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 25x^2 - 36y^2 \\ &= (5x)^2 - (6y)^2 \\ &= (5x + 6y)(5x - 6y) \quad [\because a^2 - b^2 = (a - b)(a + b)] \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad p^2 - \frac{1}{49q^2} \\ &= (p)^2 - \left(\frac{1}{7q}\right)^2 \\ &= \left(p - \frac{1}{7q}\right)\left(p + \frac{1}{7q}\right) \quad [\because a^2 - b^2 = (a - b)(a + b)] \end{aligned}$$

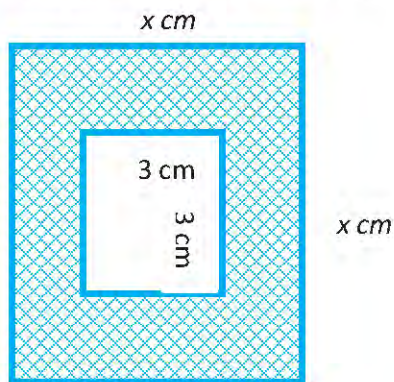
Exercise 10.1.2

1. Factorise the following using the formula of $a^2 - b^2$:

(a) $x^2 - 4$ (b) $a^2 - 4b^2$ (c) $9x^2 - y^2$
 (e) $5x^2 - 20y^2$ (g) $13a^2 - 17b^2$ (f) $25 - \frac{1}{9y^2}$
 (g) $121x^2 - \frac{1}{y^2}$ (h) $2p^2 - \frac{50}{q^2}$ (i) $72 - 2b^2$

- (j) $121 - 25y^2$ (k) $\frac{15}{a^2} - 60a^2$ (l) $81 - 64y^2$
- (m) $4x^3y - 81xy^3$ (n) $169 - 196z^2$ (o) $ab^3 - 9a^3b$
- (p) $\frac{49}{121}x^2 - \frac{64}{9}y^2$ (q) $zx^2 - zy^2$ (r) $(x - 2)^2 - 4$
- (s) $256 - \frac{x^2}{4}$ (t) $1 - \frac{81p^2}{121q^2}$ (u) $3(x - y)^2 - 12$
- (v) $9(x - 1)^2 - 16(x - 2)^2$

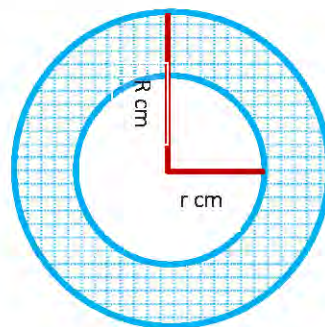
2. What is the area of shaded part in the given figure?



3. There is a pond of length 6 meters inside a garden of length x meter. What will be the area of garden excluding the pond?

4. Observe the figure given aside:

- (a) What is the area of shaded part?
(Where, the area of circle $= \pi r^2$)
- (b) If $R = 5 \text{ cm}$ and $r = 3 \text{ cm}$, find the area of shaded part.



Answer

1. (a) $(x+2)(x-2)$ (b) $(a+2b)(a-2b)$ (c) $(3x-y)(3x+y)$
 (c) $5(x+2y)(x-2y)$ (d) $13(a+3b)(a-3b)$ (e) $(5+\frac{1}{3y})(5-\frac{1}{3y})$
 (f) $(11x-\frac{1}{y})(11x+\frac{1}{y})$ (g) $2(p+\frac{5}{q})(p-\frac{5}{q})$
 (h) $2(6+b)(6-b)$ (i) $(11+5y)(11-5y)$
 (j) $15(\frac{1}{a}+2a)(\frac{1}{a}-2a)$ (k) $(9+8y)(9-8y)$
 (k) $xy(2x+9y)(2x-9y)$ (l) $(13+14z)(13-14z)$
 (m) $ab(b+3a)(b-3a)$ (n) $(\frac{7x}{11}-\frac{8y}{3})(\frac{7x}{11}+\frac{8y}{3})$
 (o) $z(x-y)(x+y)$ (p) $x(x+4)$
 (q) $(16+\frac{x}{2})(16-\frac{x}{2})$ (r) $(1+\frac{9p}{11q})(1-\frac{9p}{11q})$
 (s) $3(x-y+2)(x-y-2)$ (t) $-(7x+5)(x+11)$
2. $(x+3)(x-3)$
3. $(x+6)(x-6)$
4. $\pi(R+r)(R-r)$
5. 50.28 cm^2

10.1.3 Factorization of trinomial expressions reducible to perfect square

Activity 4

Discuss the following questions:

- (a) What is the expanded form of $(a+b)^2$?
- (b) What is the expanded form of $(a-b)^2$?

Here,

$$(a+b)^2 = a^2 + 2ab + b^2 \text{ and } (a-b)^2 = a^2 - 2ab + b^2$$

This can be expressed as

$$a^2 + 2ab + b^2 = (a+b)^2 = (a+b)(a+b)$$

$$\text{And } a^2 - 2ab + b^2 = (a-b)^2 = (a-b)(a-b)$$

- (a) An algebraic expression of the form $a^2 + 2ab + b^2$ can be written as $(a+b)^2$ and also $(a+b)(a+b)$ in the factorized form.

- (b) An algebraic expression of the form $a^2 - 2ab + b^2$ can be written as $(a - b)^2$ and also $(b - a)^2$ in the factorized form.

Example 1

What should be filled in the blank space to make the following expression perfect square?

(a) $x^2 + \dots + 16$ (b) $9a^2 - \dots + 49b^2$

Solution

(a) Here, $x^2 + \dots + 16$
 $= x^2 + \dots + (4)^2$ (Writing the first and third terms in square forms)
 $= (x)^2 + 2 \times x \times 4 + (4)^2$ [Writing in the form of $a^2 + 2ab + b^2$]
 $= (x + 4)^2$

$\therefore$ To make $x^2 + \dots + 16$ a perfect square, the blank space should be filled with $2 \times x \times 4 = 8x$.

(b) $9a^2 - \dots + 49b^2$

Solution

(a) Here, $9a^2 - \dots + 49b^2$
 $= (3a)^2 + \dots + (7b)^2$ (Writing the first and third terms in square forms)
 $= (3a + 7b)^2$ [Writing in the form of $a^2 + 2ab + b^2$]

$\therefore$ To make $9a^2 - \dots + 49b^2$ a perfect square, the blank space should be filled with $2 \times 3a \times 7b = 42ab$.

Example 2

Factorize:

(a) $4x^2 - 12x + 9$ (b) $36a^2 - 48ab + 16b^2$

Solution

(a) Here, $4x^2 - 12x + 9$
 $= (2x)^2 + 2 \times 2x \times (-3) + (-3)^2$ [Comparing with $a^2 + 2ab + b^2 = (a + b)^2$]
 $= (2x - 3)^2$
 $= (2x - 3)(2x - 3)$

(b) $36a^2 - 48ab + 16b^2$

Solution

$$\begin{aligned}
 &= 4(9a^2 - 12ab + 4b^2) \\
 &= 4\{(3a)^2 - 2 \times 3a \times 2b + (2b)^2\} \\
 &= 4(3a - 2b)^2 \quad [\text{Comparing with } a^2 - 2ab + b^2 = (a - b)^2] \\
 &= 4(3a - 2b)(3a - 2b)
 \end{aligned}$$

Example 3

Factorize:

(a) $16x^2 + 40xy + 25y^2$

Solution

$$\begin{aligned}
 &\text{Here, } 16x^2 + 40xy + 25y^2 \\
 &= (4x)^2 + 2 \cdot 4x \cdot 5y + (5y)^2 \\
 &= (4x + 5y)^2 \\
 &= (4x + 5y)(4x + 5y) \\
 &= (4x + y + 5)(4x + y + 5)
 \end{aligned}$$

Exercise 10.1.3

1. Complete the perfect square by filling the appropriate term in the blank space:

(a) $x^2 + \dots + 16$	(b) $4a^2 + \dots + y^2$	(c) $p^2 - \dots + 36$
(d) $9a^2 - \dots + 16b^2$	(e) $25p^2 - \dots + 49q^2$	(f) $p^2 + \dots + \frac{4}{p^2}$
(g) $225x^2 - \dots + 64y^2$	(h) $1 + \dots + 36y^2$	(i) $p^2 + \dots + \frac{1}{p^2}$

2. Factorize:

(a) $a^2 + 12a + 36$	(b) $y^2 + 14y + 49$	(c) $p^2 + 22p + 121$
(d) $4a^2 + 20a + 25$	(e) $9r^2 + 60r + 100$	(f) $36x^2 + 84x + 49$
(g) $x^2 - 8x + 16$	(h) $a^2 - 18a + 81$	(i) $p^2 - 26p + 169$

$$\begin{array}{lll}
 \text{(j)} \ 9a^2 - 30a + 25 & \text{(k)} \ 25y^2 - 60y + 36 & \text{(l)} \ 49r^2 - 70r + 25 \\
 \text{(m)} \ 4p^2 + 24pq + 36q^2 & \text{(n)} \ 9a^2 + 42ab + 49b^2 & \text{(o)} \ \frac{x^2}{16} - xy + 4y^2 \\
 \text{(p)} \ 25a^2 - 40ab + 16b^2 & \text{(q)} \ 49q^2 - 70q + 25r^2 & \text{(r)} \ 25x^2 - 2xy - \frac{y^2}{25}
 \end{array}$$

3. Factorize:

$$\begin{array}{ll}
 \text{(a)} \ a^2 + 12a + 36 - b^2 & \text{(b)} \ y^2 + 16y + 64 - z^2 \\
 \text{(c)} \ p^2 + 26p + 169 - 9q^2 & \text{(d)} \ 4a^2 - b^2 + 20b - 100 \\
 \text{(e)} \ 9r^2 - s^2 - 6s - 9
 \end{array}$$

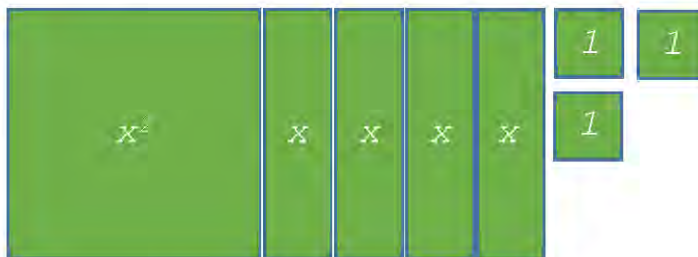
Answer:

1. (a) $8x$ (b) $4ay$ (c) $12p$
 (d) $24ab$ (e) $70pq$ (f) 4
 (g) $240xy$ (h) $12y$ (i) 2
2. (a) $(a + 6)^2$ (b) $(y + 7)^2$ (c) $(p + 11)^2$
 (d) $(2a + 5)^2$ (e) $(3r + 10)^2$ (f) $(6x + 7)^2$
 (g) $(x - 4)^2$ (h) $(a - 9)^2$ (i) $(p - 13)^2$
 (j) $(3a - 5)^2$ (k) $(5y - 6)^2$ (l) $(7r - 5)^2$
 (m) $(2p + 6q)^2$ (n) $(3a + 7b)^2$ (o) $(\frac{x}{4} - 2y)^2$
 (p) $(5a - 4b)^2$ (q) $(7q - 5r)^2$ (r) $(5x - \frac{y}{5})^2$
3. (a) $(a - b + 6)(a - b + 6)$ (b) $(y - z + 8)(y - z + 8)$
 (c) $(p + 3q + 13)(p - 3q + 13)$ (d) $(2a - b - 10)(2a - b + 10)$
 (e) $(3r - s + 3)(3r - s - 3)$

10.1.4 Factorization of the expressions of the form $ax^2 + bx + c$

Activity 5

How can the given algebraic tiles be arranged to construct a rectangle?
Discuss in your group.



In the figure, the number of x^2 , x and 1 (unit square) are 4, 3 and 1 respectively.
In the rectangle constructed by arranging the tiles,

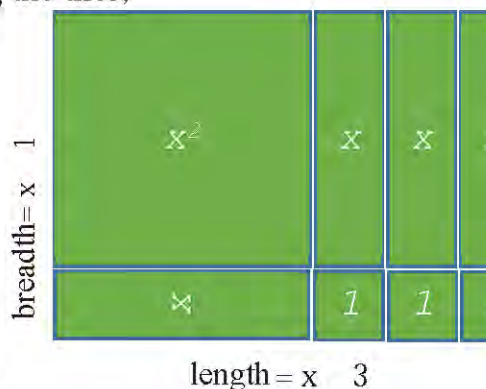
$$\text{Total area} = x^2 + 4x + 3$$

$$\text{Length} = x^2 + 4x + 3$$

$$\text{Breadth} = x + 1$$

Therefore, the factorization of

$$x^2 + 4x + 3 \text{ is } x^2 + 4x + 3 = (x + 1)(x + 3)$$



$$\text{Here, } (x + 1)(x + 3) = x(x + 1) + 3(x + 1)$$

$$= x^2 + x + 3x + 3$$

$$= x^2 + (1 + 3)x + 3$$

$$= x^2 + 4x + 3$$

Writing the process in reverse way,

$$x^2 + 4x + 3$$

$$= x^2 + (1 + 3)x + 3$$

$$= x^2 + x + 3x + 3$$

$$= x(x + 1) + 3(x + 1)$$

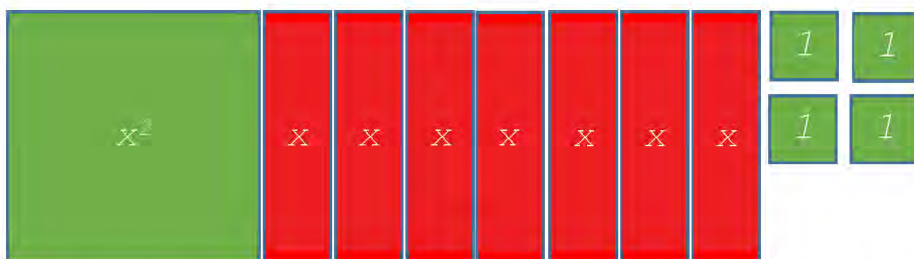
$$= (x + 1)(x + 3)$$

The coefficient of x in the second term of $x^2 + 4x + 3$ which can be written as $4 = (1+3)$. How to decide it?

While taking the product and sum, $1 \times 3 = 3$ and $1 + 3 = 4$. So, the coefficient 4 is divided into two parts so that their sum becomes 4 and product becomes 3.

Activity 6

Perform an activity to construct a rectangle using the following algebraic tiles. Discuss in your group and find the length and breadth of the rectangle so formed.



Here, the number of positive x^2 is 1, the number of negative x is 7 and the number of positive 1 (unit square) is 6 in the figure.

$$\text{Total} = x^2 - 7x + 6$$

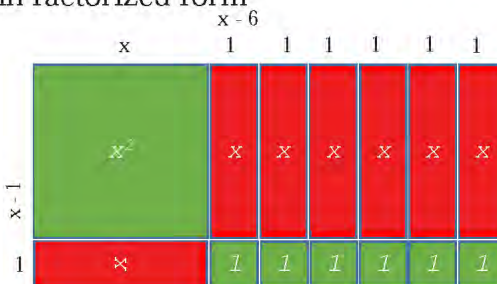
Rearranging the tiles as shown in the figure,

$$\text{Total area of rectangles} = (x-6)(x-1)$$

Therefore, $x^2 - 7x + 6 = (x-6)(x-1)$ in factorized form

Now,

$$\begin{aligned} & x^2 - 7x + 6 \\ &= x^2 - (6+1)x + 6 \\ &= x^2 - 6x - x + 6 \\ &= x(x-6) - 1(x-6) \\ &= (x-6)(x-1) \end{aligned}$$



Activity 7

Perform an activity to construct a rectangle using the following algebraic tiles. Discuss in your group and factorize the expression so obtained.



The number of positive x^2 is 1, the number of positive x is 5 and the number of negative 1 (unit square) is 6 in the figure.

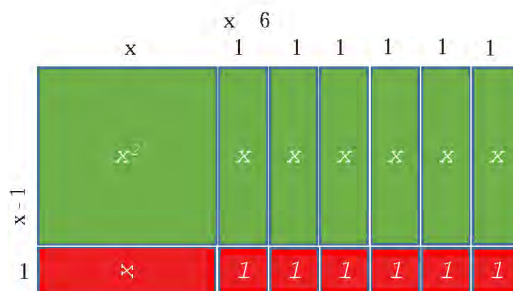
In this case, the rearrangement of the tiles cannot form a rectangular shape. So, the tiles of one positive x and one negative x are added.

The total area of rectangles = $x^2 + 5x - 6$

After rearrangement, the total area of rectangles = $(x+6)(x-1)$

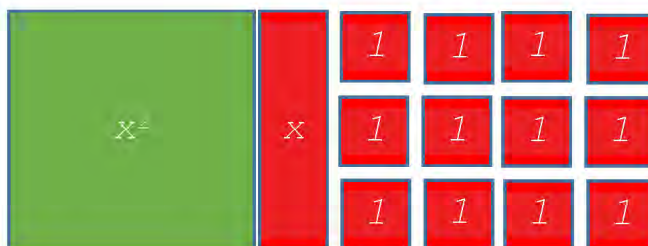
Therefore, $x^2 + 5x - 6$ in factorized form

$$\begin{aligned}
 x^2 + 5x - 6 &= (x + 6)(x - 1) \\
 x^2 + 5x - 6 \\
 &= x^2 + (6 - 1)x - 6 \\
 &= x^2 + 6x - x - 6 \\
 &= x(x + 6) - 1(x + 6) \\
 &= (x + 6)(x - 1)
 \end{aligned}$$



Activity 8

Perform an activity to construct a rectangle using the following algebraic tiles. Discuss in your group and factorize the expression so obtained.



The number of positive x^2 is 1, the number of negative x is 1 and the number of negative 1 (unit square) is 12 in the figure.

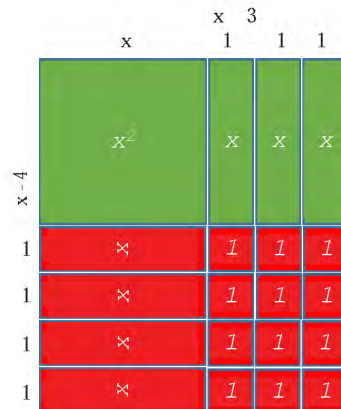
In this case, the rearrangement of the tiles cannot form a rectangular shape. So, the tiles of 3 positive x and 3 negative x are added. The total quantity remains same by the addition of two equal values of opposite sign.

The total area of rectangles = $x^2 - x - 12$

Therefore, $x^2 - x - 12$ in factorized form

$$x^2 - x - 12 = (x - 4)(x + 3)$$

$$\begin{aligned} & x^2 - x - 12 \\ &= x^2 - (4 - 3)x - 12 \\ &= x^2 - 4x + 3x - 12 \\ &= x(x - 4) + 3(x - 4) \\ &= (x - 4)(x + 3) \end{aligned}$$



The algebraic expression of the form $ax^2 + bx + c$ can be factorized as the following way.

- At first find the product of a and c .
- Find two numbers whose product equal to ac and sum or difference equal to b (sum equal to b if c is positive and difference equal to $-b$, if c is negative.)

For example:

- In the expression $x^2 + 5x + 6$,

$$a = 1, b = 5 \text{ and } c = 6$$

The product of 1 and 6 is 6 and $c=6$ is positive. So, we choose two numbers 3 and 2 so that their product is 6 and sum is 5.

- In the expression $x^2 + 5x - 6$,

$$a = 1, b = 5 \text{ and } c = -6$$

The product of 1 and 6 is 6 and $c=-6$ is negative. So, we choose two numbers 6 and 1 so that their product is 6 and difference is 5.

- Take a common factor from first two terms and also from last two terms. Again, take a common factor from the terms thus obtained. Then, express in factorized form.

Example 1

Find the two numbers whose product is P and sum is S.

(a) $P = 12, S = 7$

(b) $P = 24, S = 11$

Solution

Here,

	Product (P)	Sum (S)	Required Numbers
$p = 12$	1 12	1 12 = 13	3 and 4
	2 6	2 6 = 8	
	3 4	3 4 = 7	
$p = 24$	1 24	1 24 = 25	3 and 8
	2 12	2 12 = 14	
	3 8	3 8 = 11	
	4 6	4 + 6 = 10	

Example 2

Factorize:

(a) $x^2 - 5x + 6$ (b) $x^2 + 6x + 5$

Solution

$$\begin{aligned}
 &\text{Here, } x^2 - 5x + 6 \\
 &= x^2 - (2 + 3)x + 6 \quad (\text{Two numbers having product equal to 6 and sum equal to 5 are 2 and 3}) \\
 &= x^2 - 2x - 3x + 6 \\
 &= x(x - 2) - 3(x - 2) \\
 &= (x - 2)(x - 3)
 \end{aligned}$$

$$\begin{aligned}
 &\text{(b) } x^2 + 6x + 5 \\
 &= x^2 + (1 + 5)x + 5 \quad (\text{Two numbers having product equal to 5 and sum equal to 6 are 1 and 5}) \\
 &= x(x + 1) + 5(x + 1) \\
 &= (x + 1)(x + 5)
 \end{aligned}$$

Example 3

Find the two numbers whose product is P and difference is D.

- (a) $P = 12, D = 1$ (b) $P = 24, D = 10$

Solution

Here,

	Product (P)	Difference (D)	Required numbers
$p = 12$	1 12	$12 - 1 = 11$	4 and 3
	2 6	$6 - 2 = 4$	
	3 4	$4 - 3 = 1$	
$p = 24$	1 24	$24 - 1 = 23$	12 and 2
	2 12	$12 - 2 = 10$	
	3 8	$8 - 3 = 5$	
	4 6	$6 - 4 = 2$	

Example 4

Factorize:

- (a) $x^2 - 5x - 6$ (b) $x^2 - 6x + 16$

Solution

Here, $x^2 - 5x - 6$ (Two numbers having product -6 and difference 5 are 6 and 1)

$$= x^2 - 6x + x - 6$$

$$= x(x - 6) + 1(x - 6)$$

$$= (x - 6)(x + 1)$$

- (b) $x^2 - 6x + 16$

$= x^2 - (8 - 2)x + 16$ (Two numbers having product 16 and difference 6 are 8 and 2)

$$= x^2 - 8x + 2x + 16$$

$$= x(x - 8) + 2(x - 8)$$

$$= (x - 8)(x + 2)$$

Example 3

Factorize

(a) $3x^2 - 11x + 6$ (b) $2x^2 - 5x + 18$

Solution

$$\begin{aligned}\text{Here, } 3x^2 - 11x + 6 \\&= 3x^2 - (9 - 2)x + 6 \\&= 3x^2 - 9x + 2x + 6 \\&= 3x(x - 3) + 2(x - 3) \\&= (x - 3)(3x + 2)\end{aligned}$$

Two numbers having product 18 and sum 11 are 9 and 2

(b) $2x^2 - 5x + 18$

Solution

$$\begin{aligned}\text{Here, } 2x^2 - 5x + 18 \\&= 2x^2 - (9 - 4)x + 18 \\&= 2x^2 - 9x + 4x + 18 \\&= x(2x - 9) + 2(2x - 9) \\&= (2x - 9)(x + 2)\end{aligned}$$

Two numbers having product 36 and difference 5 are 9 and 4

Exercise 10.1.4

1. Find the two numbers whose product is P and sum is S.

- | | | |
|----------------------|----------------------|-----------------------|
| (a) $P = 2, S = 3$ | (b) $P = 3, S = 4$ | (c) $P = 8, S = 6$ |
| (d) $P = 15, S = 8$ | (e) $P = 10, S = 7$ | (f) $P = 20, S = 9$ |
| (g) $P = 45, S = 18$ | (h) $P = 56, S = 18$ | (i) $P = 160, S = 28$ |

2. Find the two numbers whose product is P and difference is S.

- | | | |
|---------------------|----------------------|----------------------|
| (a) $P = 2, D = 1$ | (b) $P = 4, D = 3$ | (c) $P = 8, D = 2$ |
| (d) $P = 16, D = 6$ | (e) $P = 20, D = 1$ | (f) $P = 20, D = 8$ |
| (g) $P = 36, D = 5$ | (h) $P = 80, D = 16$ | (i) $P = 96, D = 10$ |

3. Factorize:

(a) $x^2 + 5x + 4$

(b) $x^2 + 3x + 2$

(c) $x^2 - 5x + 6$

(d) $y^2 + 5y + 6$

(e) $x^2 + 7x + 12$

(f) $a^2 - 3a + 2$

(g) $a^2 - 6a + 8$

(h) $b^2 - 5b + 6$

(i) $b^2 + 13b + 42$

(j) $b^2 - 13b + 40$

(k) $z^2 - 13z + 36$

(l) $x^2 - 15x + 56$

(m) $x^2 - 15x + 54$

(n) $z^2 + 15z + 44$

(o) $b^2 - 12b + 36$

(p) $b^2 + 15b + 56$

(q) $z^2 - 12z + 27$

(r) $x^2 - 23x + 102$

(s) $(a - b)^2 + 11(a - b) + 18$

(t) $(x - y)^2 - 15(x - y) + 36$

4. Factorize:

(a) $x^2 + 4x - 21$

(b) $x^2 - x - 20$

(c) $x^2 + 3x - 28$

(d) $y^2 - 6y - 27$

(e) $x^2 + 7x - 18$

(f) $a^2 + 10a - 39$

(g) $a^2 - a - 132$

(h) $b^2 - 8b - 65$

(i) $b^2 + 3b - 108$

(j) $b^2 - 7b - 120$

(k) $z^2 - 29z - 132$

(l) $x^2 - xy - 240y^2$

(m) $35 - 2x - x^2$

(n) $96 - 4z - z^2$

(o) $72 + b - b^2$

(p) $(a - b)^2 + 5(a - b) - 36$

(q) $(x - y)^2 - 9(x - y) - 112$

5. Factorize:

(a) $3x^2 + 5x + 2$

(b) $3x^2 - 4x + 1$

(c) $7x^2 - 30x + 8$

(d) $4a^2 - 8a + 3$

(e) $15p^2 - 13p + 2$

(f) $12a^2 - 32a + 5$

(g) $5x^2 - 14x - 3$

(h) $10x^2 - 3x - 1$

(i) $15p^2 - 13p + 2$

(j) $6b^2 - 4b - 10$

(k) $21x^2 + 25x + 4$

(l) $12a^2 + 28ab - 5b^2$

(m) $16a^2 + 24ab + 9b^2$

(n) $6x^2 - xy - 7y^2$

(o) $3a^2 - ab - 10b^2$

(p) $6p^2q + 30pq + 36q$

(q) $6a^2 + 35ab - 6b^2$

(r) $6a^2 - 5ab - 6b^2$

(s) $4 + 17x - 15x^2$

(t) $6 - 13a + 6a^2$

(u) $28 - 31b - 5b^2$

Project Work

- Factorize $x^2 + 4x + 16$ using algebraic tiles and paste on a chart paper. Also, present your work in the classroom.
- Factorize $x^2 - 10x + 21$ using algebraic tiles and paste on a chart paper. Also, present your work in the classroom.
- Factorize $x^2 + 2x + 15$ using algebraic tiles and paste on a chart paper. Also, present your work in the classroom.
- Factorize $2x^2 + x + 3$ using algebraic tiles and paste on a chart paper. Also, present your work in the classroom.

Answer

- (a) 1 and 2 (b) 1 and 3 (c) 4 and 2 (d) 3 and 5 (e) 2 and 5
(f) 4 and 5 (g) 3 and 15 (h) 14 and 4 (i) 8 and 20
- (a) 2 and 1 (b) 4 and 1 (c) 4 and 2 (d) 8 and 2 (e) 5 and 4
(f) 10 and 2 (g) 9 and 4 (h) 20 and 4 (i) 16 and 6
- (a) $(x + 1)(x + 4)$ (b) $(x + 1)(x + 2)$ (c) $(x - 2)(x - 3)$
(d) $(y + 2)(y + 3)$ (e) $(x + 3)(x + 4)$ (f) $(a - 1)(a - 2)$
(g) $(a - 4)(a - 2)$ (h) $(b - 2)(b - 3)$ (i) $(b + 6)(b + 7)$
(j) $(b - 8)(b - 5)$ (k) $(z - 4)(z - 9)$ (l) $(x - 7)(x - 8)$
(m) $(x - 6)(x - 9)$ (n) $(z + 11)(z + 4)$ (o) $(b - 6)(b - 6)$
(p) $(b + 7)(b + 8)$ (q) $(z - 3)(z - 9)$ (r) $(x - 6)(x - 17)$
(s) $(a - b + 2)(a - b + 9)$ (t) $(x - y - 3)(x - y - 12)$
- (a) $(x + 7)(x - 3)$ (b) $(x + 5)(x - 4)$ (c) $(x + 7)(x - 4)$
(d) $(y - 9)(y + 3)$ (e) $(x + 9)(x - 2)$ (f) $(a - 13)(x + 3)$
(g) $(a - 12)(a + 11)$ (h) $(b - 13)(b + 5)$ (i) $(b + 12)(b - 9)$
(j) $(b + 8)(b - 15)$ (k) $(z - 33)(z + 4)$ (l) $(x + 16)(x - 15)$
(m) $(5 - x)(x + 7)$ (n) $(8 - z)(z + 12)$ (o) $(8 + b)(9 - b)$
(p) $(a - b - 4)(a - b + 9)$ (q) $(x - y - 16)(x - y + 7)$

5. (a) $(3x + 2)(x + 1)$ (b) $(3x - 1)(x - 1)$ (c) $(x - 4)(7x - 2)$
 (d) $(2a - 3)(2a - 1)$ (e) $(5p - 1)(3p - 2)$ (f) $(2a - 5)(6a - 1)$
 (g) $(5x + 1)(x - 3)$ (h) $(5x + 1)(2x - 1)$ (i) $(3p - 2)(5p - 1)$
 (j) $(3b - 5)(2b + 2)$ (k) $(21x + 4)(x + 1)$ (l) $(2a + 5b)(6a - b)$
 (m) $(4a + 3b)^2$ (n) $(6x + 7y)(x - y)$ (o) $(3a + 5b)(a - 2b)$
 (p) $6q(p + 2)(p + 3)$ (q) $(6a - b)(a + 6b)$ (r) $(3a + 2b)(2a - 3b)$
 (s) $(4 - 3x)(1 + 5x)$ (t) $(2 - 3a)(3 - 2a)$ (u) $(7 + b)(4 - 5b)$

10.2 Highest common factor of algebraic expressions

Activity 9

We have already learnt to find the highest common factor (HCF) of given numbers by two different methods at class 7. Discuss the two methods in your group and find the HCF of 12 and 18.

H.C.F. of 12 and 18 by prime factorization method:

$$12 = 2 \times 2 \times 3$$

$$18 = 2 \times 3 \times 3$$

$$\text{H.C.F.} = \text{common factor} = 2 \times 3 = 6$$

Again, H.C.F. of 12 and 18 by division method:

$$\begin{array}{r} 12 \overline{) 18(1} \\ \underline{-12} \\ 6 \overline{) 18(3} \\ \underline{-18} \\ 0 \end{array}$$

Hence, HCF of 12 and 18 is 6.

Activity 10

Discuss the following questions to find the HCF of x^3 and x^5 .

- What are the factors of x^3 ?
- What are the factors of x^5 ?
- What are the common factors of x^3 and x^5 ?
- Which is the greatest common factor of x^3 and x^5 ?
- What is the greatest common factor of given expressions called?

Here, the factors of x^3 are 1, x , x^2 , x^3

Factors of x^5 are 1, x , x^2 , x^3 , x^4 , x^5 .

The common factors of x^3 and x^5 are 1, x , x^2 , x^3 .

The greatest common factor of x^3 and x^5 is x^3 .

The greatest common factor of given expressions is called highest common factor (H.F)

Therefore, the HCF of x^3 and x^5 is x^3 .

HCF of x^3 and x^5 by factorization method

$$x^3 = x \cdot x \cdot x$$

$$x^5 = x \cdot x \cdot x \cdot x \cdot x$$

$$\text{Common factor} = x \cdot x \cdot x = x^3$$

$$\therefore \text{HCF} = x^3$$

The greatest common factor of given algebraic expressions is called Highest Common Factor (HCF). In the factorization method, each expression is written in the factorized form and the HCF is obtained by multiplying their common factors.

Example 1

Find the HCF of $9x^2y^3$ and $15xy^2$

Solution

Here,

$$\text{First expression} = 9x^2y^3$$

$$= 3 \times 3 \times x \cdot x \cdot y \cdot y \cdot y$$

$$\text{Second expression} = 15xy^2$$

$$= 3 \times 5 \times x \cdot y \cdot y$$

$$\therefore \text{HCF} = \text{Common factor} = 3 \times x \cdot y \cdot y = 3xy^2$$

Example 2

Find the HCF of $x^2 - 2xy + y^2$ and $x^2 - y^2$

Solution

$$\begin{aligned}\text{Here, first expression} &= x^2 - 2xy + y^2 \\ &= x^2 - xy - xy + y^2 \\ &= (x - y)(x - y)\end{aligned}$$

$$\begin{aligned}\text{Second expression} &= x^2 - y^2 \\ &= (x - y)(x + y)\end{aligned}$$

$$\therefore \text{HCF} = \text{Common factor} = (x - y)$$

Example 3

Find the HCF of $x^2 - 6x + 8$, $x^2 - 4$ and $x^2 - 4x + 4$

Solution

$$\begin{aligned}\text{Here, first expression} &= x^2 - 6x + 8 \\ &= x^2 - 4x - 2x + 8 \\ &= x(x - 4) - 2(x - 4) \\ &= (x - 4)(x - 2)\end{aligned}$$

$$\begin{aligned}\text{Second expression} &= x^2 - 4 \\ &= x^2 - 2^2 \\ &= (x - 2)(x + 2)\end{aligned}$$

$$\begin{aligned}\text{Third expression} &= x^2 - 4x + 4 \\ &= x^2 - 2 \cdot 2 + 2^2 \\ &= (x - 2)^2 \\ &= (x - 2)(x - 2)\end{aligned}$$

$$\therefore \text{H.C.F.} = \text{the common factor of all three expressions} = (x - 2)$$

Exercise 10.2.1

1. Find the highest common factor (HCF):

a) $4x^2y$ and xy^2

c) $25x^2y^3$ and $15xy^2$

e) a^2b , b^2ac and c^2ab

b) $x^2 - 4$ and $3x + 6$

d) $x^2 - y^2$ and $xy - y^2$

f) p^2q , q^2p and $2p^2 - 2pq$

- g) $3a - b$ and $15a + 5b$ h) $x^2 + 4x + 4$ and $x^2 - 4$
 i) $x^2 - 11x + 30$ and $x^2 - 36$ j) $x^2 - 9$ and $x^2 - 6x + 9$
 k) $x^2 + 16x + 60$ and $x^2 + 20x + 100$
 l) $a^2 + 5a + 6$ and $a^2 - a - 6$ m) $x^2 - 11x + 10$ and $x^3 - x$
 n) $a^2 - 2ab - b^2$ and $a^4 - b^4$ o) $x^2 - x^2y^2$ and $y^2 - y^4$
 p) $x^2 - a^2$ and $x^2 - 2ax - a^2$ q) $x^2 - y^2$ and $x^2y - y^2x$
 r) $a^3 - ab^2$ and $a^2b - ab^2$ s) $x^2 + 5x + 6$ and $x^2 - x - 6$
 t) $a^2 + 2a - 3$ and $a^2 - 3a + 2$ u) $x^2 + 7x + 10$ and $x^2 - x - 6$
 v) $x^2 - 7x + 12$ and $3x^2 - 27$ w) $a^2 - 3a + 2$ and $2a^2 - 9a + 10$
 x) $a^2 + 5a + 6$ and $a^2 - 4$

Answer

- a) xy b) $5xy^2$ c) abc d) $x + 2$ e) $x - y$
 f) $p(p - q)$ g) $3a - b$ h) $x + 2$ i) $x - 6$ j) $x - 3$
 k) $x + 10$ l) $a + 3$ m) $x - 1$ n) $a - b$ o) $1 - y^2$
 p) $x - a$ q) $x - y$ r) $a(a - b)$ s) $x + 3$ t) $a - 1$
 u) $x + 2$ v) $x - 3$ w) $a - 2$ x) $a + 2$

10.3 LCM of algebraic expressions

Activity 11

We have already learnt to find the lowest common multiple (LCM) of given algebraic expressions by two different methods at class 7. Discuss the two methods in your group and find the LCM. of 12 and 18.

Here, prime factorization method and division method are given to find the LCM.

LCM of 12 and 18 by prime factorization method:

$$12 = 2 \times 2 \times 3$$

$$18 = 2 \times 3 \times 3$$

LCM = common factor remaining factor

$$= 2 \times 3 \times 2 \times 3 = 36$$

LCM of 12 and 18 by division method

$$\begin{array}{r|l} 2 & 12, 18 \\ \hline 3 & 6, 9 \\ \hline & 2, 3 \end{array}$$

Hence, LCM of 12 and 18 $= 2 \times 3 \times 2 \times 3 = 36$

Activity 12

Discuss the following questions in your group to find the LCM of x^2 and x^3

- (a) Write the factors of x^2 .
- (b) Write the factors of x^3 .
- (c) What are the common factors of x^2 and x^3 ?
- (d) Write the remaining factors of x^2 and x^3 ?

Here,

$$x^2 = x \times x$$

$$x^3 = x \times x \times x$$

Common factors $= x \times x = x^2$

Remaining factor $= x$

$\therefore$ LCM = Common factors $\times$ Remaining factors $= x^2 \times x = x^3$

The lowest common multiple (LCM) of given algebraic expressions is the smallest expressions which are exactly divisible by the given expressions. It is obtained by multiplying the common factors and remaining factors of the expressions.

Example 1

Find the LCM of $3xy^2$ and $6x^2y$

Solution

Here, First expression = $3xy^2$

$$= 3 \times x \times y \times y$$

Second expression = $6x^2y$

$$= 2 \times 3 \times x \times x \times y$$

Common factor = $3 \times x \times y = 3xy$

Remaining factor = $2 \times x \times y = 2xy$

$\therefore$ LCM = common factor $\times$ remaining factor = $3xy \times 2xy = 6x^2y^2$

Example 2

Find the LCM of $3x^3 - 15x^2$ and $2x^3 - 50x$.

Solution

Here, First expression = $3x^3 - 15x^2$

$$= 3x^2(x - 5)$$

Second expression = $2x^3 - 50x$

$$= 2x(x^2 - 25)$$

$$= 2x(x^2 - 5^2)$$

$$= 2x(x + 5)(x - 5)$$

Common factor = $x(x - 5)$

Remaining factor = $3x \times 2(x + 5) = 6x(x + 5)$

$$= (x - 2)(x - 2)$$

$\therefore$ LCM = common factor $\times$ remaining factor

$$= x(x - 5) \times 6x(x + 5)$$

$$= 6x^2(x - 5)(x + 5)$$

Example 3

Find the LCM of $x^2 - x - 20$ and $x^2 - 25$.

Solution

$$\begin{aligned}\text{Here, First expression} &= x^2 - x - 20 \\ &= x^2 + 5x - 4x - 20 \\ &= x(x + 5) - 4(x + 5) \\ &= (x + 5)(x - 4)\end{aligned}$$

$$\begin{aligned}\text{Second expression} &= x^2 - 25 \\ &= x^2 - 5^2 \\ &= (x + 5)(x - 5)\end{aligned}$$

$$\text{Common factor} = (x + 5)$$

$$\text{Remaining factor} = (x - 4)(x - 5)$$

$$\begin{aligned}\therefore \text{LCM} &= \text{Common factor} \times \text{Remaining factor} \\ &= (x + 5)(x - 4)(x - 5)\end{aligned}$$

Exercise 10.3.1

1. Find the lowest common multiple (LCM):

- a) $2x$ and 4 b) $3x^2y$ and $6xy^2$ c) $5xy$ and $10y^2$
d) $6a^2b$ and $6ab^2$ e) $2a$ and $2a + 4$ f) $3x^2 - 3$ and $x^2 - 1$
g) $x - y$ and $x^2 - xy$ h) $x^2 + 4x + 4$ and $x^2 + 2x$ i) $5x - 20$ and $x^2 - 16$
j) $p^2 - pq$ and $pq - q^2$ k) $3x^3 + 15x^2$ and $2x^3 - 50x$
l) $x^3 - 4x$ and $x^2 + 7x + 10$ m) $3x^2 + 7x + 2$ and $2x^2 + 3x - 2$
n) $y^2 + 2y - 48$ and $y^2 - 9y + 18$ o) $a^2 + 4ab + 4b^2$ and $a^2 - 4b^2$
p) $9x^2 - 24xy + 16y^2$ and $3x^2 - xy - 4y^2$ q) $a^2 - 1$ and $a^2 - a - 2$
r) $x^2 - 4$ and $x^2 + 3x + 2$ s) $x^2 - x - 6$ and $x^2 + 2x - 3$
t) $4x^2 + 12xy + 9y^2$ and $4x^2 - 12xy + 9y^2$
u) $6x^3 + 5x^2 - 6x$, and $3x^3 - 5x^2 + 2x$ v) $x^3 - x^2 - 42x$ and $x^4 + 4x^3 - 12x^2$

Answer

1. a) $4x$ b) $6x^2y^2$ c) $10xy^2$ d) $6a^2b^2$ e) $2a(a+2)$
f) $3(x^2-1)$ g) $x(x-y)$ h) $x(x+2)^2$ i) $5(x^2-16)$ j) $pq(p-q)$
k) $6x^2(x^2-25)$ l) $x(x^2-4)(x+5)$ m) $(x+2)(3x+1)(2x-1)$
n) $(y-6)(y-3)(y-8)$ o) $(a-2b)(a+2b)^2$
p) $(3x-4y)^2(x+y)$ q) $(a^2-1)(a+2)$
r) $(x+1)(x^2-4)$ s) $(x-1)(x-2)(x+3)$
t) $(2x-3y)^2(2x+3y)^2$ u) $x(2x+3)(3x-2)(x-1)$
v) $x^2(x-2)(x+6)(x-7)$

11.0 Review

Discuss the given questions in group:

- What is fraction?
- Write down the numerator and denominator of the given fraction.
 $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{5}{12}$
- What is an algebraic expression? Provide an example of algebraic expression.
- If the area of a square field is $x^2 - 2xy + y^2$, what is the length of the square field?

11.1 Algebraic Fraction

Activity 1

Discuss the given question in group:

What kinds of numbers are 3, 5, $\frac{2}{3}$, $\frac{6}{7}$?

What are in these numerator and denominator of $\frac{a}{b}$, $\frac{x}{x-1}$?

What do we call for the fraction in which an algebraic expression in both the numerator and denominator is used?

Here, 3, 5, $\frac{2}{3}$, $\frac{6}{7}$ are rational number. Similarly, $\frac{a}{b}$, $\frac{x}{x-1}$ are also rational numbers, whose numerator and denominator contains algebraic expression.

A fraction whose numerator and denominator are algebraic expressions is called algebraic fraction.

Example 2

Simplify:

(a) $\frac{x^3 - x}{x^2 - 1}$

Solution

$$\text{Here } \frac{x^3 - x}{x^2 - 1} = \frac{x(x^2 - 1)}{x(x - 1)}$$

Eliminating common factor x .

$$= \frac{(x - 1)(x + 1)}{(x - 1)}$$

$$[a^2 - b^2 = (a - b)(a + b)]$$

$$= (x + 1)$$

(Eliminating common factor $(x - 1)$)

(b) $\frac{x^2 - 5x + 6}{x^2 - 2x}$

Solution

$$\text{Here } \frac{x^2 - 5x + 6}{x^2 - 2x}$$

$$= \frac{(x - 2)(x - 3)}{x(x - 2)}$$

$$= \frac{(x - 3)}{x}$$

$$\begin{aligned} x^2 - 5x + 6 &= x^2 - 3x - 2x + 6 \\ &= x(x - 3) - 2(x - 3) \\ &= (x - 3)(x - 2) \end{aligned}$$

Exercise 11.1

1. Simplify:

$$(a) \frac{3a^2}{4a^3}$$

$$(b) \frac{5x^2y}{10xy^2}$$

$$(c) \frac{a^2+2ab+b^2}{a^2-b^2}$$

$$(d) \frac{5a^3-45a}{4a^2-12a}$$

$$(e) \frac{(x-3)^3}{2x-6}$$

$$(f) \frac{x^2+6x+9}{x^2-9}$$

$$(g) \frac{a^2+6a+8}{a^2-16}$$

$$(h) \frac{x^2+x-12}{x^2-x-6}$$

$$(i) \frac{(2x+3)^2}{4x^2-9}$$

$$(j) \frac{x^2+2x-15}{x^2+9x+20}$$

$$(k) \frac{x^2+5x+6}{(x+3)^2}$$

$$(l) \frac{x^2-9x+18}{x^2-7x+6}$$

$$(m) \frac{x^2-1}{x^2-6x+5}$$

$$(n) \frac{3xyz}{3x^2-12x}$$

$$(a) \frac{x^2-4x+4}{x-2}$$

Answer

1. (a) $\frac{3}{4a}$ (b) $\frac{x}{2y}$ (c) $\frac{a+b}{a-b}$ (d) $\frac{5(a+3)}{4}$ (e) $\frac{(x-3)^3}{2}$
- (f) $\frac{x+3}{x-3}$ (g) $\frac{a+2}{a-4}$ (h) $\frac{x+4}{x+2}$ (i) $\frac{2x+3}{2x-3}$ (j) $\frac{x-3}{x+4}$
- (k) $\frac{x+2}{x+3}$ (l) $\frac{x-3}{x-1}$ (m) $\frac{x+1}{x-5}$ (n) $\frac{yz}{x-4}$ (o) $x-2$

11.2 Addition and subtraction of Algebraic Fraction having same Denominator

Activity 2

Simplify the given algebraic expression. Discuss with your friend the process to be followed for simplification.

$$(a) \frac{2}{3} + \frac{4}{3} \quad (b) \frac{a}{x} + \frac{b}{x} \quad (c) \frac{x}{x-y} + \frac{y}{x-y}$$

When simplifying fractions, we should check whether the given fractions are like or unlike. If they are like fractions we write same denominator and simplify. If they are unlike fractions, we make them like and simplify.

Like :

$$(a) \frac{2}{3} + \frac{4}{3} \text{ These are fractions of the same denominator. So,}$$

$$\frac{2}{3} + \frac{4}{3} \quad [\text{Adding process done by keeping one denominator}]$$

$$= \frac{2+4}{3}$$

$$= \frac{6}{3} = 2$$

$$(b) \frac{a}{x} + \frac{b}{x}$$

$$\frac{a}{x} + \frac{b}{x} \text{ These are fractions of the same denominator. So,}$$

$$= \frac{a+b}{x} \quad [\text{Adding numerator by keeping single denominator.}]$$

$$(c) \frac{x}{x-y} + \frac{y}{x-y}$$

$$\begin{aligned}
 &= \frac{x}{x} - \frac{y}{y} \\
 &= \frac{x-y}{x-y} = 1
 \end{aligned}$$

If the of algebraic fraction are like then, we only add or subtract the numerator keeping the denominator as it is.

Example 1

Simplify:

$$(a) \frac{x}{a} - \frac{y}{a} \qquad (b) \frac{3a}{a} - \frac{9}{a}$$

solution

$$(a) \text{ Here, } \frac{x}{a} - \frac{y}{a}$$

$$= \frac{x-y}{a}$$

$$(b) \frac{3a}{a} - \frac{9}{a}$$

$$= \frac{3a-9}{a}$$

$$= \frac{3(a-3)}{a}$$

$$= 3$$

Example 2

Simplify:

$$(a) \frac{x^2}{x} - \frac{y^2}{y} \qquad (b) \frac{a^2}{a} - \frac{6a-9}{a}$$

solution

$$(a) \text{ Here, } \frac{x^2}{x} - \frac{y^2}{y}$$

$$\begin{aligned}
 &= \frac{x^2 - y^2}{x - y} \\
 &= \frac{(x + y)(x - y)}{x - y} \\
 &= (x + y)
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad &\frac{a^2}{a - 3} - \frac{6a - 9}{a - 3} \\
 &= \frac{(a^2 + 6a - 9)}{a - 3} \\
 &= \frac{a^2 + 2a - 3}{(a - 3)} \\
 &= \frac{(a - 3)^2}{(a - 3)}
 \end{aligned}$$

Exercise 11.2

1. Simplify:

$$\text{(a)} \quad \frac{2a}{y} + \frac{3b}{y}$$

$$\text{(b)} \quad \frac{5p}{r^2} + \frac{7q}{r^2}$$

$$\text{(c)} \quad \frac{5a}{6b} - \frac{a}{6b}$$

$$\text{(d)} \quad \frac{3}{x+2} - \frac{2}{x+2}$$

$$\text{(e)} \quad \frac{x+1}{2} + \frac{x+2}{2}$$

$$\text{(f)} \quad \frac{x+y}{a+1} - \frac{y}{a+1}$$

$$\text{(g)} \quad \frac{6}{y-3} - \frac{3y}{y-3}$$

$$\text{(h)} \quad \frac{3x}{x+1} + \frac{3}{x+1}$$

$$\text{(i)} \quad \frac{mn}{m+n} - \frac{mn}{m+n}$$

2. Simplify:

$$\text{(a)} \quad \frac{(x+2)}{(x+3)} + \frac{(x-2)}{(x+3)}$$

$$\text{(b)} \quad \frac{3x+1}{x^2+2} - \frac{x+1}{x^2+2}$$

$$\text{(c)} \quad \frac{y-15}{y^2-9} + \frac{18}{y^2-9}$$

$$(d) \frac{ax^2+bx}{x+a} + \frac{c}{x+a} \quad (e) \frac{x^2-4x}{x^2-4} - \frac{4}{x^2-4} \quad (f) \frac{y^2+3y}{y+3} + \frac{5y+15}{y+3}$$

$$(g) \frac{5p^2}{4-p} - \frac{35p-60}{4-p}$$

$$(h) \frac{p^4}{(p+3)^2} + \frac{81-18p^2}{(p+3)^2}$$

$$(i) \frac{3x^2}{x+y} + \frac{6xy+3y^2}{x+y}$$

$$(j) \frac{a^2+b^2}{(a-b)^2} - \frac{2ab}{(a-b)^2}$$

$$(k) \frac{m^2}{m^2+5m+6} + \frac{2m}{m^2+5m+6}$$

$$(l) \frac{x^2}{x^2-4x+3} - \frac{3x}{x^2-4x+3}$$

Answer

$$1. (a) \frac{2a+3b}{y} \quad aa \quad \frac{5p+7q}{r^2} \quad (c) \frac{2a}{3b} \quad (d) \frac{1}{x+2} \quad (e) \frac{2x+3}{2}$$

$$(f) \frac{x}{a+1} \quad (g) \frac{6-3y}{y-3} \quad (h) 3 \quad (i) 0$$

$$2. (a) \frac{2x}{x+3} \quad (b) \frac{2x}{x^2+2} \quad (c) \frac{1}{y-3} \quad (d) \frac{ax^2+bx+c}{x+a}$$

$$(a) \frac{x-2}{x+2} \quad (a) y+5 \quad (a) 15-5p \quad (a) (p-3)^2$$

$$(e) 3(x-y) \quad (f) 1 \quad (g) \frac{m}{m+3} \quad (h) \frac{x}{x-1}$$

11.3 Addition and subtraction of algebraic fraction having different denominator

Activity 3

Simplify the given algebraic fractions. Discuss with friends about the process to be followed while simplifying.

$$(a) \frac{2}{3} - \frac{4}{5} \quad (b) \frac{a}{x} - \frac{b}{y} \quad (c) \frac{x}{x-y} - \frac{y}{x-y} \quad (d) \frac{1}{x-y} - \frac{x}{x^2-y^2}$$

While simplifying the above mentioned fractions, we should check, whether the denominators of fractions are same or not. If denominator is same, we can write only one denominator by doing necessary actions with numerator, if denominator is not same, then we should make them the same.

For example:

$$\begin{aligned} (a) \quad & \frac{2}{3} - \frac{4}{5} \quad [\text{denominators are not same, for making the fraction having} \\ & \hspace{15em} \text{same denominators} \\ & = \frac{2}{3} \cdot \frac{5}{5} - \frac{4}{5} \cdot \frac{3}{3} \quad [\text{multiplying the numerator and denominator} \\ & \hspace{15em} \text{of one fraction by denominator of other fraction.} \\ & = \frac{10}{15} - \frac{12}{15} \quad [\text{Now, these are fraction of same denominator}] \\ & = \frac{10-12}{15} \quad [\text{adding in numerator by keeping only one denominator.}] \\ & = \frac{-2}{15} \end{aligned}$$

$$(b) \frac{a}{x} - \frac{b}{y}$$

Here, $\frac{a}{x} - \frac{b}{y}$ [Denominators are not same. Now, for making the denominator same.

$$\begin{aligned} & = \frac{a}{x} \cdot \frac{y}{y} - \frac{b}{y} \cdot \frac{x}{x} \quad [\text{Same as above, multiplying the numerator and} \\ & \hspace{15em} \text{denominator of one fraction by denominator of} \\ & \hspace{15em} \text{other fraction} \\ & = \frac{ay}{xy} - \frac{bx}{xy} \quad \text{Now, these are fractions of same denominator, then} \\ & = \frac{ay-bx}{xy} \quad [\text{Adding nominator by keeping only the denominator.}] \end{aligned}$$

$$(c) \quad \frac{x}{x-y} - \frac{y}{x-y} \quad [\text{Denominators are not same. Now, for making fraction of same denominator.}]$$

$$\begin{aligned}
&= \frac{x(x+y)}{(x+y)(x-y)} - \frac{y(x-y)}{(x+y)(x-y)} \quad \text{[multiplying the numerator and denominator of one fraction by denominator of other fraction]} \\
&= \frac{x(x+y) - y(x-y)}{(x+y)(x-y)} \\
&= \frac{x^2 - xy - x y + y^2}{(x+y)(x-y)} \\
&= \frac{x^2 + y^2}{x^2 - y^2}
\end{aligned}$$

(d) $\frac{1}{x-y} - \frac{x}{x^2-y^2}$ [denominators are same. Now, for making the denominator same]

$$= \frac{1(x+y)}{(x-y)(x+y)} - \frac{x}{(x-y)(x+y)}$$

$$= \frac{x+y-x}{(x-y)(x+y)}$$

$$= \frac{y}{(x-y)(x+y)}$$

Denominator of first fraction = $(x-y)$	for making equal denominators
Denominator of second fraction = $(x-y)(x+y)$	

$\times (x+y)$
$\times 1$

It can be done in this way,

Denominator of first fraction = $(x-y)$

Denominator of second fraction = $(x-y)(x+y)$

L.C.M. = $(x-y)(x+y)$

$$= \frac{1}{x-y} - \frac{x}{(x-y)(x+y)}$$

$$= \frac{(x+y) - x}{(x-y)(x+y)} \quad \text{[Multiplying numerator, after keeping LCM in denominators and dividing that LCM by denominator]}$$

$$= \frac{(x+y) - x}{(x-y)(x+y)}$$

$$= \frac{y}{(x-y)(x+y)}$$

While Simplifying algebraic fraction,

- (A) If the algebraic fraction are not equal, we need to make them equal first before simplifying the fractions.
- (B) We need to find the LCM of the denominators and divide it by the denominators of each fraction. Then, we need to multiply both the denominators and numerators of the fractions with corresponding quotient thus obtained.

Example 1

Simplify:

(a) $\frac{x}{3} - \frac{x}{2}$ (b) $\frac{x+3}{x-2} - \frac{x+2}{x-3}$

Solution

Here, a) $\frac{x}{3} - \frac{x}{2}$

$$= \frac{x \times 2}{3 \times 2} - \frac{x \times 3}{2 \times 3} \quad \begin{array}{l} \text{[denominator of one frac-} \\ \text{tion is multiplied with} \\ \text{numerator and denominator} \\ \text{of other fraction.} \end{array}$$

$$= \frac{2x + 3x}{6}$$

$$= \frac{5x}{6}$$

Alternative method

$$\frac{x}{3} - \frac{x}{2}$$

LCM of 3 and 2 = 3 × 2 = 6

$$= \frac{x \times 2}{3 \times 2} - \frac{x \times 3}{2 \times 3}$$

$$= \frac{2x + 3x}{6}$$

$$= \frac{5x}{6}$$

(b) $\frac{x+3}{x-2} - \frac{x+2}{x-3}$

$$= \frac{(x+3)(x-3)}{(x-2)(x-3)} - \frac{(x+2)(x-2)}{(x-3)(x-2)}$$

$$= \frac{(x^2 - 9)}{(x-2)(x-3)} - \frac{x^2 - 4}{(x-3)(x-2)}$$

Alternative Method

$$\frac{x+3}{x-2} - \frac{x+2}{x-3}$$

LCM of (x-2) and (x-3).

$$= (x-2)(x-3)$$

$$= \frac{(x+3)(x-3) - (x+2)(x-2)}{(x-2)(x-3)}$$

$$= \frac{(x^2 - 9) - (x^2 - 4)}{(x-3)(x-2)}$$

$$= \frac{x^2 - 9 - x^2 + 4}{(x-3)(x-2)}$$

$$= \frac{-5}{(x-3)(x-2)}$$

$$\begin{aligned}
 &= \frac{(x^2 - 9)}{(x - 2)(x - 3)} - \frac{x^2 - 4}{(x - 3)(x - 2)} \\
 &= \frac{(x^2 - 9) - (x^2 - 4)}{(x - 3)(x - 2)} \\
 &= \frac{x^2 - 9 - x^2 + 4}{(x - 3)(x - 2)} \\
 &= \frac{-5}{(x - 3)(x - 2)}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{(x+3)(x-3) - (x+2)(x-2)}{(x-2)(x-3)} \\
 &= \frac{(x^2 - 9) - (x^2 - 4)}{(x-3)(x-2)} \\
 &= \frac{x^2 - 9 - x^2 + 4}{(x-3)(x-2)} \\
 &= \frac{-5}{(x-3)(x-2)}
 \end{aligned}$$

Example 2

Simplify:

(a) $\frac{x}{x^2+3x+2} - \frac{2}{x^2-1}$

solution

Here $\frac{x}{x^2+3x+2} - \frac{2}{x^2-1}$

$$\begin{aligned}
 &= \frac{x}{(x+1)(x+2)} - \frac{2}{(x+1)(x-1)} \\
 &= \frac{x(x-1)}{(x+1)(x+2)(x-1)} - \frac{2(x+2)}{(x+1)(x-1)(x+2)} \\
 &= \frac{x(x-1) - 2(x+2)}{(x+1)(x+2)(x-1)} \\
 &= \frac{x^2 - x - 2x - 4}{(x+1)(x+2)(x-1)} \\
 &= \frac{x^2 - 3x - 4}{(x+1)(x+2)(x-1)} \\
 &= \frac{(x-4)(x+1)}{(x+1)(x+2)(x-1)} \\
 &= \frac{(x-4)}{(x+2)(x-1)}
 \end{aligned}$$

First expression

$$\begin{aligned}
 &x^2 - 3x - 2 \\
 &= x^2 - 2x - x - 2 \\
 &= x(x - 2) - 1(x - 2) \\
 &= (x - 2)(x - 1)
 \end{aligned}$$

Second expression

$$x^2 - 1 = (x - 1)(x + 1)$$

Denominator of first fraction = $(x+2)(x-1)$	for making equal denominators $\times (x-y)$ $\times (x-2)$
Denominator of second fraction = $(x-1)(x-1)$	

Alternative method

$$= \frac{x}{(x+1)(x+2)} - \frac{2}{(x+1)(x-1)}$$

$$\text{LCM of } (x-1)(x+2) \text{ and } (x-1)(x-1) \\ = (x-1)(x-1)(x+2)$$

$$= \frac{x(x-1)-2(x+2)}{(x+1)(x+2)(x-1)}$$

$$= \frac{x^2-3x-4}{(x+1)(x+2)(x-1)}$$

$$= \frac{(x-4)(x+1)}{(x+1)(x+2)(x-1)}$$

$$= \frac{(x-4)}{(x+2)(x-1)}$$

$$\begin{aligned} x^2 - 3x - 4 \\ = x^2 - 4x + x - 4 \\ = x(x-4) + 1(x-4) \\ = (x-4)(x+1) \end{aligned}$$

Exercise 11.3

1. Simplify:

(a) $\frac{a}{3} + \frac{a}{4}$

(b) $\frac{2}{x} + \frac{3}{2x}$

(c) $\frac{1}{2a} - \frac{1}{3a}$

(d) $\frac{3x}{2y} - \frac{2x}{3y}$

(e) $\frac{2}{ab} + \frac{3}{2bc}$

(f) $\frac{4}{ax} - \frac{3}{bx}$

(g) $4x + \frac{3x}{7}$

(h) $\frac{x^2}{4} + \frac{y^2}{3}$

(i) $\frac{2}{a} - \frac{3}{ab}$

(j) $\frac{3}{7} - \frac{5}{3y}$

(k) $\frac{x^2}{y} - 4y$

(k) $\frac{x}{2-x} - \frac{2-x}{x}$

2. Simplify:

$$(a) \frac{2}{x-y} + \frac{3}{x+y} \quad (b) \frac{1}{a-b} - \frac{1}{a+b} \quad (c) \frac{2}{p-2q} + \frac{1}{p+2q}$$

$$(d) \frac{x}{2(x-2)} - \frac{1}{(x-2)} \quad (e) \frac{a}{a+b} + \frac{b}{a-b} \quad (f) \frac{3}{x-a} + \frac{4}{x+a}$$

$$(g) \frac{x}{x^2-1} + \frac{1}{x-1} \quad (h) \frac{x+3}{x-5} - \frac{x+5}{x-3} \quad (i) \frac{x+7}{x-7} - \frac{x}{7-x}$$

$$(j) \frac{2x+1}{6} + 2 \cdot 2x \quad (k) \frac{x}{2(x+y)} - \frac{2}{3(x+y)} \quad (l) \frac{1}{x+6} - \frac{x}{x+9}$$

$$(m) \frac{x+2}{x^2+x} - \frac{3}{x^2-x-2} \quad (n) \frac{1}{x-3} + \frac{3x-5}{x^2-5x+6}$$

$$(o) \frac{2x-1}{x^2+4x} - \frac{x-2}{x^2+2x-8} \quad (p) \frac{2a}{a-1} - \frac{a^2+3}{a^2-1}$$

$$(q) \frac{a^2+b^2}{a^2-b^2} - \frac{a-b}{a+b} \quad (r) \frac{a}{a^2+3a+2} - \frac{2}{a^2-1}$$

Answer

$$1. (a) \frac{7a}{12} \quad (b) \frac{7}{2x} \quad (c) \frac{1}{6a} \quad (d) \frac{5x}{6y}$$

$$(e) \frac{4c+3a}{2abc} \quad (f) \frac{4b-3a}{abx} \quad (g) \frac{31x}{7} \quad (h) \frac{3x^2+4y}{12}$$

$$(i) \frac{2b-3}{ab} \quad (j) \frac{9y-35}{21v} \quad (k) \frac{x^2-4y^2}{v} \quad (l) \frac{4x-4}{2x-x^2}$$

$$2. (a) \frac{5x-y}{(x^2-y^2)} \quad (b) \frac{2b}{a^2-b^2} \quad (c) \frac{3p+2q}{p^2-4q^2} \quad (d) \frac{1}{2}$$

(e) $\frac{a^2+b^2}{a^2-b^2}$	(f) $\frac{7x-a}{x^2-a^2}$	(g) $\frac{2x+1}{x^2-1}$	(h) $\frac{16}{(x-5)(x-3)}$
(i) $\frac{2x+7}{x-7}$	(j) $\frac{14x+1}{6}$	(k) $\frac{3x-4}{6(x+y)}$	(l) $\frac{9-5x-x^2}{(x+9)(x+6)}$
(m) $\frac{x+4}{x^2-2x}$	(n) $\frac{4x-3}{(x-3)(x-2)}$	(o) $\frac{x-1}{x(x+4)}$	(p) $\frac{a+3}{a-1}$
(q) $\frac{2ab}{a^2-b^2}$	(r) $\frac{a-4}{(a+2)(a-1)}$		

11.4 Multiplication and Division of Algebraic Fraction

Activity 5

Discuss in group and solve the following questions.

Find the product when $\frac{3}{4}$ multiplied by $\frac{2}{3}$?

Find the product when $\frac{a^2b^2}{a^2ab}$ multiplied by $\frac{a^2}{a^2b^2}$

Find the product when $\frac{4}{5}$ divided by $\frac{2}{3}$?

Find the quotient $\frac{a^2b^2}{b^2}$ multiplied by $\frac{a^2ab}{ab}$?

Here, $\frac{3}{4}$ quotient $\frac{2}{3}$

$$\frac{3}{4} \times \frac{2}{3} = \frac{3 \times 2}{4 \times 3} = \frac{1}{2}$$

$$\begin{aligned}
 \text{(b)} \quad & \frac{a^2-b^2}{a^2+ab} \text{ multiplied by } \frac{a^2}{ab-b^2} \\
 &= \frac{a^2-b^2}{a^2+ab} \times \frac{a^2}{ab-b^2} \\
 &= \frac{(a+b)(a-b) \times a^2}{a(a+b) \times b(a-b)} \\
 &= \frac{a}{b}
 \end{aligned}$$

While multiplying between fractions, numerator should be multiplied with numerator, and denominator should be multiplied with denominator.

$$\begin{aligned}
 \text{(c)} \quad & \frac{4}{5} \text{ divide by } \frac{2}{3} \\
 &= \frac{4}{5} \div \frac{2}{3} \\
 &\text{Any number divided by 1, the quotient and the dividend are same} \\
 &= \frac{4}{5} \times \frac{3}{2} \div \frac{2}{3} \times \frac{3}{2} \quad (\text{Divisor 1 making}) \\
 &= \frac{4}{5} \times \frac{3}{2} \div 1 \\
 &= \frac{4}{5} \times \frac{3}{2} \\
 &= \frac{6}{5}
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad & \frac{a^2-b^2}{b^2} \text{ divided by } \frac{a^2+ab}{ab} \\
 &= \frac{a^2-b^2}{b^2} \div \frac{a^2+ab}{ab} \\
 &= \frac{(a^2-b^2)}{b^2} \times \frac{ab}{a^2+ab} \div \frac{a^2+ab}{ab} \times \frac{ab}{a^2+ab} \quad \text{making divisor 1} \\
 &= \frac{(a+b)(a-b)}{b^2} \times \frac{ab}{a(a+b)} \div 1
 \end{aligned}$$

$$= \frac{(a+b)(a-b)}{b^2} \times \frac{ab}{a(a+b)} \quad \text{What conclusion can be drawn?}$$

$$= \frac{(a-b)}{b}$$

The result will be same when one fraction is divided by other fraction and multiplied the fraction by the reciprocal of fractional divisor.

Example 1

Simplify:

(a) $\frac{4}{x^2+3x+2} \div \frac{2}{x^2-1}$

Solution

Here, $\frac{4}{x^2+3x+2} \div \frac{2}{x^2-1}$

$$= \frac{4}{x^2+3x+2} \times \frac{x^2-1}{2}$$

$$= \frac{4}{(x+2)(x+1)} \times \frac{(x+1)(x-1)}{2}$$

$$= \frac{2}{(x+2)} \times \frac{(x-1)}{1}$$

$$= \frac{2(x-1)}{(x+2)}$$

first denominator

$$x^2 - 3x - 2$$

$$= x^2 - 2x - x - 2$$

$$= x(x - 2) - 1(x - 2)$$

$$= (x - 2)(x - 1)$$

second denominator

$$x^2 - 1 = (x - 1)(x + 1)$$

Example 2

Simplify:

(a) $\frac{x^2-6x+9}{x^2+3x+2} \div \frac{x^2-5x+6}{x^2-x-2}$

Solution

Here, $\frac{x^2-6x+9}{x^2+3x+2} \div \frac{x^2-5x+6}{x^2-x-2}$

$$\begin{aligned}
 &= \frac{x^2-6x+9}{x^2+3x+2} \times \frac{x^2-x-2}{x^2-5x+6} \\
 &= \frac{(x-3)(x-3)}{(x+1)(x+2)} \times \frac{(x-2)(x+1)}{(x-2)(x-3)} \\
 &= \frac{(x-3)}{(x+2)}
 \end{aligned}$$

$$\begin{aligned}
 &x^2 - 6x + 9 \\
 &= x^2 - 2 \cdot x \cdot 3 + 3^2 \\
 &= (x-3)(x-3)
 \end{aligned}$$

$$\begin{aligned}
 &x^2 - 5x + 6 \\
 &= x^2 - 3x - 2x + 6 \\
 &= x(x-3) - 2(x-3) \\
 &= (x-3)(x-2)
 \end{aligned}$$

$$\begin{aligned}
 &x^2 - x - 2 \\
 &= x^2 - 2x + x - 2 \\
 &= x(x-2) + 1(x-2) \\
 &= (x-2)(x+1)
 \end{aligned}$$

Exercise 11.4

1. Simplify:

(a) $\frac{a^2}{b} \times \frac{2}{b}$

(b) $\frac{3x^2}{4y^2} \times \frac{4y}{3x}$

(c) $\frac{7a^2b}{8c} \times \frac{4c^2}{14ab^2}$

(d) $\frac{x-y}{x+y} \times \frac{x}{y}$

(e) $\frac{a-3}{3} \times \frac{6}{a-3}$

(f) $\frac{x-3}{x+2} \times \frac{(x+2)^2}{(x-3)^2}$

2. Simplify:

(a) $\frac{a^2}{b^2} \div \frac{a}{b}$

(b) $\frac{3xy}{4ab} \div \frac{6y}{5b}$

(c) $\frac{x}{7} \div \frac{x^2}{14}$

(d) $\frac{6a^2b}{7x^2y} \div \frac{6ab^2}{7y^2}$

(e) $\frac{a^2-b^2}{a} \div \frac{a-b}{b}$

(f) $\frac{x^2-1}{y^2} \div \frac{x-1}{y}$

3. Simplify:

(a) $\frac{x^2-y^2}{x+y} \times \frac{x+y}{(x-y)^2}$

(b) $\frac{x^2+2xy+y^2}{x^2-y^2} \times \frac{x-y}{x+y}$

$$(c) \frac{x^2-4x+4}{3y-xy} \times \frac{4x-12}{x-2}$$

$$(d) \frac{a^2-b^2}{a^2+2a+ab+2b} \times \frac{a+2}{a+3}$$

$$(e) \frac{y^2+10y+24}{y^2+2y-8} \times \frac{y-3}{y+6}$$

$$(f) \frac{x^2-3x-10}{x^2-5x+6} \times \frac{bx-3b}{cx-5c}$$

$$(g) \frac{x^2-11x+30}{x^2-7x+10} \times \frac{5x-10}{x^2-8x+12}$$

$$(h) \frac{x^2-9}{x^2+4x} \times \frac{x^2+2x-8}{x^2+x-6}$$

$$(i) \frac{x^2-5x+6}{x^2-6x+9} \times \frac{x^2-2x-3}{x^2-3x+2}$$

4. Simplify:

$$(a) \frac{x^2-y^2}{x+y} \div \frac{x-y}{x+y}$$

$$(b) \frac{x^2-5x+6}{x^2-9} \div \frac{x-3}{x+3}$$

$$(c) \frac{x^2+12x+36}{x^2-16} \div \frac{3x+18}{2x^2+8x}$$

$$(d) \frac{3x^2-4x-7}{3x^2-7x} \div \frac{x^2-1}{x-4}$$

$$(e) \frac{x^2+2x-15}{x-2} \div \frac{3(x^2+4x-5)}{x^2-3x+2}$$

$$(f) \frac{x^2+12x+27}{x^2+x-6} \div \frac{x^2+4x-45}{9(x^2-4x-5)}$$

$$(g) \frac{xy-x+2y-2}{3y+2x+xy+6} \div \frac{xy-x+5y-5}{x^2+8x+15}$$

$$(h) \frac{y^2+4y-12}{y^2-5y+6} \div \frac{y^2+3y-18}{y^2-9}$$

$$(i) \frac{x^2-8x+15}{x^2-14x+45} \div \frac{x^2-2x-15}{x^2-8x-9}$$

$$(j) \frac{a^2+3a+2}{a^2-4a-12} \div \frac{a^2-a-6}{a^2-9a+18}$$

5. Simplify:

$$(a) \quad \frac{2x}{5y} \times \left(\frac{2y}{5} \div \frac{y}{3} \right)$$

$$(b) \quad \left(\frac{x}{(x-1)} - \frac{1}{(x+1)} \right) \div \frac{x-1}{x^2-1}$$

$$(c) \quad \left(\frac{3x}{(x-1)} \times \frac{1}{(x+1)} \right) \div \frac{3}{x^2-1}$$

$$(d) \quad \frac{x-4}{x+4} \times \frac{x-3}{x+3} \div \frac{x^2-7x+12}{x^2+7x+12}$$

$$(e) \quad \left(\frac{a+b}{a-b} - \frac{a-b}{a+b} \right) \times \frac{a^2-b^2}{4ab}$$

Project work

There is a rectangle having breadth $(x-2)$ unit and area of (x^2-3x-6) sq. unit.

There is another rectangle of same breadth having area (x^2-3x-6) sq. unit.

(a) What is the length both rectangle? Find

(b) If breadth of both rectangles is arranged together, what will be the total length then? Draw a figure in chart paper and present.

Answer

1. (a) $\frac{2a^2}{b^2}$ (b) $\frac{x}{y}$ (c) $\frac{ac}{4b}$ (d) $\frac{x(x-y)}{y(x+y)}$ (e) 2 (f) $\frac{x}{x}$
2. (a) $\frac{a}{b}$ (b) $\frac{5x}{8a}$ (c) $\frac{2}{x}$ (d) $\frac{ay}{bx^2}$ (e) $\frac{b(a+b)}{a}$ (f) $\frac{x+1}{y}$
3. (a) $\frac{x+y}{x-y}$ (b) 1 (c) $\frac{-4(x-2)}{y}$ (d) $\frac{a-b}{a+3}$ (e) $\frac{y-3}{y-2}$
 (f) $\frac{b(x+2)}{c(x-2)}$ (g) $\frac{5}{x-2}$ (h) $\frac{x-3}{x}$ (i) $\frac{x+1}{x-1}$
4. (a) $x-y$ (b) $\frac{x-2}{x-3}$ (c) $\frac{2x(x+6)}{3(x-4)}$ (d) $\frac{x-4}{x(x-1)}$ (e) $\frac{x-3}{3}$
 (f) $\frac{9(x+1)}{x-2}$ (g) $\frac{x+2}{y+2}$ (h) $\frac{y+3}{y-3}$ (i) $\frac{(x-3)(x+1)}{(x-5)(x+3)}$ (j) $\frac{a+1}{a+2}$
5. (a) $\frac{12x}{25y}$ (b) $\frac{x^2+1}{x-1}$ (c) x (d) 1 (e) 1

12.0 Review

Discuss in a group and answer the following questions:

Santosh plucked the x number of oranges and y number of guava from his garden. If the total number of fruits he plucked is 12, then

1. Explain the above mathematical statement as an equation.
2. How many oranges and guava would Santosh bring to make a total of 12? Write the possible answers in a table.
3. Show the values of the variables in the graph.

12.1 Simultaneous equations with two variables

Activity 1

Nirmala bought 6 Mangoes.

1. If she distributed the mangoes to her daughters, Asmi and Anisha, how many mangoes would each of them get?
2. If Nirmala has given 2 more mangoes to Asmi than Anisha, how many mangoes does she give to each of her daughters? Discuss with friends and find the answer.

Here,

Let, the number of mangoes Asmi gets is x , and

the number of mangoes Anisha would gets = y

presenting it at the table,

Asmi (x)	5	4	3	2	1
Anisha (y)	1	2	3	4	5
Total mango	6	6	6	6	6

In the table above, the total number of mangoes both Asmi and Anisha get is 6 in all cases.

So, it is $x + y = 6$ (i)

Again, if Asmi was given two more mangoes than Anisha, then how many mangoes works each of them get? This information can be shown as in the table below.

Anisha (y)	0	1	2	3	4
Asmi (x)	2	3	4	5	6
Total mango	2	4	6	8	10

In the table above, the number of mangoes given to Asmi is more than that of Anisha by two, so,

$$x = 2 + y$$

It is, $x - y = 2$ (ii)

From the table above,

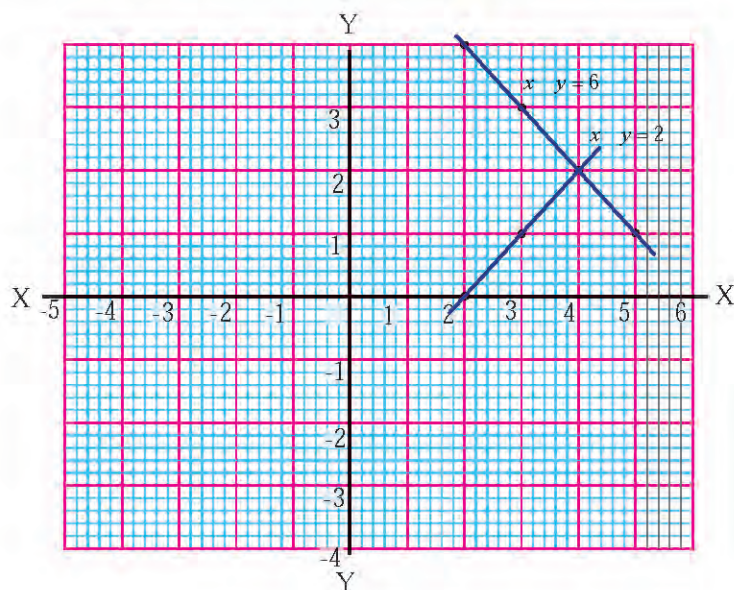
If Nirmala gave more mangoes to Asmi than to Anisha out of the 6 mangoes with her, it would be 2 mangoes to Anisha and 4 mangoes to Asmi.

Plotting these two equations on graph paper,

$x + y = 6$ and $x - y = 2$ lines representing the equations are intersected at point (4,2)

Thus, the number of mangoes given to Asmi (x) = 4

The number of mangoes given to Anisha (y) = 2



Two equations are said to be simultaneous equations if the lines representing the equations in the graph intersect at a point. The coordinates of the point of intersection is the solution to the equations. The method of representing the equations in the graph to find the solution is called the graphical method.

Example 1

Solve the following by the graphical method and test if your answers are correct or not.

$$3x - y = 7 \text{ and } x - 2y = 1$$

Solution

Here, $3x - y = 7$. (i)

and $x - 2y = 1$. (ii)

From equation (i)

$$3x - y = 7$$

Substituting $x = 2, 3$ and 4 in $y = 3x - 7$

x	2	3	4
y	1	2	5

Thus, the points of the lines are $(2, -1)$, $(3, 2)$ and $(4, 5)$

Similarly, from Equation (ii),

$$x - 2y = 1$$

Substituting $y = 1, 2$ and 3 in $x = 2y - 1$

x	1	3	5
y	1	2	3

Thus, the points of the lines are: $(1, 1)$, $(3, 2)$ and $(5, 3)$

Now, plotting these points on the graph,

In the graph, the lines representing the equations are intersected at a point (3,2). Hence $x=3$ and $y=2$ is the solution to the equation i and ii.

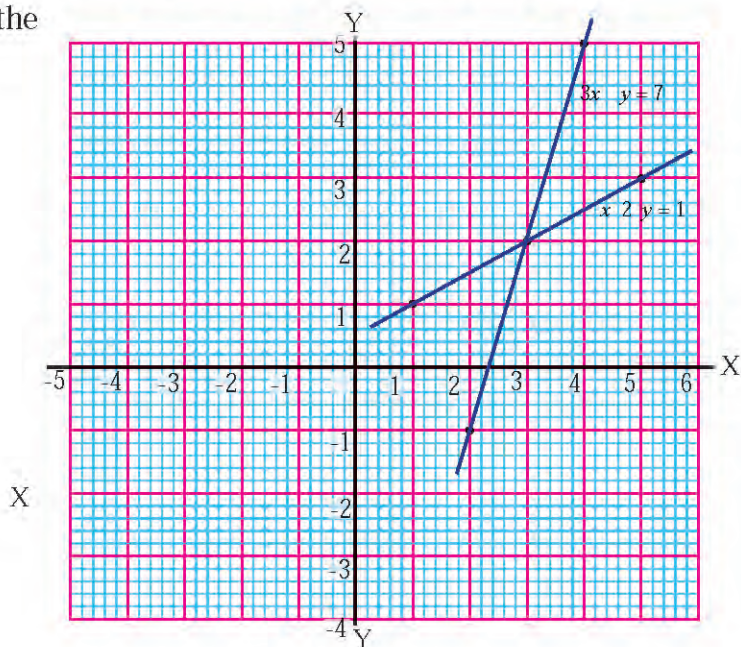
Testing these points on the

equations, $3x - y = 7$

$$\begin{aligned}\text{LHS} &= 3x - y \\ &= 3 \times 3 - 2 \\ &= 9 - 2 \\ &= 7 = \text{RHS}\end{aligned}$$

In $x + 2y = 1$

$$\begin{aligned}\text{LHS} &= x + 2y \\ &= 3 + 2 \times 2 \\ &= 3 + 4 \\ &= 7 = \text{RHS}\end{aligned}$$



Example 2

The father's present age exceeds two times his daughter's age by 10 years. If the difference in age between the father and the daughter is 25 years, then find their present ages.

solution

Let the present age of the father be x .

And daughter's present age = y

According to the question,

$$x = 2y + 10 \dots\dots\dots(i) \text{ and}$$

$$x - y = 25 \dots\dots(ii)$$

From equation (i)

Substituting $y = 0, 5$, and 15 in $x = y + 10$

x	10	20	40
y	0	5	15

Thus, the points of the lines are:

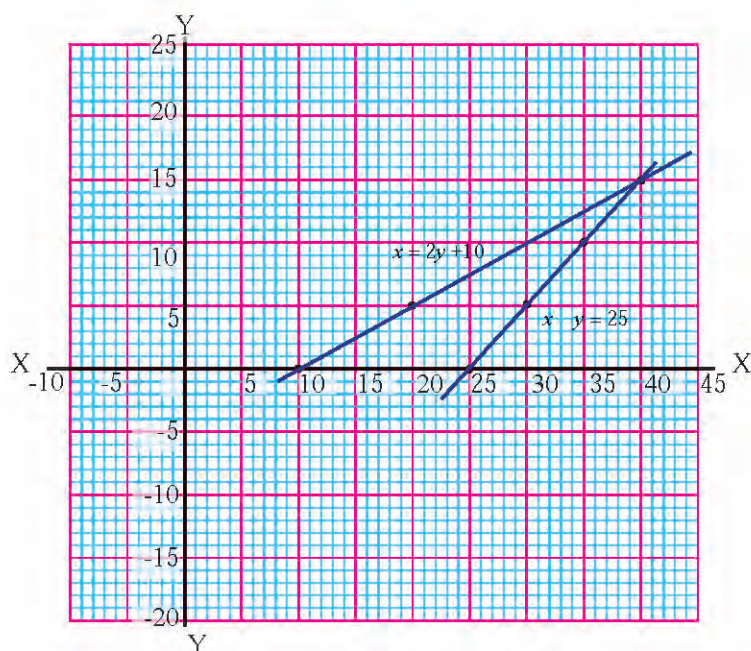
Similarly, from equation (ii)

Substituting $y = 0, 5$, and 10 in $x = y + 25$

x	25	30	35
y	0	5	10

Thus, the points of the lines are: $(25, 0)$, $(30, 5)$ and $(35, 10)$

Now, plotting these points on the graph,



In the graph, the lines representing the equations intersect at the point $(40, 15)$. Hence $x=40$ and $y=15$ is the solution to the equation (i) and (ii).

Hence, the present age of the father (x) = 40 years

And, the present age of daughter (y) = 15 years

Things to consider while solving verbal problems with two unknowns using a graphical method.

1. Read the given problem carefully and use variables to make the equation.
2. Find the value of another variable for the assigned value of the first variable and tabulate it.
3. Plot the points on the graph paper from the tables and draw straight lines.
4. The coordinate of the point of intersection is the required solution.

Exercise 12.1

1. Solve the following pairs of equations using the graphical method and check the answer.

- | | |
|--------------------------------|--------------------------------|
| (a) $x - y = 5, x - y = 3$ | (b) $3x - y = 7, x = 2y$ |
| (c) $x - y = 13, 2x = y - 8$ | (d) $x - y = 6, x - y = 2$ |
| (e) $x - y = 8, x - y = 4$ | (f) $4x - y = 2, 3x - 2y = 7$ |
| (g) $x - 2y = 6, 2y - x = 2$ | (h) $3x - 2y = 4, x - 3y = 5$ |
| (i) $2x = 5 - 3y, 5y = 2x - 3$ | (j) $2x - 1 = y, 3x - 2y = 0$ |
| (k) $x - 3 = 2y, 2x - y = 14$ | (l) $x - 2y = 5, 2x - 3y = 10$ |

2. Solve the following pair of equations using the graphical method and check the answer.

- a. If the sum of two numbers is 15 and the difference between them is 5, find the numbers.
- b. The sum of two numbers is 12. If the greater is 3 times more than the smaller, find the numbers.
- c. The difference between the two numbers is 5. If five times the smaller is equal to 4 times the greater, then find the numbers.
- d. The cost of buying 3 notebooks and 4 pens is Rs. 200. The cost of buying 5 notebooks and 2 pens is Rs. 240. Find the cost of a notebook and a pen.
- e. Father's present age is 3 less than three times the present age of the daughter. If the difference between the age of father and daughter is 37 years, find their present age.

- f. The present age of Kamala is 5 years more than that of Bimala. The age of Kamala after five years will be two times the age of Bimala. Find their present age.
- g. Bipin is 6 years older than Bipana. Two years ago, Bipin's age was two times more than Bipana's age. Find their present age.
- h. The age difference between Kusum and her father is 20 years. If the father's age is more than two times the age of Kusum by 4 years, then find their age.
- i. In an exam, Ram scored 20 marks more than Shyam. If Ram's score is two times Shyam's score, then find their scores.

Answer

1. (a) (4,1) (b) (2,1) (c) (7,6) (d) (4,2)
 (e) (6,2) (f) (1,-2) (g) (2,2) (h) (2,-1)
 (i) (4,1) (j) (2,3) (k) (5,4) (l) (5,0)
2. (a) (10,5) (b) (3,9) (c) (20,25) (d) Rs. 40, Rs. 20
 (e) (20,57) (f) (15 years, 10 years) (g) (10 years, 6 years)
 (h) (16 years, 36 years) (i) (40, 20)

12.2 Solving quadratic equations by factorization method

Activity 2

Study the following equations and discuss the questions that follow.

(a) $x - 4 = 0$ (b) $x^2 - 2x - 3 = 0$ (c) $x^2 - 25 = 0$

- i. How many variables are there in the equation given above?
- ii. What is the highest degree of x in the equation given above?
- iii. What are the values of the variables?
- iv. What distinguishes these equations from one another?

Here, in the first equation, the highest power of variable x is 1. In the second

and third equations, the power of variable x is 2. The first equation is one variable linear equation and the second and third equations are quadratic equations.

(a) $x - 4 = 0$

or, $x = 4$

(b) $x^2 - 2x - 3 = 0$

or, $x^2 - (3 + 1)x - 3 = 0$

or, $x^2 - 3x - x - 3 = 0$

or, $x(x - 3) - 1(x - 3) = 0$

or, $(x - 3)(x + 1) = 0$

If the product of two factors is 0, then any one of them must be zero

either $(x - 3) = 0$ or $(x + 1) = 0$

if $x - 3 = 0$ then $x = 3$ and

if $x + 1 = 0$ then $x = -1$

Therefore, $x = 3, -1$

(c) $x^2 - 25 = 0$

or, $(x)^2 - (5)^2 = 0$

or, $(x - 5)(x + 5) = 0$

If the product of two factors is 0, then any one of them must be zero

either, $(x + 5) = 0$

or, $(x - 5) = 0$

$x + 5 = 0$ then $x = -5$

if $(x - 5) = 0$ then $x = 5$

A quadratic equation is an equation of a single variable with degree 2. It is in the form $ax^2 + bx + c = 0$ where $a \neq 0$. For example: $x^2 - 5x + 6 = 0$.

Example 1

Solve:

(a) $x^2 - 6x + 8 = 0$

(b) $4x^2 - 25 = 0$

Solution

(a) Here, $x^2 - 6x + 8 = 0$

or, $x^2 - (4 + 2)x + 8 = 0$

or, $x^2 - 4x - 2x + 8 = 0$

or, $x(x - 4) - 2(x - 4) = 0$

or, $(x - 4)(x - 2) = 0$

either, $(x - 4) = 0 \quad \therefore x = 4$

or, $(x - 2) = 0 \quad \text{or, } (x - 2) = 0$

If $x = 4$, then $x = 4$

Hence, the value of x are 4 and 2.

(b) $4x^2 - 25 = 0$

or, $(2x)^2 - (5)^2 = 0$

or, $(2x + 5)(2x - 5) = 0$

either, $(2x + 5) = 0 \quad \therefore x = -\frac{5}{2} = -2\frac{1}{2}$

or, $(2x - 5) = 0 \quad \therefore x = \frac{5}{2} = 2\frac{1}{2}$

Hence, the value of x are $-2\frac{1}{2}$ and $2\frac{1}{2}$.

Example 2

Find a quadratic equation having the values of x are 2 and 3.

Solution

Here,

The value of x is 2 so, $x = 2$

or, $x - 2 = 0$

Again another value of x is 3 i.e. $x = 3$

$$x - 3 = 0$$

Now, $(x - 2)(x - 3) = 0$

or, $x(x - 3) - 2(x - 3) = 0$

or, $x^2 - 3x - 2x + 6 = 0$

Hence, $x^2 - 5x + 6 = 0$ is the required quadratic equation.

Exercise 12.2

1. Solve:

(a) $x^2 - 3x = 0$ (b) $2x^2 - x = 0$ (c) $9x^2 - 3x = 0$

(d) $9y^2 - 4 = 0$ (e) $5x - 9x^2 = 0$ (f) $4y^2 - 7y = 0$

(g) $x^2 - 49 = 0$ (h) $169x^2 - 96 = 0$ (i) $\frac{x^2}{4} - 36 = 0$

(j) $5x^2 - 125 = 0$ (k) $x^2 - 7 = 29$ (l) $x^2 - 4x = 0$

2. Solve:

(a) $x^2 - 2x + 1 = 0$ (b) $y^2 - y - 2 = 0$ (c) $x^2 - x - 2 = 0$

(d) $x^2 - 4x + 4 = 0$ (e) $x^2 - 10x - 24 = 0$ (f) $x^2 - 9x + 18 = 0$

(g) $x^2 - 11x + 30 = 0$ (h) $x^2 - 2x - 3 = 0$ (i) $x^2 - 8x + 16 = 0$

(j) $x^2 - 8x + 16 = 0$ (k) $x^2 + 10x + 25 = 0$ (l) $x^2 - 8x + 15 = 0$

(m) $x^2 - 6x + 8 = 0$ (n) $2x^2 - x - 6 = 0$ (o) $y^2 - 7y + 12 = 0$

(p) $7x^2 - 13x - 2 = 0$ (q) $x^2 - 9x - 22 = 0$ (r) $x^2 - 18x + 77 = 0$

(s) $2x^2 - 11x + 12 = 0$ (t) $3x^2 - 11x - 20 = 0$ (u) $10x^2 - 19x + 6 = 0$

(v) $12x^2 - 11x + 2 = 0$ (w) $3z^2 - 11z + 6 = 0$ (x) $(x - 1)^2 - 4 = 0$

(y) $(p - 3)^2 - 16 = 0$ (z) $(x - 6)^2 - 36 = 0$ (2a) $(x - 7)^2 - 64 = 0$

(b) $100 - (x - 5)^2 = 0$

3. Find the quadratic equation whose roots (values of x) are 1 and 2.

4. Find the quadratic equation whose roots (values of x) are 3 and -2.

5. Find the quadratic equation whose roots (values of y) are 3 and -2.

Answer

1.

- (a) 0,3 (b) $0, \frac{1}{2}$ (c) $\frac{1}{3}, 0$ (d) $\frac{2}{3}$ (e) $0, -\frac{5}{9}$
 (f) $0, \frac{7}{4}$ (g) 7 (h) $\frac{14}{13}$ (i) 12 (j) 5
 (k) 6 (l) 0,4

2. (a) -1, 1 (b) -1, 2 (c) 1, -2 (d) -2, 2 (e) 12, -2
 (f) 3, 6 (g) 5, 6 (h) -3, 1 (i) -4 (j) 4
 (k) -5 (l) 5, 3 (m) 2, 4 (n) $2, -\frac{3}{2}$ (o) -4, -3
 (p) $\frac{1}{7}, -2$ (q) 2, -11 (r) 7, 11 (s) $-4, -\frac{3}{2}$ (t) $5, \frac{4}{3}$

Mixed Exercise

- Write the expanded form of $a^2 \cdot b^2$
 - Simplify: $x^{(a \cdot b)} \cdot x^{(b \cdot c)} \cdot x^{(c \cdot a)}$
- Simplify: $\frac{2b}{a} \cdot \frac{8b^2}{a^2 \cdot 4 \cdot b^2}$
 - If $3x^2 - 8x - 16 = 0$, then find the values of x .
Check both the result to conclude whether they satisfy the equation.
- Solve the following using graphical method $2x - y = 5$ & $x - y = 1$
- Which of the following is incorrect?
 - $x^m \times x^n = x^{m+n}$
 - $x^m \div x^n = x^{m-n}$
 - $(x^m)^n = x^{m+n}$
 - $x^{-m} = \frac{1}{x^m}$
 - Find the LCM and HCF of the following expressions.
 $x^2 - 5x - 6$ & $x - 3$

5. The sum of two numbers is 12 and their difference is 4.
 - a. If x represents the greater number and y represents the smaller, then write the equations for the given statement.
 - b. Solve the equations using the graphical method.
6. If the two algebraic expressions are $x^2 - 5x + 6$ and $x^2 + 4x - 12$ then,
 - a. Find the Highest Common Factor of the given expressions
 - b. Find the Lowest Common Multiple of the given expressions.
 - c. For what values of x , the value of the expression $x^2 + 4x - 12$ is zero.
7. If the two algebraic expressions are $x^2 - 5x + 6$ and $x^2 - 2x - 1$ then,
 - a. Find the Highest Common Factor of the given expressions.
 - b. Find the Lowest Common Multiple of the given expressions.
 - c. For what values of x , the value of the expression $x^2 - 5x + 6$ is zero.
8. If the two algebraic expressions are $x^3 - 8x^2 + 16x$ and $x^3 - x^2 - 12x$ then,
 - a. Find the Highest Common Factor of the given expressions.
 - b. Find the Lowest common multiple of the given expressions.
 - c. For what values of x , the value of the expression $x^3 - 8x^2 + 16x$ is zero.
9. If the two algebraic expressions are $x^2 - 5x + 6$ and $x^2 - 7x + 12$ then,
 - a. Find the Highest Common Factor of the given expressions.
 - b. Find the Lowest Common Multiple of the given expressions.
 - c. For what values of x , the value of the expression $x^2 - 5x + 6$ is zero.
 - d. Prove that:
$$\frac{1}{x^2 + 7x + 12} + \frac{1}{x^2 + 5x + 6} = \frac{2}{x^2 + 6x + 8}$$
10. The algebraic fraction given: $\frac{x^2 - y^2}{y^2} \div \frac{x^2 + xy}{xy}$
 - a. How would we write the fraction by replacing $\div$ sign with multiply sign?
 - b. Factorise the expressions on both numerators and denominators
11. The algebraic fraction given: $\frac{a}{a^2 + 3a + 2} - \frac{2}{a^2 - 1}$

- a. Factorise its denominators.
 - b. Find the LCM of the denominators of the fractions and convert them into the like fractions.
 - c. Simplify the like fractions obtained in part b and write in simplest form.
12. a. Solve the following equations using the graphical method.
 $x - 2y = 8$ and $x - y = 5$.
- b. What type of equations are they?
13. a. Solve the following equations using the graphical method.
 $x - 2y = 6$ and $2y - x = 2$
- b. What type of equations are they?
14. a. What is the degree of the equation given below?
 $x^2 - 7x + 12 = 0$
- b. What types of equations are they?
- c. Find the roots of the equation
15. The total price of the pen and notebook shown in the figure is given below.



Total price Rs. 320



Total price Rs. 300

- a. If the price of a notebook is Rs. x and the price of a pen is Rs. y write the equation for both cases.
- b. Find the price of a notebook and a pen.
- c. You have Rs. 450. You have to buy equal number of pens and notebooks. Determine the number of pens and notebooks that you can purchase in equal quantities.
- d. If the price of both pen and notebook is increased by 10%, find the total cost for 2 pens and 3 notebooks.

16. If there are some cats and chickens on the ground, and you count a total of 10 heads and 26 legs, what is the number of cats and chickens on the ground?
17. Prove that: $\frac{1}{(x+y)^{-1}} - \frac{1}{(y-z)^{-1}} - \frac{1}{(z+x)^{-1}} = 0$
18. Prove that: $(a^{(x+y)})^{(x-y)} (a^{(y-x)})^{(y+x)} = 1$

Answer

1. a) $(x-b)(a-b)$ b) 1 2. a) $\frac{2b}{a-2-b}$ b) Show to your teacher
3. Show to your teacher 4. a) b) $(x-3)$ and $(x-2)(x-3)$
5. a) $x-y=12, x-y=4$ b) $x=8, y=4$
6. a) $(x-2)$ b) $(x^2-4)(x-3)$ c) 2
7. a) $(x-1)$ b) $(x-1)^2(x-6)$ c) 6, 1
8. a) $x(x-4)$ b) $x(x-4)^2(x-3)$ c) 0, 3 and 4
9. a) $(x-3)$ b) $(x-2)(x-3)(x-4)$ c) 2 and 3
10. a) $\frac{x^2-y^2}{y^2} - \frac{xy}{x^2-xy}$ b) $\frac{x-y}{y}$
11. (a) $a^2-3a-2 = (a-2)(a-1), a^2-1 = (a-1)(a+1)$
- (b) $\frac{a(a-1)}{(a+2)(a+1)(a-1)} - \frac{2(a+2)}{(a+2)(a+1)(a-1)}$ (c) $\frac{(a-4)}{(a+2)(a-1)}$
12. (a) Linear equation b) (2, 3)
13. (a) Linear equation b) (2, 2)
14. (a) Degree 2 b) Quadratic equation c) (3, 4)
15. The price of a pen is Rs.40 and the price of a notebook is Rs 50
16. 3 cats and 7 chicken
- 17 and 18: Show to your teacher

13.0 Review

Look at the following figures. Discuss where parallel lines are formed in the figure.



a. The edge of door



b. Two banks of the river



c. Two edges of the road



d. Two edges of the bridge



e. The opposite edge of the table



f. The inner and outer edges of a circular garden

If two straight lines do not intersect when extended infinitely in both directions and the perpendicular distance between them is constant throughout then the lines are said to be parallel lines.



In the figure alongside the lines, AB and CD are parallel. It is written as $AB \parallel CD$. If the perpendicular distance between two lines is constant throughout, then the lines are parallel.

13.1 Straight lines and Transversal

Activity 1

Which are the transversal in the given figure? Discuss the reason.

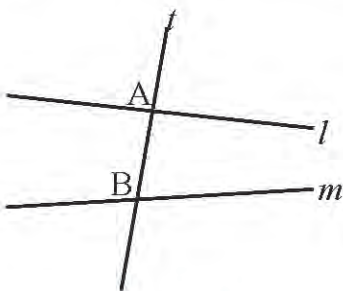


Figure (a)

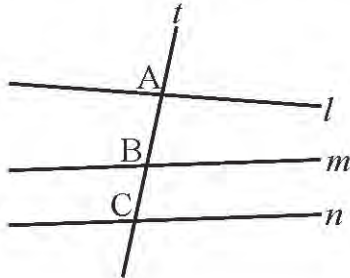


Figure (b)

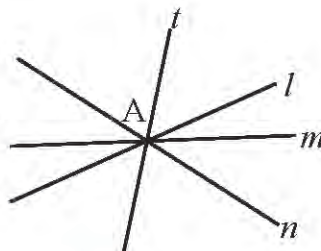


Figure (c)

Here

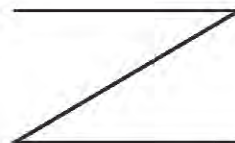
In the first figure, l and m are two straight lines and the line t is a transversal. Line t intersects lines l and m at points A and B . l , m , and n are three straight lines in figure (b) and the line t is the transversal that intersects the lines l , m , and n at points A , B , and C . But in the third figure, t is not a transversal. Why?

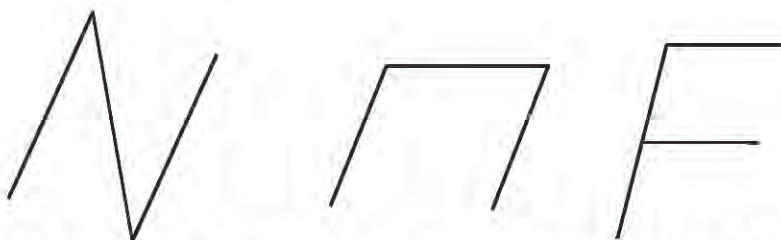
The line that intersects two or more lines at different points is called transversal.

13.1.1 Angles formed when a transversal intersects straight lines

Activity 2

Take some straw of equal and different lengths. Try to make the English alphabets Z, C, and F facing to different directions using the straws





Trace out the shapes in your notebook and name each of the line segments. What happens if you extend the line that joins two straight lines on either side?

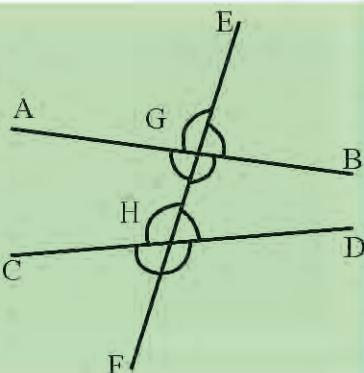
Now, extend these lines that join two lines in every figure. Discuss in pairs the following questions about the angle formed and their characteristics and fill out the table given below.

Figure number	angle pair	Characteristics		
		Angles at the same side of the transversal	Adjacent or Non-adjacent	Interior/exterior

- What is the name of the pair of non-adjacent interior angles that lies on both side of the transversal?
- What is the name of the pair of non-adjacent, one interior and other exterior angles that lie on the same side of the transversal?
- What is the name of the pair of non-adjacent interior angles that lies on the same side of the transversal?
- How would the English alphabets Z, C, and F be helpful to identify the angle pair?

Alternate angles: When two lines are transverses by a transversal, the non-adjacent, interior angles that lie on opposite sides of the transversal are called alternate angles. $\angle AGH$ and $\angle DHG$; $\angle BGH$ and $\angle CHG$.

Corresponding angles: When two lines are transverses by a transversal, the non-adjacent, one



interior other exterior angles that lie on the same sides of the transversal are called corresponding angles. $\angle AGE$ and $\angle CHG$, $\angle BGE$ and $\angle DHG$, $\angle CHF$ and $\angle AGH$, $\angle BGH$ and $\angle DHF$.

Co-interior angles: When two lines are transverses by a transversal, the non-adjacent, interior angles that lie on the same sides of the transversal are called co-interior angles. In the given figure, $\angle AGH$ & $\angle CHG$ and $\angle BGH$ and $\angle DHG$ are corresponding angles.

13.2 Parallel lines and transversal

13.2.1 Relationship between the alternate angles formed when two parallel lines are transverses by the transversal

Activity 3

Which are the transversal in the given figure? Discuss the reason in group.

Draw three different figures on your notebook in which two straight lines are transverses by the transversal, as shown below.

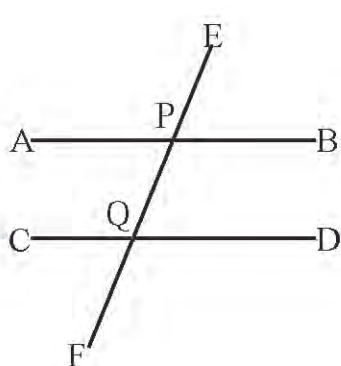


Figure (a)

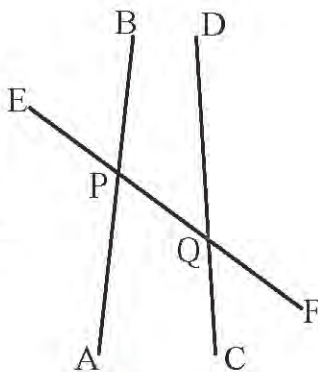


Figure (b)

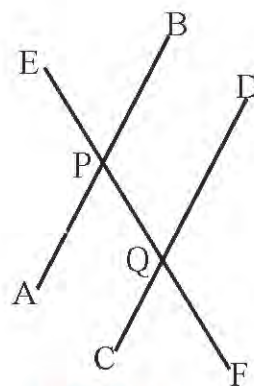


Figure (c)

Measure the alternate angles in the figure above with a protractor and record the results in a table. Using set square, indicate if the lines are parallel or not and record it in the table.

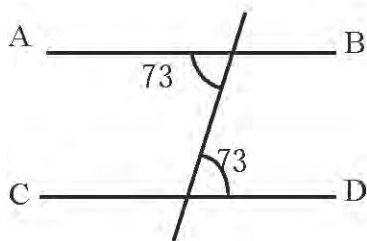
Fig-ure	Pair I		pair II		Result (relationship between Alternate angles)	Lines are (Parallel/ Not Parallel)
(a)	$\angle APQ$	$\angle PQD$	$\angle BPQ$	$\angle PQC$		
(b)						
(c)						
Conclusion: When two parallel lines are transversed by a transversal, then the alternate angles are equal.						

Question to think: What is the relationship between the two alternative angles if the two lines are not parallel?

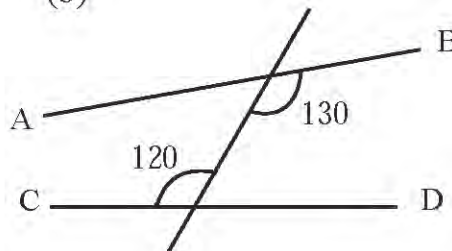
Example 1

Determine if the lines are parallel or not based on the alternate angle from the figures given below.

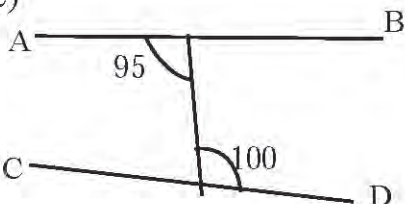
(a)



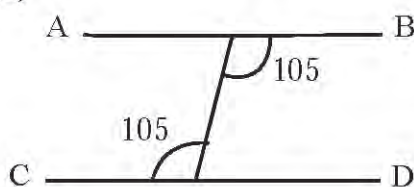
(b)



(c)



(d)



Solution

a. Yes

b. no

c. no

d. Yes

13.2.2 Relationship between the Corresponding angles formed when two parallel lines are transversal

Activity 4

Draw three different figures on your notebook in which two straight lines are transversal, as shown below.

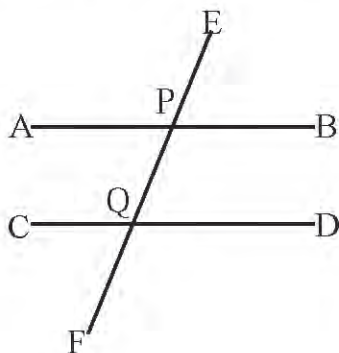


Figure (a)

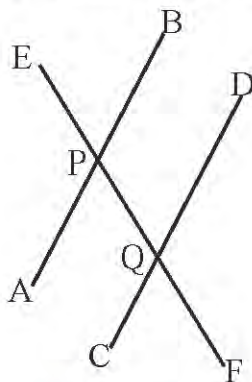


Figure (b)

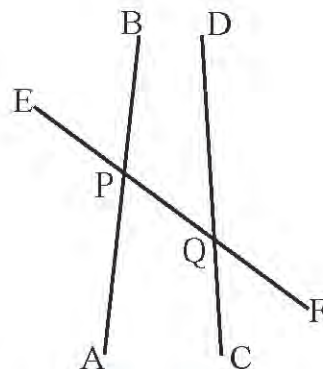


Figure (c)

Measure the corresponding angles in the figure above with a protractor and record the results in the table. Using set square, indicate if the lines are parallel or not, and record it in the table.

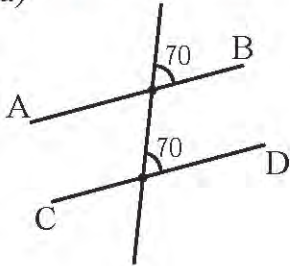
Figure	Pair I		pair II		Pair III		Pair IV		Result (relationship between Corresponding angles)	Lines are (Parallel/ Not parallel)
(a)	$\angle EPA$	$\angle PQC$	$\angle EPB$	$\angle PQD$	$\angle APQ$	$\angle CQF$	$\angle BPQ$	$\angle DQF$		
(b)										
(c)										
Conclusion: When two parallel lines are transversal, then the corresponding angles are equal.										

Question to think: What is the relationship between the two corresponding angles if the two lines are not parallel?

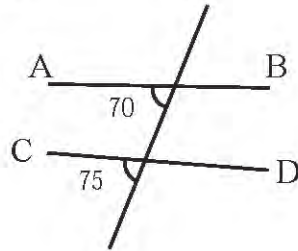
Example 2

Determine if the lines are parallel or not based on the corresponding angle from the figures given below.

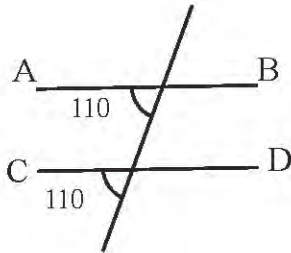
(a)



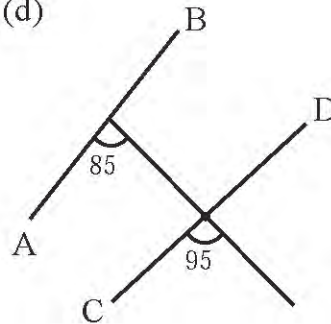
(b)



(c)



(d)



Solution

a. Yes

b. no

c. Yes

d. No

13.2.3 Relationship between the Co-interior angles formed when two parallel lines are transversal

Activity 5

Draw three different figures on your notebook in which two straight lines are transversal by the transversal, as shown below.

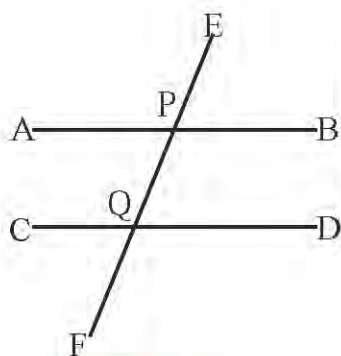


Figure (a)

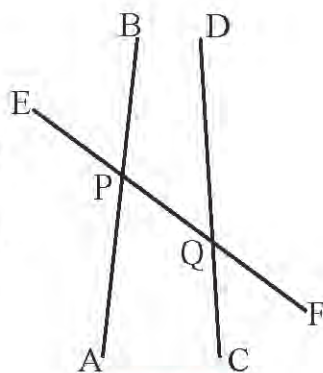


Figure (b)

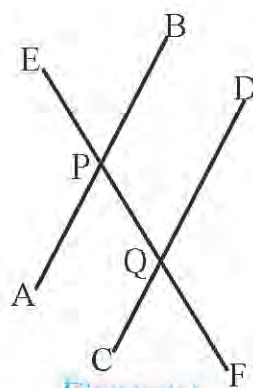


Figure (c)

Measure the co-interior angles in the figure above with a protractor and record the results in the table. Using set square, indicate if the lines are parallel or not and record it in the table.

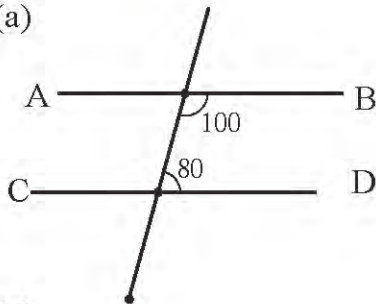
Figure	Pair I		$\angle APQ + \angle PQC$		Pair II		$\angle BPQ + \angle PQD$		Result (relationship between Co-interior angles)	Lines are (Parallel/ Not Parallel)
(a)	$\angle APQ$	$\angle PQC$			$\angle BPQ$	$\angle PQD$				
(b)										
(c)										
Conclusion: When two parallel lines are transversed by a transversal, then the sum of the co-interior angles is two right angles.										

Question to think: What is the sum of Co-interior angles if the two lines are not parallel?

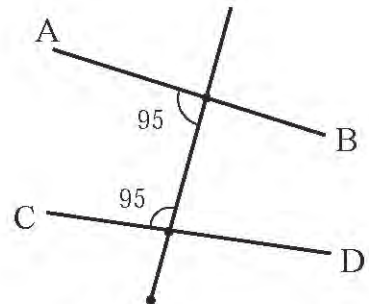
Example 3

Determine if the lines are parallel or not based on the co-interior angle from the figures given below.

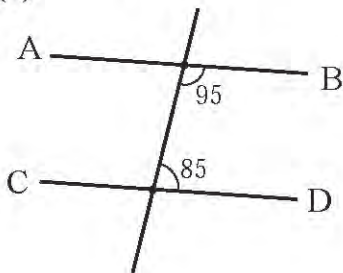
(a)



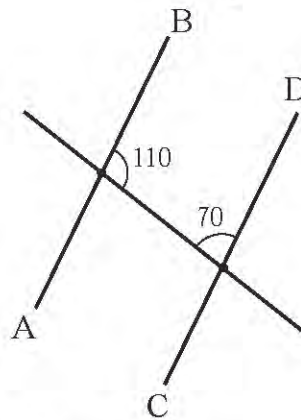
(b)



(c)



(d)



Solution

a. Yes

b. No

c. Yes

d. Yes

Example 4

AB and CD are parallel lines in the given figure. Find the measurement of the unknown angles based on the known angles.

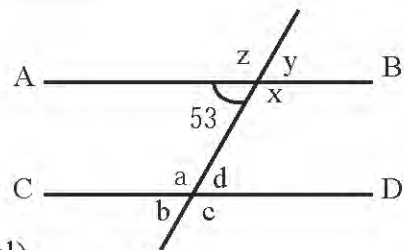
Solution:

Let us suppose the other unknown angles be a , b , c , d , x , y , z as shown in the figure alongside.

$$b = 53^\circ \quad (\text{Corresponding angles are equal})$$

$$d = 53^\circ \quad (\text{Alternate angles are equal})$$

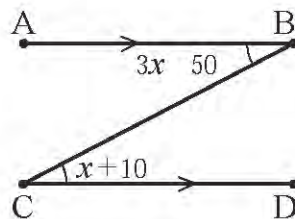
$$y = d = 53^\circ \quad (\text{Corresponding angles are equal})$$



$a + 53^\circ = 180^\circ$	(Sum of co-interior angles is 180°)
$z = a = 127^\circ$	(Corresponding angles are equal)
$x = a = 127^\circ$	(Alternate angles are equal)
$c = a = 127^\circ$	(Vertically opposite angles are equal)

Example 5

In the given figure, $AB \parallel CD$. Find the value of $\angle ABC$ and $\angle DCB$



Solution

Here,

$$3x - 50^\circ = x + 10^\circ \quad (\text{Alternate angles are equal})$$

$$\text{or, } 3x - x = 50^\circ + 10^\circ$$

$$\text{or, } 2x = 60^\circ$$

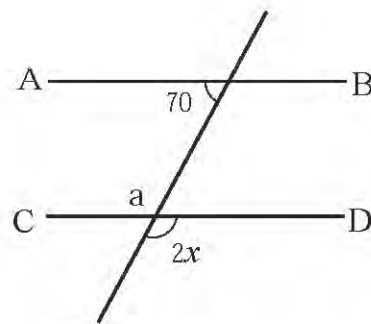
$$\therefore x = 30^\circ$$

$$\begin{aligned} \therefore \angle ABC &= 3x - 50^\circ = (3 \times 30^\circ) - 50^\circ \\ &= 90^\circ - 50^\circ = 40^\circ \end{aligned}$$

$$\therefore \angle ABC = \angle DCB = 40^\circ$$

Example 6

For what value of x are the lines AB and CD parallel in the figure?



Solution:

Here,

$$a = 2x \quad (\because \text{Vertically opposite angles are equal.})$$

$$\text{To be } AB \parallel CD, a + 70^\circ = 180^\circ$$

$$\text{or, } a + 70^\circ = 180$$

$$\text{or, } 2x = 180^\circ - 70$$

$$\text{or, } 2x = 110^\circ$$

$$\therefore x = \frac{110}{2}$$

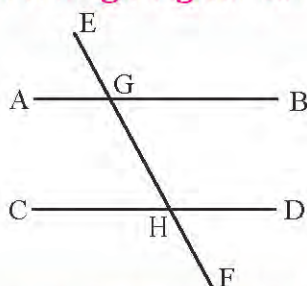
Hence, if $x = 55^\circ$, then, the lines are parallel.

Exercie 13.1

1. Fill in the blanks.

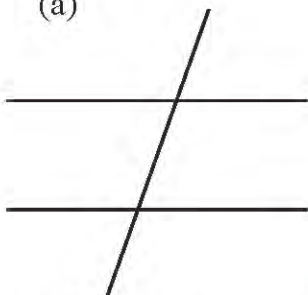
- If the two lines are parallel, the alternate angles are
- If the two lines are parallel, the corresponding angles are.....
- If the two lines are parallel, the sum of the co-interior angles is

2. Write the pair of alternate angles, corresponding angles, and co-interior angles from the figure given below.



3. Determine if the lines are parallel or not by finding the measurement of the angles from the figures given below.

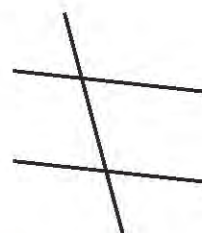
(a)



(b)



(c)



4. Draw a transversal line on each of the following figure. Determine if the lines are parallel or not by finding the measurement of the angles.

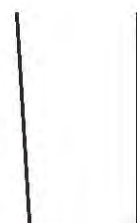
(a)



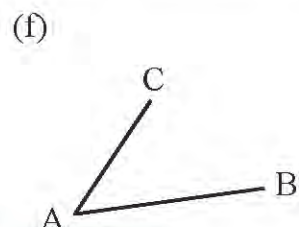
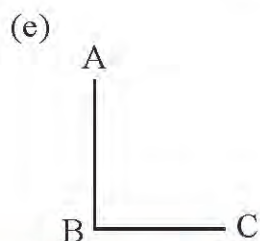
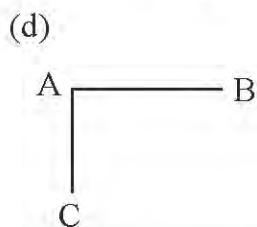
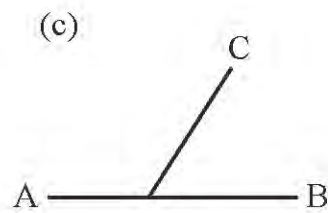
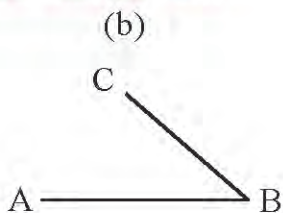
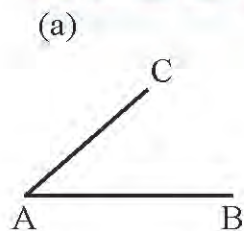
(b)



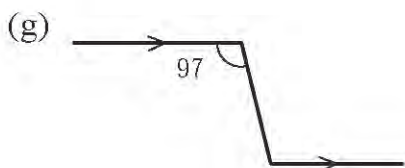
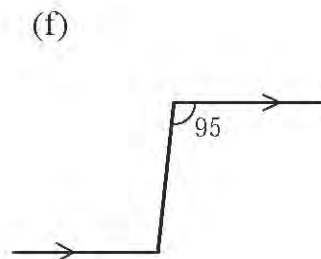
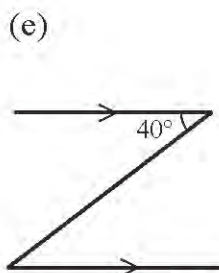
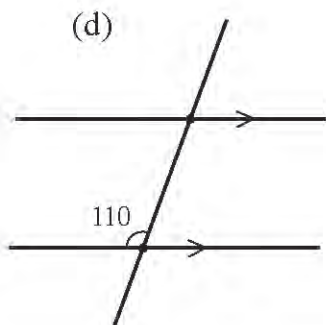
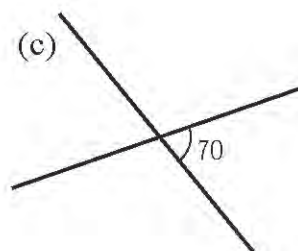
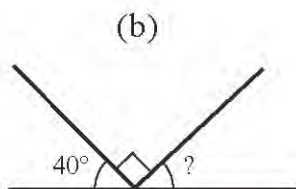
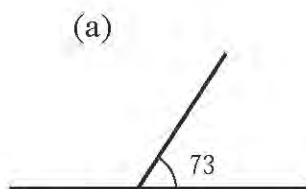
(c)



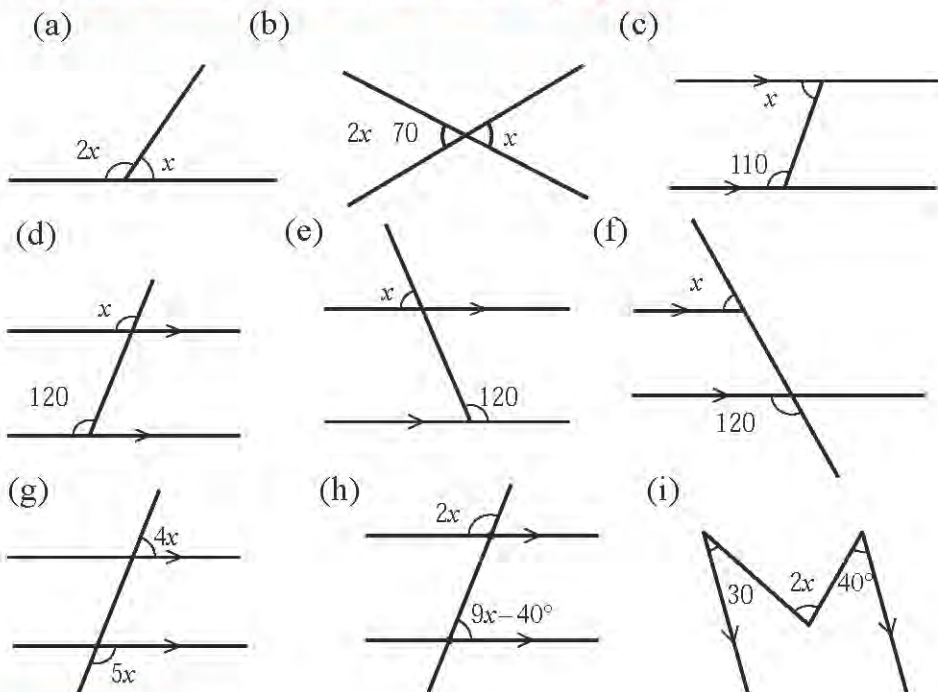
5. Draw the line CD such that $AB \parallel CD$



6. Find the value of unknown angle from the given figures.



7. Find the value of x from the following figures.



Project Work

1. Take a long bamboo sticks or thick sticks and tie them to each other so that they remain on the top of two straight sticks. Rotate each stick turn by turn so that they make different angles.
2. Indicate alternate angles, corresponding and co-interior in such case.
3. Note down the condition of important angles while rotating the stick.
4. Write your findings on the chart paper for presentation.
5. Paste the chart paper on the classroom wall and present it. Note down your friend's new findings and discuss.

Answers

1-5 Show to your teacher.

6. (a) 107° (b) 50° (c) Show to your teacher (d) Show to your teacher
 (e) 40° (f) 95° (g) 97°
 7. (a) 60° (b) 70° (c) 70° (d) 120° (e) 60°
 (f) 20° (g) 20° (h) 20° (i) 35°

14.0 Review

Classify the following triangles on the basis of side and angles.

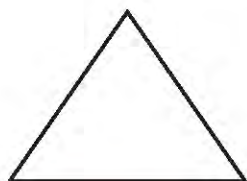


Figure (a)

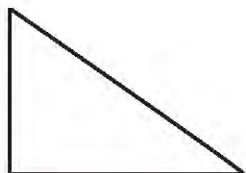


Figure (b)

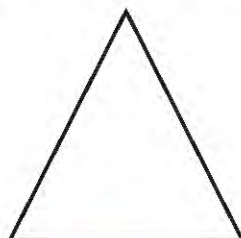


Figure (c)

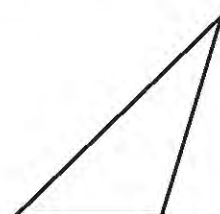


Figure (d)

14.1 Identification and verification of the properties of triangle

(a) Verification of the sum of interior angle.

Activity 1

Take a chart paper and construct a triangle of a certain measurement. Cut its angles as shown in the figure (b) and arrange them as shown in the figure (c). What type of angle is formed from their combination? Discuss and present it in your classroom.

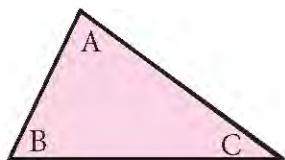


Figure (a)



Figure (b)



Figure (c)

The arrangement of angles of a given triangle forms a straight angle. The measurement of an angle in a straight line is 180° .

The sum of three interior angles of a triangle is 180° .

Activity 2

Construct three triangles of different shape and size as shown in the figure.

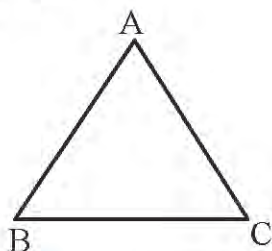


Figure (a)

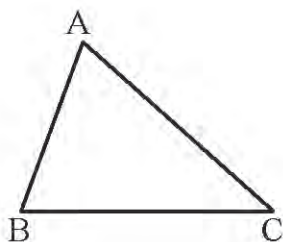


Figure (b)

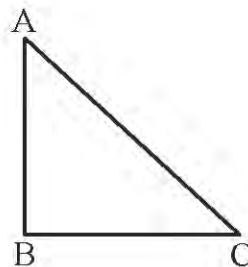


Figure (c)

Measure each angle of the triangles using a protractor and fill in the following table. Find the conclusion from the table.

figure	$\angle BAC$	$\angle ABC$	$\angle BCA$	$\angle BAC + \angle ABC + \angle BCA$	Result
(a)					
(b)					
(c)					
Conclusion:					

b Verification of the angles of an equilateral triangle

Activity 3

Draw an equilateral triangle on a chart paper. Cut the piece of triangle and remove it from the chart paper. Fold the triangle as shown in the figure and discuss the following questions.

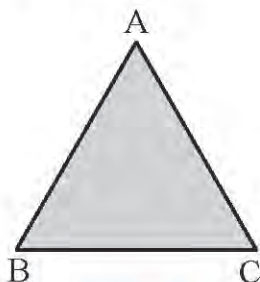


Figure (a)

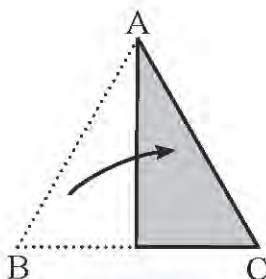


Figure (b)

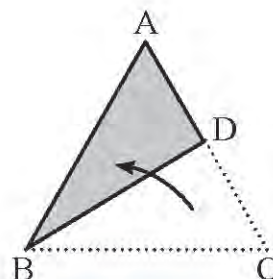


Figure (c)

- What is the relationship between the angles B and C, when the triangle is folded in such a way that the vertex B coincides with vertex C?
- What is the relationship between the angles C and A, when the triangle is folded in such a way that the vertex C coincides with A?
- Are all the angles of an equilateral triangle equal? Discuss and present the conclusion so obtained.

All the angles of an equilateral triangle are equal. So, each angle of an equilateral triangle is 60° .

Activity 4

Draw three equilateral triangles with different measurement as shown in the figure.

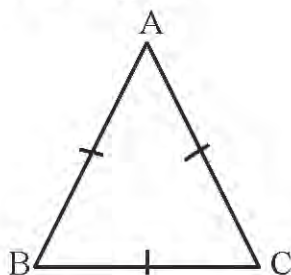


Figure (a)

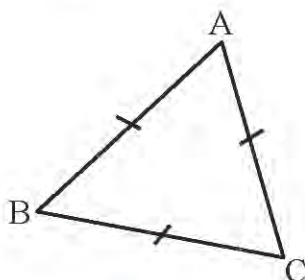


Figure (b)

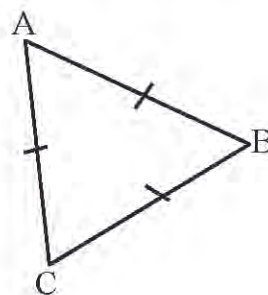


Figure (c)

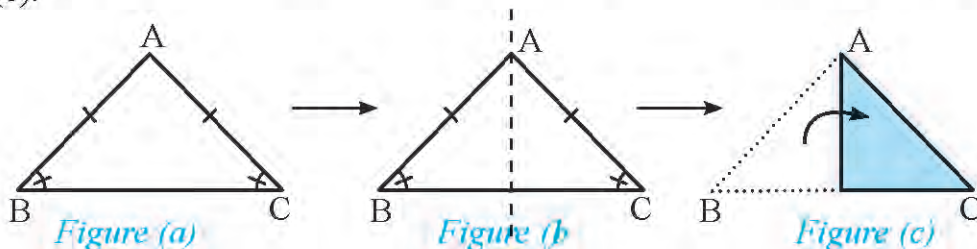
Measure each angle of the triangles using a protractor and fill in the following table. Write the conclusion obtained from the experiment.

Figure	$\angle CAB$	$\angle ABC$	$\angle BCA$	
(a)				
(b)				
(c)				
Conclusion:				

c Experimental verification of the base angles of an isosceles triangle

Activity 5

Take a chart paper and draw a triangle having two sides equal. Take out the piece of triangle cutting it with scissors. Mark the equal sides and the base angles of the triangle as shown in the figure (a). Fold the triangle in such a way that the vertex B coincides with vertex C as shown in the figure (b) and (c).

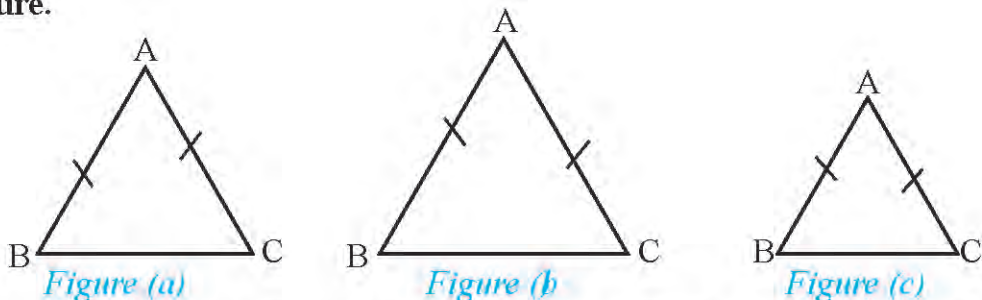


Discuss the relationship between the angles B and C in your group.

The base angles of an isosceles triangle are equal.

Activity 6

Draw three isosceles triangles of different measurement as shown in the figure.



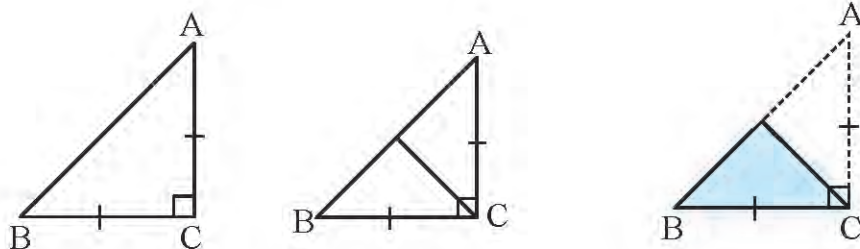
Measure the base angles using a protractor and fill in the following table. Write the conclusion obtained from the experiment.

Figure	$\angle ABC$	$\angle ACB$	Result
(a)			
(b)			
(c)			
Conclusion:			

(d) Experiential verification of the base angles of a right angled isosceles triangle

Activity 7

Draw a right angled isosceles triangle on a paper and take out the triangle cutting it with a scissor. Fold the triangle as shown in the figure and discuss the following questions.



- (a) In the right angled isosceles triangle ABC, $\angle ACB = 90^\circ$ and $BC=AC$. The triangle is folded in such a way that the vertex A coincides with Vertex B. What is the relationship between the angles A and B?
- (b) What is the relationship between the angles made by the equal sides with base of a right angled isosceles triangle?
- (c) What are the measurements of the base angles of a right angled isosceles triangle?

The base angles of a right angled isosceles triangle are equal and each is 45° .

Activity 8

Three right angled isosceles triangles are given in the following figures.

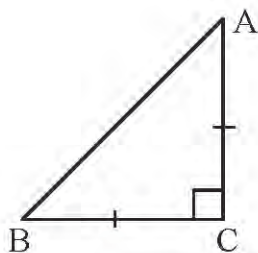


Figure (a)

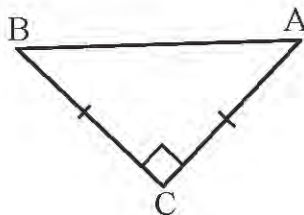


Figure (b)

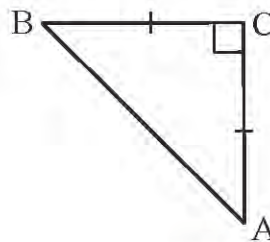


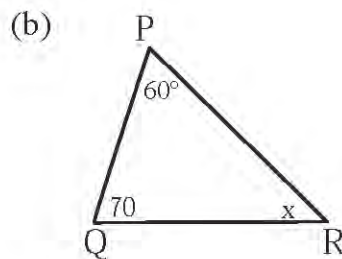
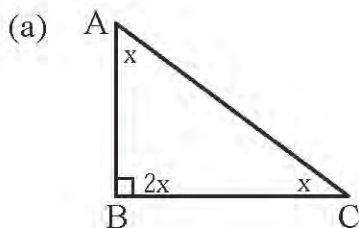
Figure (c)

Measure each base angle using a protractor and fill in the table. Write the conclusion obtained from the experiment.

Figure	$\angle CAB$	$\angle CBA$	Result
(a)			
(b)			
(c)			
Conclusion:			

Example 1

Find the value of x from the following figures.



Solution

Here, (a) $\angle ABC + \angle BCA + \angle CAB = 180^\circ$ [$\because$ The sum of three interior angles of a triangle is 180°].

$$\text{or, } 2x + x + x = 180^\circ$$

$$\text{or, } 4x = 180^\circ$$

$$\therefore x = 45^\circ$$

Hence, the value of x is 45° .

Alternative method,

ABC is a right angled isosceles triangle.

So, $\angle ABC = 90^\circ$ and $\angle BCA = \angle CAB = 45^\circ$

Hence, the value of x is 45° .

- (b) $\angle PQR + \angle QRP + \angle RPQ = 180^\circ$ [$\because$ The sum of three interior angles of a triangle is 180°].

$$\text{or, } 70^\circ + x + 60^\circ = 180^\circ$$

$$\text{or, } x + 130^\circ = 180^\circ$$

$$\text{or, } x = 180^\circ - 130^\circ$$

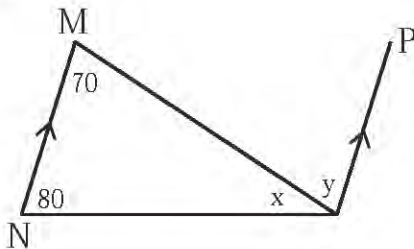
$$\therefore x = 50^\circ$$

$$\text{Hence, } x = 50^\circ$$

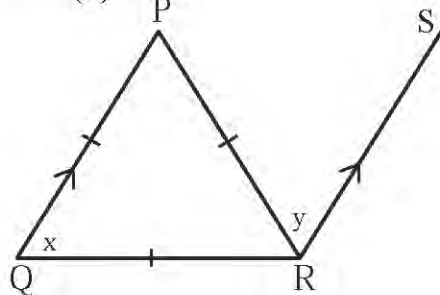
Activity 2

Find the values of x and y from the following figures.

(a)



(b)



Solution:

Here, (a) $\angle MOP = \angle OMN$ [$\because$ The alternate angles are equal when the transversal line OM cuts the lines. The alternate angles are equal when the transversal line OM cuts the lines NM//OP.]

$$\text{or, } y = 70^\circ$$

Similarly,

$\angle MNO + \angle NMO + \angle MON = 180^\circ$ [$\because$ The sum of three interior angles of a triangle is 180° .]

$$\text{or, } 80^\circ + 70^\circ + x = 180^\circ$$

$$\text{or, } 150^\circ + x = 180^\circ$$

$$\text{or, } x = 180^\circ - 150^\circ$$

$$\therefore x = 30^\circ$$

Hence, the values of x and y are 30° and 70° respectively.

- (b) The given triangle PQR is an equilateral triangle. All the angles of an equilateral triangle are equal.

$$\text{So, } \angle PQR = \angle QRP = \angle RPQ = 60^\circ.$$

$$\therefore x = 60^\circ$$

And, $\angle QRP = \angle PRS$ [$\because$ Alternate angles are equal when the parallel lines $QP \parallel RS$ are cut by the transversal PR].

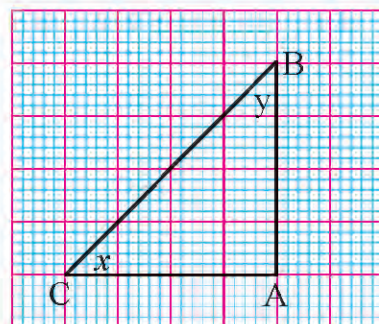
$$\therefore y = 60^\circ$$

Hence, the value of x and y is 60° each.

Example 3

A triangle is given in the figure.

- What type of triangle is given in the figure on the basis of angles and sides?
- Find the values of x and y .
- Find the relation of the angles on the basis of values of x and y .



Solution:

- (a) In the triangle ABC, $AB = AC$, and $\angle BAC = 90^\circ$

So, $\triangle BAC$ is a right angled isosceles triangle.

- (b) $\angle ACB = x$ and $\angle ABC = y$ are the base angles.

Since the sum of the three angles of a triangle is 180°],

$$x + y + 90^\circ = 180^\circ$$

$$x + y = 90^\circ \dots \dots (i)$$

Again, the base angles of an isosceles triangle are equal.

$$\text{So, } \angle ACB = \angle ABC,$$

$$x = y \dots \dots (ii)$$

From (i) and (ii),

$$x + x = 90^\circ$$

$$\text{or, } 2x = 90^\circ$$

Let's $x = 45^\circ$

Also, $y = 45^\circ$

(c) Since $x = y = 45^\circ$, $\angle ACB = \angle ABC$

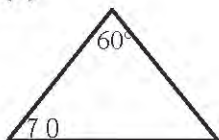
Exercise 14.1

1. Identify whether the following statements are true or false. Correct the statements if they are false.

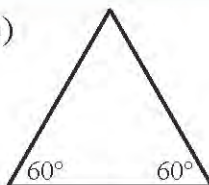
- (a) The sum of interior angles of a triangle is 360° .
- (b) All the angles of an equilateral triangle are equal.
- (c) All the angles of an isosceles triangle are equal.
- (d) Each base angle of a right angled isosceles triangle is 60° .

2. Name the following triangles and identify which of them are isosceles triangles. Also, write the name of their equal sides.

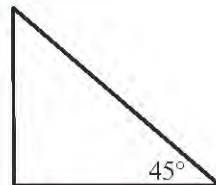
(a)



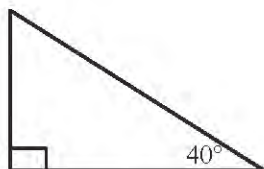
(b)



(c)



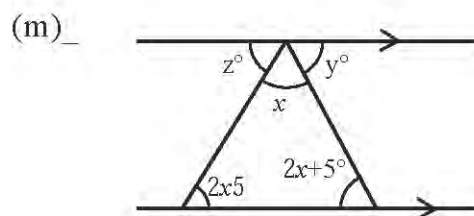
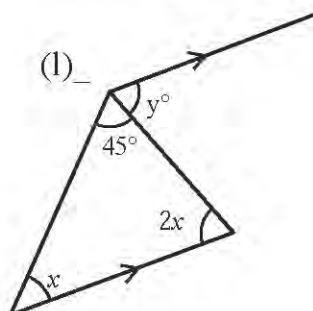
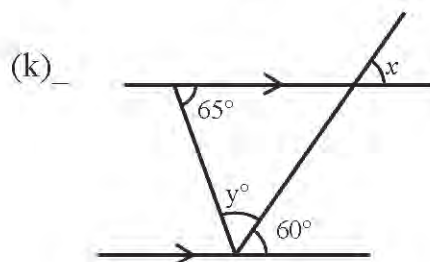
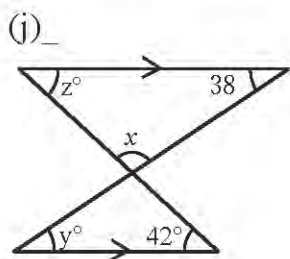
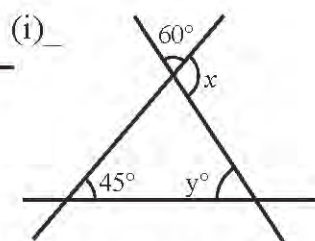
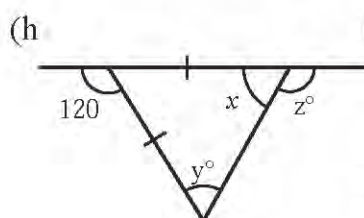
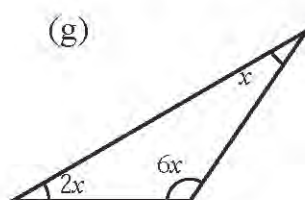
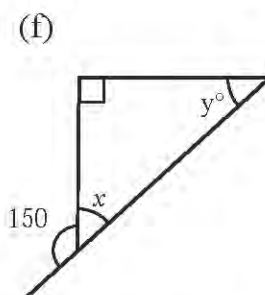
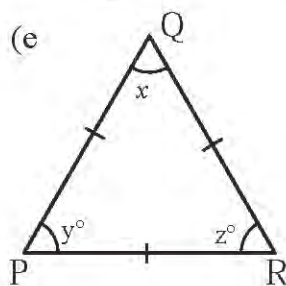
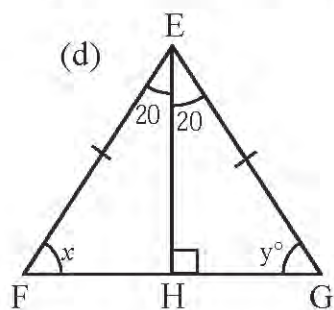
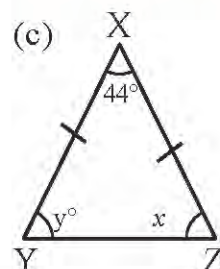
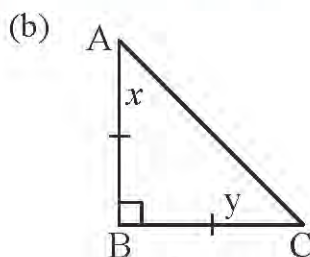
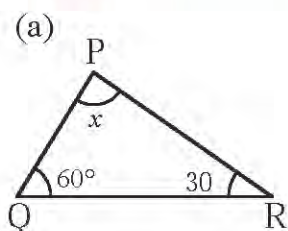
(d)



(e)



3. Find the values of x , y , and also, the unknown angles from the figures.



4. (a) Find the base angles of an isosceles triangle if its vertex angle is 70° .

(b) In the triangle $\triangle ABC$, if $\angle BAC = 45^\circ$, $\angle ABC = 2\angle BCA$,

(i) Find the measurement of $\angle ABC$ and $\angle BCA$.

(ii) What type of triangle is ABC? Answer with reason.

(c) If the interior angles of a triangle are in the ratio of 2:3:4, find the angles.

5. (a) In the given figure, the side QR of $\triangle PQR$ is extended to the point S.

(i) Find the measurement of $\angle QPR$, $\angle PQR$ and $\angle PRS$.

(ii) What is the value of $\angle QPR + \angle PQR$?

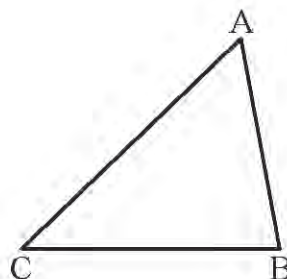
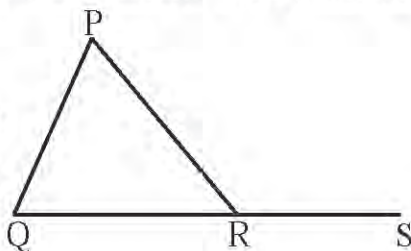
(iii) Find the relationship between $\angle QPR + \angle PQR$ and $\angle PRS$.

(b) In the figure, $\triangle ABC$ is given.

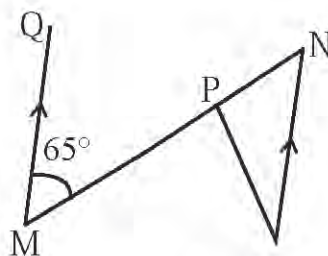
(i) Find the measurement of sides AB, BC and AC.

(ii) What is the value of $AB + BC$?

(iii) What is the relationship between $AB + BC$ and AC ?



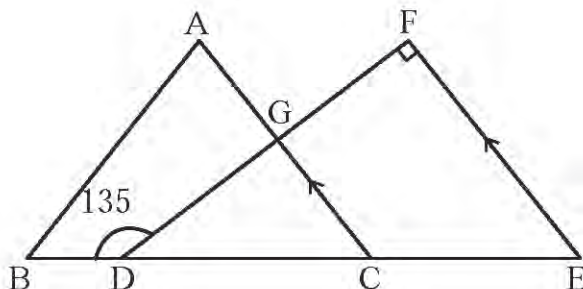
6. (a) In the given figure : $MQ \parallel ON$,
 $\angle PN : \angle NOP = 5:2$, $\angle QMN = 65^\circ$.
 Find the value of $\angle NPO$.



(b) In the given figure :

$AC \parallel FE$, $\angle BDF$
 $= 135^\circ$, $DF \perp EF$

(i) What is the value of $\angle CDG$?

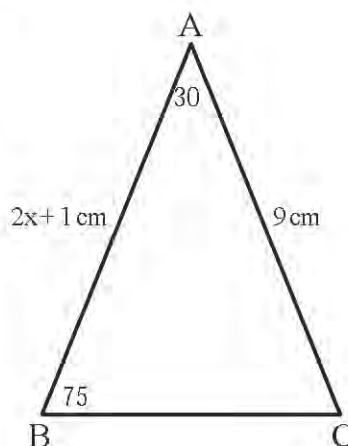


(ii) Write the relationship between the sides GD and GC.

(iii) What type of triangle is CGD?

(c) $\triangle ABC$ is given in the figure.

If $\angle BAC = 30^\circ$, $\angle ABC = 75^\circ$,
 $AB = (2x + 1)$ cm and $AC = 9$ cm, find
the value of x .



Answers

1 and 2: Show to your teacher.

3. (a) 90° (b) $x = y = 45^\circ$ (c) $x = y = 68^\circ$
(d) $x = y = 70^\circ$ (e) $x = y = z = 60^\circ$ (f) $x = 30^\circ$, $y = 60^\circ$
(g) $x = 20^\circ$ (h) $x = y = 60^\circ$, $z = 120^\circ$
(i) $x = 120^\circ$, $y = 75^\circ$ (j) $x = 100^\circ$, $y = 38^\circ$, $z = 42^\circ$
(k) $x = 60^\circ$, $y = 55^\circ$ (l) $x = 45^\circ$, $y = 90^\circ$
(m) $x = 36^\circ$, $y = 77^\circ$, $z = 67^\circ$
4. (a) 55° (b) (i) 90° and 45° (ii) Right angled isosceles
(c) 40° , 60° , 80°
5. Show to your teacher.
6. (a) 89° (b) (i) 45° (ii) equal
(iii) Right angled isosceles

14.2 Identification and verification of the properties of rhombus, trapezium and kite

Activity 9

Study the following quadrilaterals given in the figures. Discuss in your pair and identify what types of quadrilateral they are. Then, write their characteristics in the table.

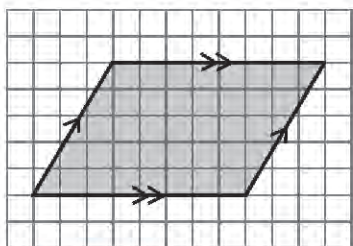


Figure (a)

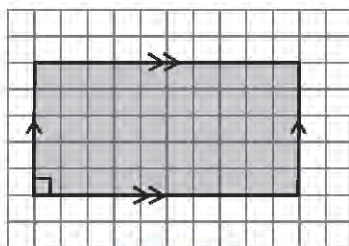


Figure (b)

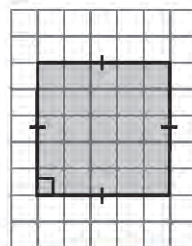


Figure (c)

Quadrilaterals and their characteristics

Figure no.	Name of quadrilateral	Definition	Characteristics

(a) Identification and verification of the properties of Rhombus

Activity 10

Collect 8 pieces of stick or straws of equal length. Construct different closed shapes with 4 straws in each. Observe the similarities and differences between the shapes and discuss in your group. Also present the conclusion in your class.

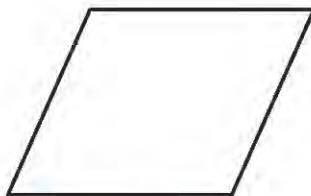


Figure (a)



Figure (b)

Both the shapes given in the figure are rhombus. The rhombus given in figure (b) is a square.

S. No.	Similarities	Differences
(i)	All four sides are equal	Each angle of a square is 90° but it may not be 90° in case of rhombus.
(ii)	The sum of all four angles is 360°	All angles of a square are equal but may not be equal in rhombus. Only the opposite angles are equal in case of rhombus.
(iii)	The opposite sides are parallel	The diagonals are equal in square but the diagonals in rhombus may not be equal.
All squares are rhombuses but all rhombuses may not be squares.		

Activity 11

There are three rhombuses with different measurements in the figures. The diagonals AC and BD are intersecting at the point O.

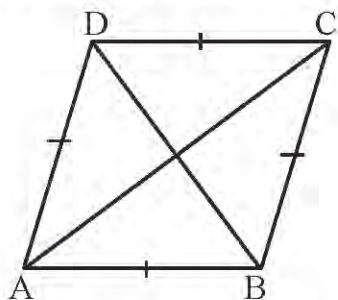


Figure (a)

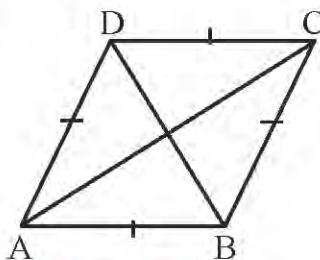


Figure (b)

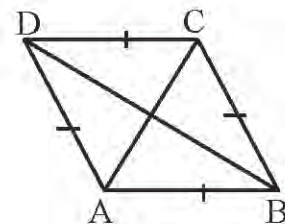


Figure (c)

Take the measurements of AO, CO, BO, DO, $\angle AOB$ and $\angle BOC$ and fill the following table. Also, write the conclusion.

Figure No.	A	C	B	D	$\angle AOB$	$\angle BOC$	Result
(a)							
(b)							
(c)							
Conclusion:							

Is your conclusion same to the following statement?

The diagonals of a rhombus bisect each other at right angle.

Activity 12

There are three rhombuses with different measurements in the given figure. Diagonals are drawn in each rhombus.

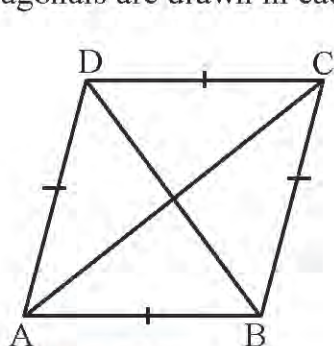


Figure (a)

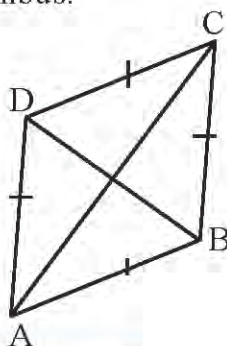


Figure (b)

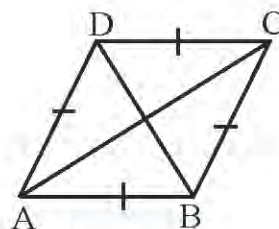


Figure (c)

Measure the angles using a protractor and fill in the table. Write the conclusion obtained from the experiment.

Figure no.	$\angle ADB$	$\angle BDC$	$\angle ACD$	$\angle ACB$	$\angle ABD$	$\angle CBD$	$\angle BAC$	$\angle CAD$	Result
(a)									
(b)									
(c)									
Conclusion:									

Is your conclusion same to the following statement?

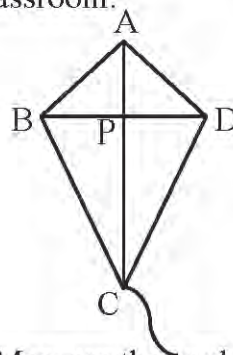
The diagonals of a rhombus bisect the vertical angles .

(b) Identification and verification of the properties of kite

Activity 13

A figure of kite is given aside. Discuss in your group how it can be made. Make a kite using paper. Draw the similar figure on a paper. Discuss the following questions and present the conclusion in your classroom.

- (i) Which sides of kite ABCD are equal?
- (ii) How many diagonals are there?
- (iii) Are the diagonals equal?
- (iv) If the diagonals are not equal, identify the longer and shorter diagonals.
- (v) Are BP and PD equal?
- (vi) What are the measurements of $\angle APB$ and $\angle APD$? Measure the angles using protractor.
- (vii) How many isosceles triangles are there in the kite? What are they?



In the kite ABCD, there are two pairs of adjacent sides $AB = AD$ and $BC = DC$. The diagonal AC is longer and BD is shorter. Also, $BP = PD$ and $\angle APB = \angle APD = 90^\circ$.

A quadrilateral having two pairs of equal adjacent sides is a Kite. The longer diagonal bisects the other at right angle. The angles between the non-equal sides are equal.

Activity 14

Draw three kites with different measurements as shown in the figure and also draw the diagonals.

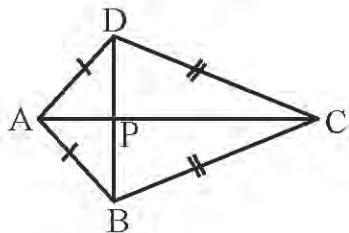


Figure (a)

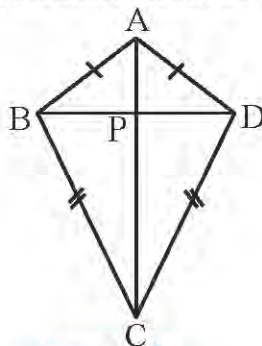


Figure (b)

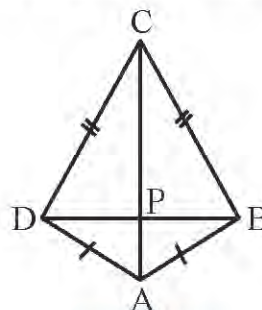


Figure (c)

Take the measurements of BP, DP, $\angle APB$ and $\angle APD$ and fill in the table. Write the conclusion obtained from the experiment.

Figure no.	BP	DP	$\angle APB$	$\angle APD$	Result
(a)					
(b)					
(c)					
Conclusion:					

Is your conclusion same to the following statement?

The longer diagonal of a kite bisects the shorter diagonal at right angle.

Activity 15

Draw three kites with different measurements as shown in the figure and also draw their diagonals.

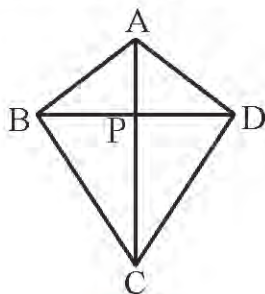


Figure (a)

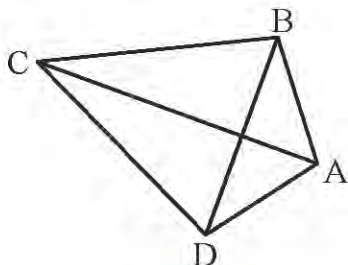


Figure (b)

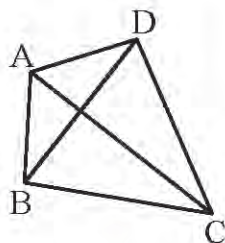


Figure (c)

Take the measurements of AB, AD, BC and CD and fill in the table. Write the conclusion obtained from the experiment.

Figure no.	In $\triangle ABD$		In $\triangle BCD$		Result
	AB	AD	BC	CD	
(a)					
(b)					
(c)					
Conclusion:					

Is your conclusion same to the following statement?

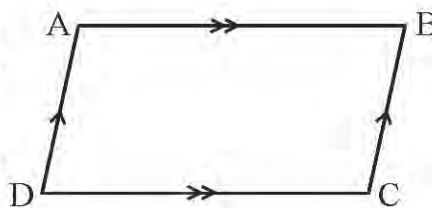
The shorter diagonal divides the kite into two isosceles triangles.

(c) Identification and verification of the properties of trapezium

Activity 16

Discuss the following questions in your group.

- (a) Which shape is given in the figure?
- (b) What is the relationship between the sides AD and BC, and AB and DC ? Are they parallel?
- (c) Are the opposite angles $\angle BAD$ and $\angle BCD$, and $\angle ADC$ and $\angle ABC$ equal?
- (d) What will be the shape if only one pair of sides is parallel?
- (e) Are the opposite angles and sides are equal in the quadrilateral so formed?



If a quadrilateral with only one pair of parallel sides is drawn, it looks like the following quadrilaterals. Such a quadrilateral is called trapezium.

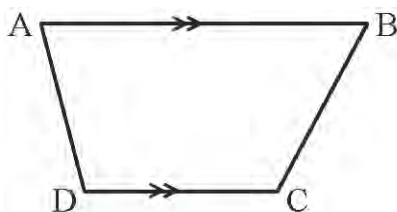


Figure (a)

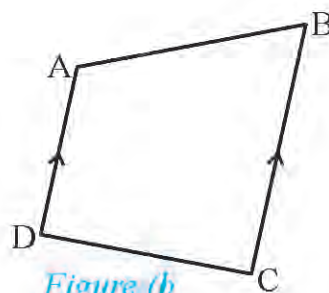


Figure (b)

A quadrilateral having a pair of parallel sides is called trapezium.

Activity 17

Draw three trapeziums with different measurements as shown in the figure.

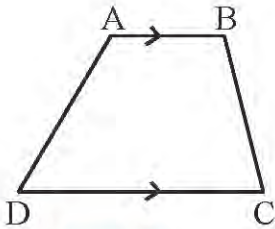


Figure (a)

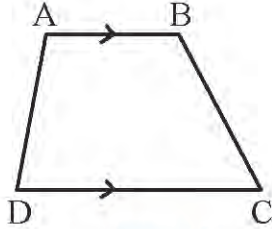


Figure (b)

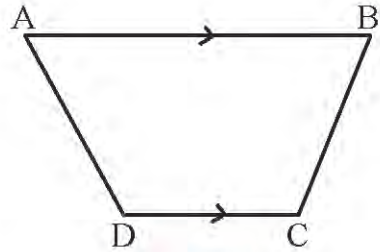


Figure (c)

Measure the co-interior angles using a protractor and fill in the following table.

Figure no.	$\angle DAB + \angle ADC$	$\angle ABC + \angle BCD$	Result
(a)			
(b)			
(c)			

Conclusion:

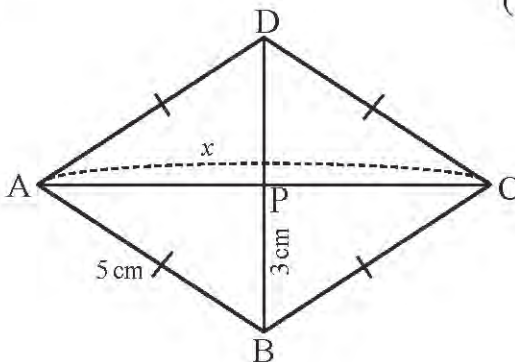
Is your conclusion same to the following statement?

The sum of co-interior angles on each of non-parallel sides is 180° . So, a pair of opposite sides in a trapezium is parallel.

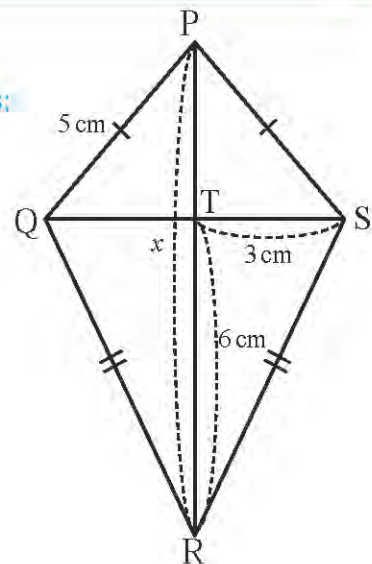
Example 1

Find the value of x in the following quadrilaterals:

(a)



(b)



Solution

- (a) The quadrilateral ABCD is a rhombus, in which $AB = 5$ cm, $BP = 3$ cm, $AC = ?$

Here, $\angle APB = \angle BPC = 90^\circ$. [$\therefore$ The diagonals of a rhombus bisect each other at right angle]

In the right angled triangle $\triangle APB$,

$$AB^2 = AP^2 + BP^2 \quad [\therefore \text{According to the pythagorus theorem } h^2 = p^2 + b^2]$$

$$\text{Or, } 5^2 = AP^2 + 3^2$$

$$\text{Or, } 25 = AP^2 + 9$$

$$\text{Or, } AP^2 = 25 - 9$$

$$\therefore AP = 4 \text{ cm}$$

$$\text{Hence, } AC = x = 2 \times 4 = 8 \text{ cm}$$

- (b) The quadrilateral PQRS is a kite, in which $PQ = 5$ cm, $ST = 3$ cm and $TR = 6$ cm. $PR = ?$

Here, $\angle PTQ = 90^\circ$ and $QT = 3$ cm. [$\therefore$ The longer diagonal of a kite bisects shorter diagonal at right angle.]

In the right angled triangle $\triangle PTQ$,

$$PQ^2 = PT^2 + QT^2 \quad [\therefore \text{According to the pythagorus theorem, } h^2 = p^2 + b^2]$$

$$\text{Or, } 5^2 = PT^2 + 3^2$$

$$\text{Or, } 25 = PT^2 + 9$$

$$\text{Or, } PT^2 = 25 - 9$$

$$\therefore PT = 4 \text{ cm}$$

$$\text{Hence, } PR = x = PT + RT = 4 + 6 = 10 \text{ cm}$$

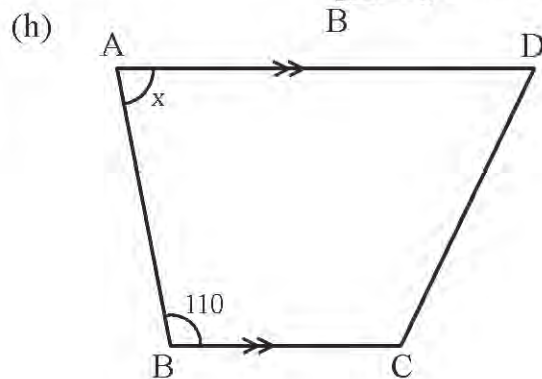
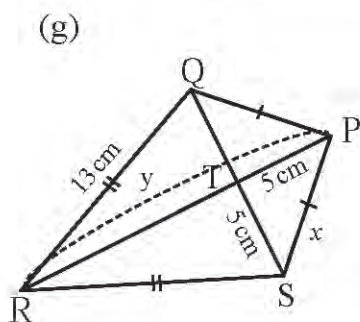
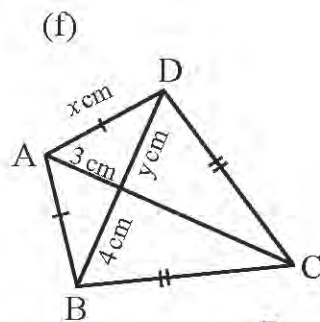
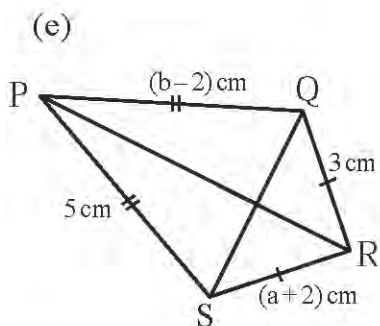
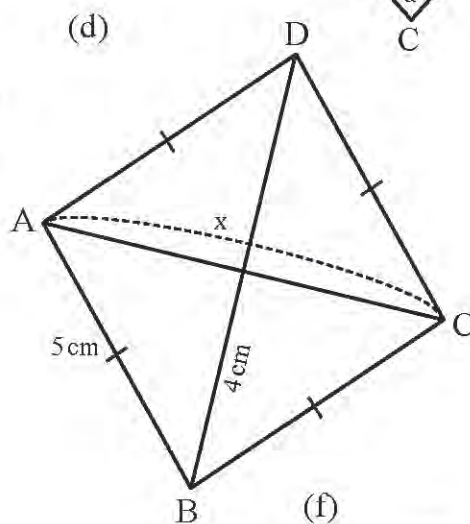
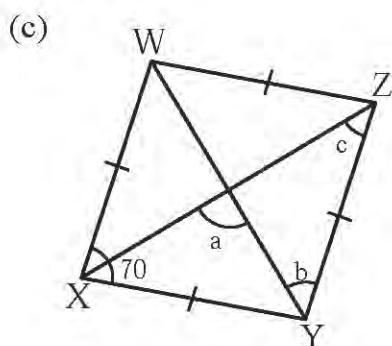
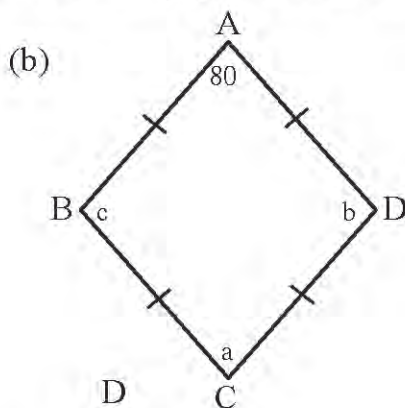
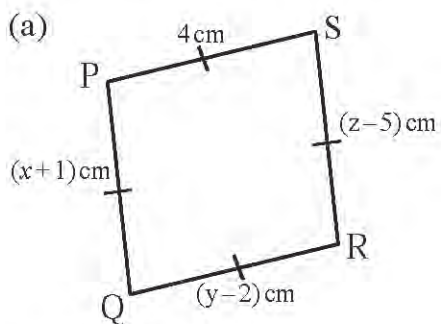
Exercise 14.2

1. Identify whether the following statements are true or false. Write the statements correctly if they are false.
 - (a) All rhombuses are square.
 - (b) All the angles of a rhombus are equal.
 - (c) The shorter diagonal of a kite bisects the longer diagonal.
 - (d) The diagonals of a rhombus bisect each other.
 - (e) The angle between the diagonals of a rhombus is 90° .
 - (f) The opposite sides of a trapezium are parallel.
 - (g) All trapeziums are parallelogram.
2. The various characteristics of quadrilateral regarding to the side, angle and parallelism are given in the following table. Put True or False marks below the quadrilaterals to each characteristic.

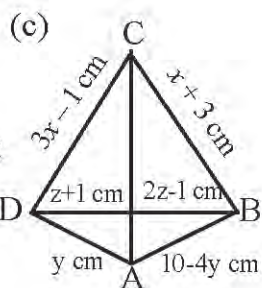
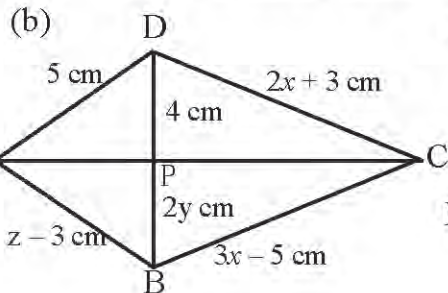
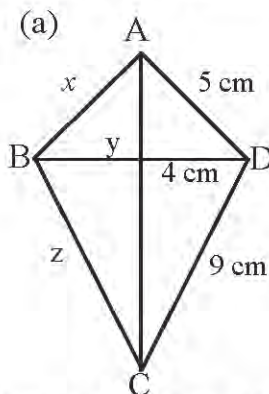
Characteristics	Quadrilaterals				
	Parallelogram	Rhombus	Trapezium	Square	Rectangle
All the angles are equal.					
All the angles are right angle.					
The pair of opposite angles is equal.					
All sides are equal.					
The pair of opposite sides are equal.					
A pair of opposite sides are parallel.					
Both the pairs of opposite sides are parallel.					

3. Using a diagram, show the relationship between the different quadrilaterals given in question no.2 on the basis of various characteristics.

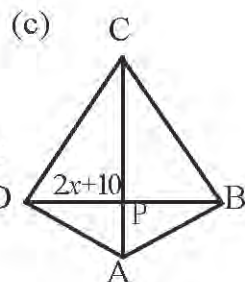
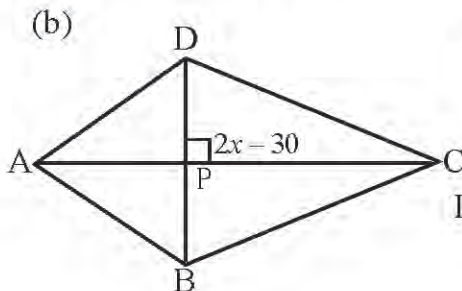
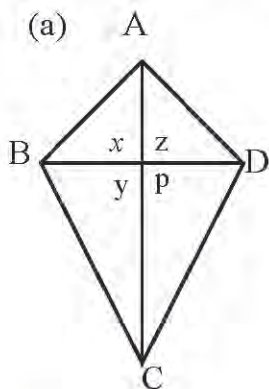
4. Find the values of a , b , c , x , y and z in the following quadrilaterals.



5. Find the values of x , y , in the following kites.



6. Find the values of x , y , in the following kites.



Answers

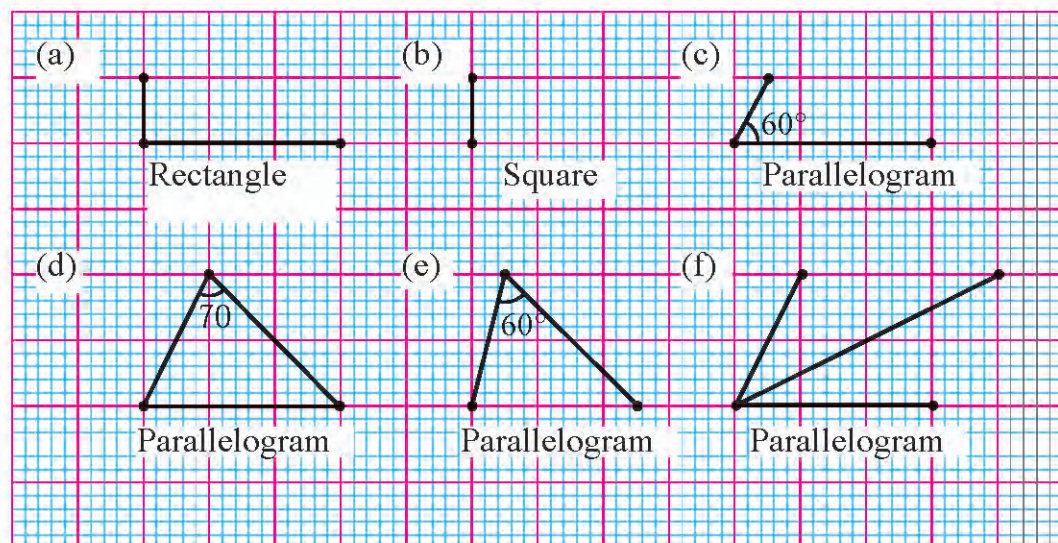
Show to the teacher for question no. 1 to 3

4. (a) $x = 3$, $y = 6$, $z = 9$ (b) $a = 80^\circ$ $b = c = 100^\circ$
 (c) $a = 90^\circ$, $b = 55^\circ$, $c = 35^\circ$ (d) $x = 6$ cm
 (e) $a = 1$, $b = 7$ (f) $x = 5$ cm $y = 4$ cm
 (g) $x = 5\sqrt{2}$ cm, $y = 17$ cm (h) $x = 70^\circ$
5. (a) $x = 5$ cm, $y = 4$ cm, $z = 9$ cm (b) $x = 8$ cm, $y = 2$ cm, $z = 8$ cm
 (c) $x = 2$ cm, $y = 2$ cm, $z = 2$ cm
6. (a) $x = y = z = p = 90^\circ$ (b) $x = 60^\circ$ (c) $x = 40^\circ$

14.3 Construction

Activity 18

Complete the shape of the following quadrilaterals.

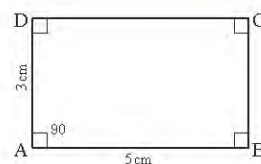


14.3.1 Construction of rectangle (When two adjacent sides are given.)

Activity 19

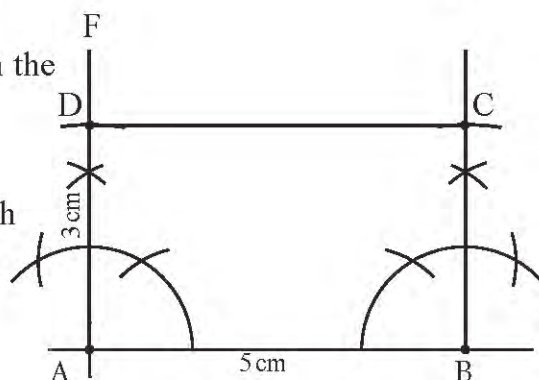
Discuss the following questions:

- What is rectangle?
- What is the measurement of each angle of a rectangle?
- What is the relationship between the adjacent sides of a rectangle?



H

Now, construct a rectangle with adjacent sides of length 5 cm and 3 cm.



The steps of construction are as follows:

1. Sketch the rectangle roughly.
2. Draw a base line and mark $AB = 5$ cm.
3. Draw an angle of 90° at A with the help of compass. Then, $\angle FAB = 90^\circ$
4. Take an arc of radius 3 cm from A and cut the line AF at the point D.
5. Draw an angle of 90° at B with the help of compass. Then, $\angle HBA = 90^\circ$
6. Take an arc of radius 3 cm from B and cut the line BH at the point C.
7. Join C and D.

Now, the construction of rectangle ABCD is completed.

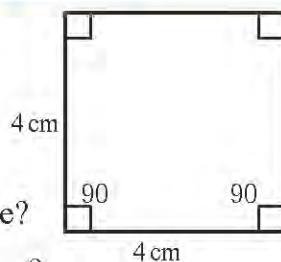
14.3.2 Construction of a square (When a side is given)

Activity 20

Discuss the following questions:

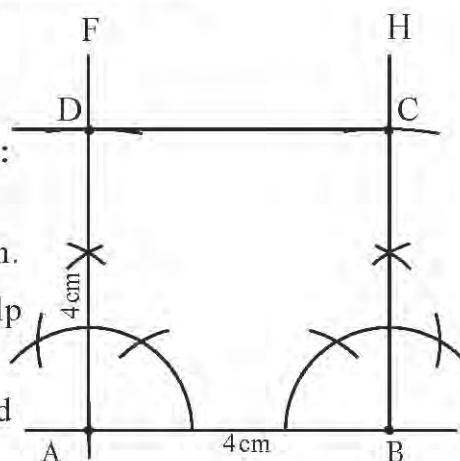
- (a) What is a square?
- (b) What is the measurement of each angle of a square?
- (c) What is the relationship between all sides of a square?

Now, construct a square having a side of length 4 cm.



The steps of construction are as follows:

1. Sketch the square roughly.
2. Draw a base line and mark $AB = 4$ cm.
3. Draw an angle of 90° at A with the help of compass. Then, $\angle FAB = 90^\circ$
4. Take an arc of radius 4 cm from A and cut the line AF at the point D.



5. Draw an angle of 90° at B with the help of compass. Then, $\angle HBA = 90^\circ$.
6. Take an arc of radius 4 cm from B and cut the line BH at the point C.
7. Join C and D.

Now, the construction of square ABCD is completed.

14.3.3 Construction of a parallelogram

Activity 21

Discuss the following questions:

- (a) What is a parallelogram?
- (b) What are the characteristics of a parallelogram?

Discuss how a parallelogram is constructed in the following conditions.

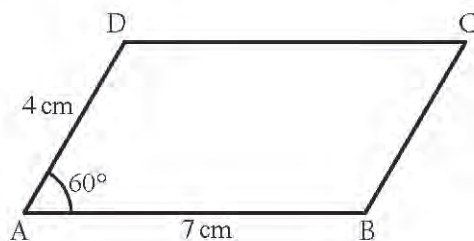
- a Construction of a parallelogram when the measurements of two adjacent sides and the angle between them are given**

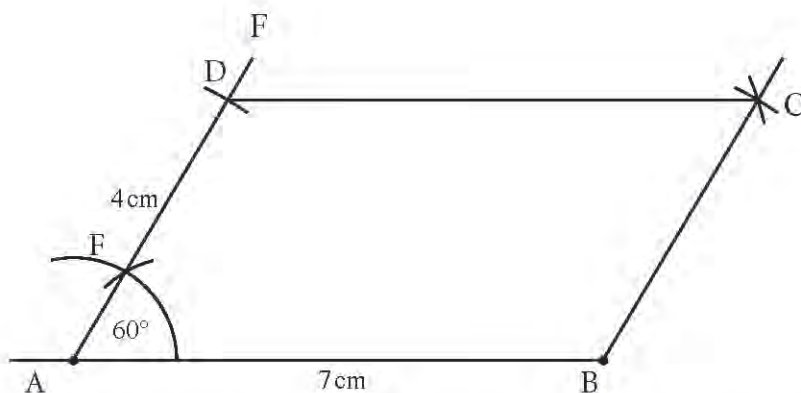
Construct a parallelogram having adjacent sides of length 7 cm and 4 cm and the angle between them is 60° .

The steps of construction are as follows:

1. Sketch the parallelogram roughly.
2. Draw a base line and mark $AB = 7$ cm.
3. Draw an angle of 60° at A with the help of compass.

Then, $\angle FAB = 60^\circ$.





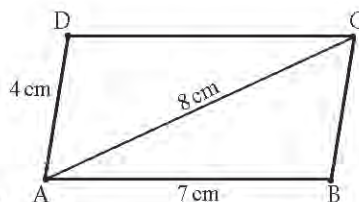
4. Take an arc of radius 4 cm from A and cut the line AF at the point D.
5. Take an arc of radius equal to AD and cut from the point B upside. Also, take an arc of radius equal to AB and cut from the point D on the right side. Mark the point of intersection of two arcs by C.
6. Join C and D. Also, join B and C.
7. Now, the construction of parallelogram ABCD is completed.

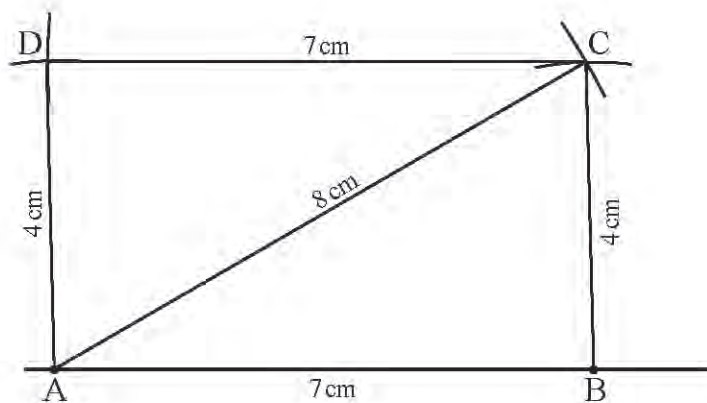
b Construction of a parallelogram when the measurements of two adjacent sides and a diagonal are given

Construct a parallelogram having adjacent sides of length 7 cm and 4 cm and a diagonal of length 8 cm.

The steps of construction are as follows:

1. Sketch the parallelogram roughly.
2. Draw a base line and mark $AB = 7$ cm.
3. Draw an arc of radius 8 cm from A and also an arc of radius 4 cm from B so that they meet at the point C.
4. Join A and C. Also, join B and C. As a result, a triangle ABC is formed.





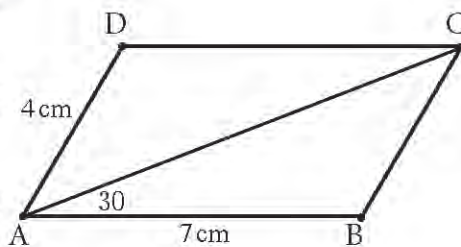
5. Draw an arc of radius 4 cm from the point A and also an arc of radius 7 cm from the point C. Mark their point of intersection by D.
6. Join A and D. Also, join C and D.
7. Now, the construction of parallelogram ABCD is completed.

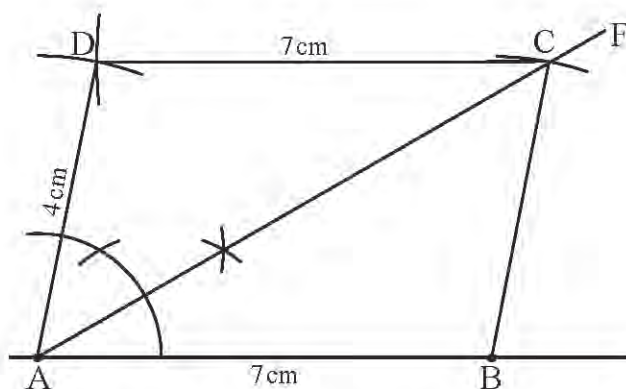
c Construction of a parallelogram when the measurements of two adjacent sides and the angle between a side and a diagonal are given

Construct a parallelogram ABCD, in which the adjacent sides are $AB = 7$ cm, $AD = 4$ cm and the angle between the side AB and a diagonal is 30° .

The steps of construction are as follows:

1. Sketch the parallelogram roughly.
2. Draw a base line and mark $AB = 7$ cm.
3. Draw an angle of 30° with the help of compass. Then, $\angle FAB = 30^\circ$.
4. Take an arc of radius 4 cm from B and cut the line AF at the point C.
5. Join B and C. As a result a triangle ABC is formed.

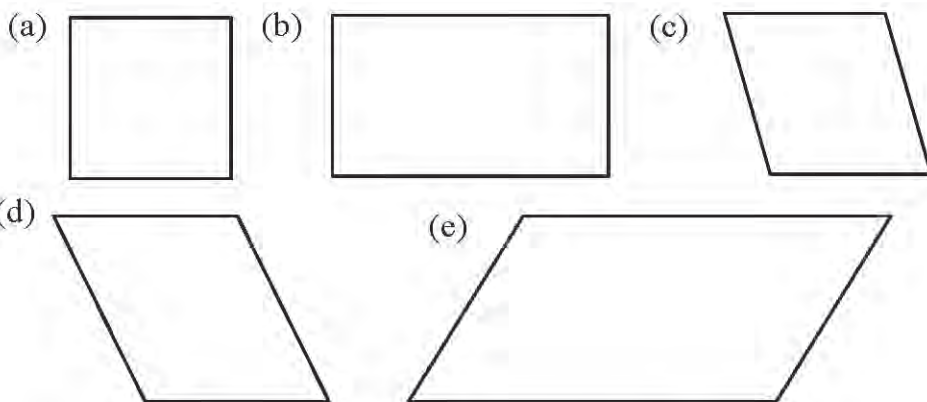




6. Draw an arc of radius 4 cm from the point A and also an arc of radius 7 cm from the point C. Mark their point of intersection by D.
7. Join A and D, and also join C and D
8. Now, the construction of a parallelogram is completed.

Exercise 14.3

1. Draw the shapes with the same measurements as given in the following figures.



2. Draw rectangles with the following measurements:
 - (a) Rectangle ABCD with $AB = 7$ cm and $AD = 4$ cm
 - (b) Length 6 cm and breadth 4 cm
 - (c) Adjacent sides with length 8 cm and 5 cm.
 - (d) Take the measurements of length and breadth yourself.

3. Construct the rectangles with the following measurements.
 - (a) Rectangle ABCD with $AB = 7$ cm and $AD = 4$ cm
 - (b) Length = 6 cm and breadth = 4 cm
 - (c) Adjacent sides with length 8 cm and 5 cm
4. Construct the squares with the following measurements.
 - (a) Length of a side is 4 cm
 - (b) Length of a side 4.5 cm
 - (c) Take a length yourself.
5. Construct the parallelograms with the following measurements.
 - (a) Adjacent sides of length 3 cm and 5 cm, and the angle between the adjacent sides is 45° .
 - (b) Adjacent sides of length 4 cm and 6 cm, and a diagonal of length 7cm
 - (c) A side of length 4 cm and diagonal of length 8 cm. The angle made by the diagonal with the side of length 4 cm is 60° .

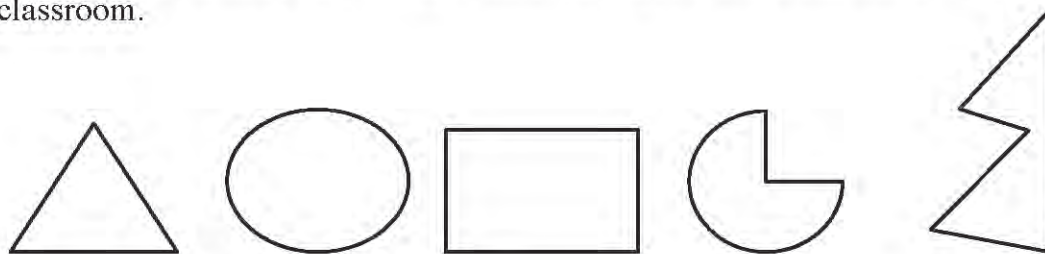
Answers

Show to your teacher.

14.4 Polygon

Activity 22

Observe the following figures. Discuss the questions regarding their shapes, number of sides and closed shape nature. Also, present the conclusion in your classroom.



The closed geometrical shapes having three or more than three sides are called polygons. The polygon having three sides is a triangle. Similarly quadrilateral, pentagon and hexagon are the polygons having four, five and six sides respectively.

14.4.1 Regular polygon

Activity 23

Using ruler and protractor, measure the sides and angles of the following polygons, and also discuss the following questions in your group.



- (a) Are the sides of polygon equal in each figure?
- (b) Are the angles of polygon equal in each figure?

(c) Which polygons have equal sides and angles?

(d) Which polygons do not have equal sides and angles?

The polygons in the figure (a), (b) and (c) have equal sides and angles. The polygon in the figure (c) has equal angles but only opposite sides equal. The polygon in the figure (e) has only base angles equal. So, the polygons given in the figure (a), (b) and (d) are regular polygons and the polygons in the figure (c) and (e) are not the regular polygons.

A polygon having all sides and all interior angles equal is called regular polygon. An equilateral triangle is a regular triangle; a square is a regular quadrilateral. Similarly, regular polygons having more than four sides are named as regular pentagon, regular hexagon etc.

14.4.2 Measurement of interior angles of regular polygon

Activity 24

There are different types of regular polygons in the figures given below. Draw diagonals to each polygon to form triangles as shown in the figure (a) and fill the number of triangles in the table.

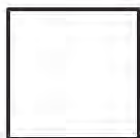


Figure (a)

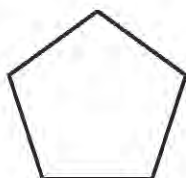


Figure (b)

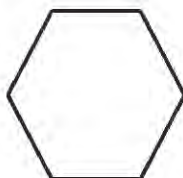


Figure (c)



Figure (d)

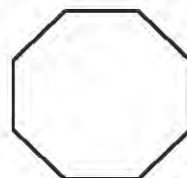
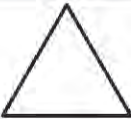
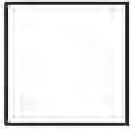
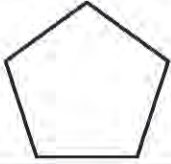
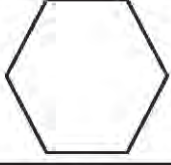
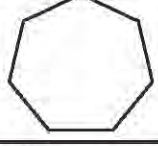


Figure (e)

Name of Regular polygon	Number of sides	Number of triangles
Quadrilateral	4	2

Activity 25

Observe the following shapes and measure their interior angles using a protractor. Fill in the following table on the basis of your observation and the result of activity 24 as well. Discuss the following questions in your pair.

Figure of regular Polygon	Name of regular polygon	Number of sides	Number of triangles	Sum of interior angles
	Triangle	3	1	180°
				
				
				
				
Polygon with sides 'n'				

- What is the relationship between the number of sides and the number of triangles formed by diagonals?
- What is the relationship between the number of triangles formed by diagonals and the sum of interior angles?
- What is the relationship between the number of sides and the sum of interior angles of a polygon?
- What is the sum of interior angles of a polygon having n sides?
- What is the sum of interior angles of an octagon?

The sum of interior angles = $(n - 2) \times 180^\circ$ and each angle of a regular polygon is $= \frac{(n-2)}{n} \times 180^\circ$

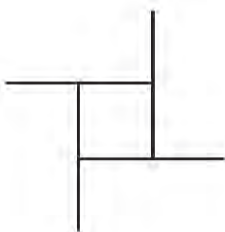
14.4.2 Measurement of exterior angles of regular polygon

Activity 26

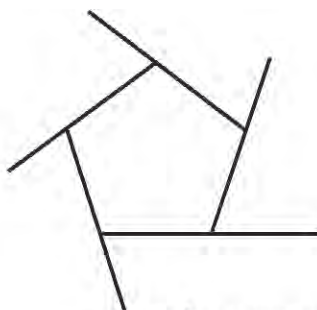
In the following figures, the sides of regular polygons are extended to form exterior angles. Draw more regular polygons to form exterior angles by extending their sides. Now, measure all the interior and exterior angles using protractor and fill in the following table.



Equilateral triangle



Square



Regular pentagon

Name of the regular polygon	Measurement of an interior angle	Measurement of an exterior angle	Sum of the exterior angles
Polygon having n side			

If the number of sides of a polygon is n,

The sum of exterior angles = 360° and each exterior angle of a regular polygon is $\frac{360^\circ}{n}$.

Example 1

Find the sum of interior angles and the sum of exterior angles of a polygon having 5 sides.

Solution

Here,

Number of sides of a polygon (n)=5

We know,

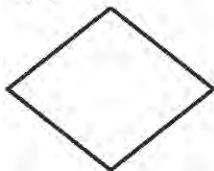
$$\begin{aligned}
 \text{Sum of the interior angles} &= (n - 2) \times 180 \\
 &= (5 - 2) \times 180 \\
 &= 3 \times 180 \\
 &= 540^\circ
 \end{aligned}$$

The sum of exterior angles = 360°

Example 14.4

1. Identify which of the followings are polygons and which are not.

(a)



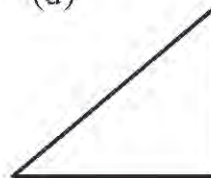
(b)



(c)

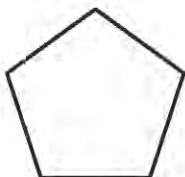


(d)

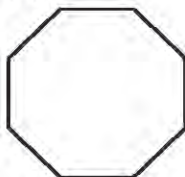


2. Write the number of sides and name of the following polygons.

(a)



(b)



(c)

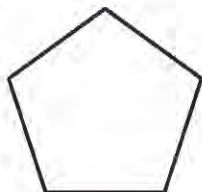


(d)

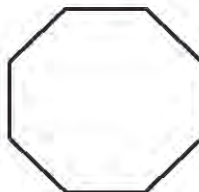


3. Draw the triangles as far as possible from the following polygons

(a)



(b)



4. Find the measurement of an interior angle of the regular polygon having the following number of sides.

(a) $n = 7$

(b) $n = 10$

(c) $n = 12$

5. Find the measurement of an exterior angle of the regular polygon having the following number of sides.

(a) $n = 6$

(b) $n = 9$

(c) $n = 11$

6. Find the sum of interior angles of the regular polygons having the following number of sides.
- (a) $n = 6$ (b) $n = 9$ (c) $n = 10$ (d) $n = 11$
7. Find the sum of exterior angles of the regular polygons having the following number of sides.
- (a) $n = 6$ (b) $n = 9$ (c) $n = 10$ (d) $n = 11$

Project work

Divide the students of your class into proper groups. Take a flag of Nepal in each group and study the different shapes in it. Discuss how the flag of Nepal can be made. Prepare the steps of construction and follow it to make the flag. Also, present your work in the classroom.

Answers

Show to your teacher.

Lesson 15

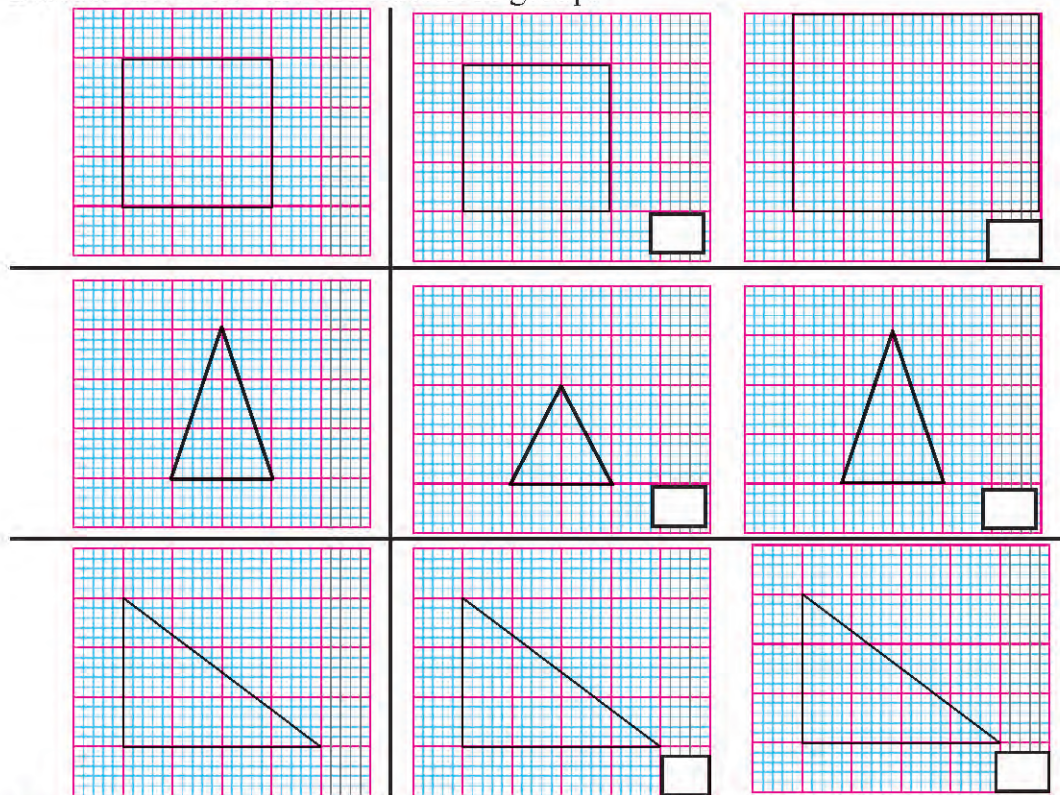
Congruency and Similarity

15.0 Review

Observe the following pictures.

- If the given figure has the same shape and equal measurements, write A.
- If the shape is the same but the measurements are not equal, write B.
- If neither the shape is the same nor the measurements are equal, write C.

Discuss and write the conclusion in group.



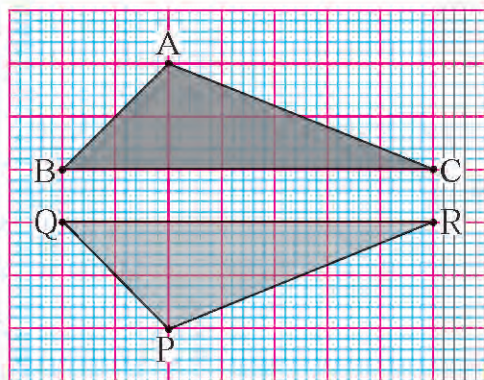
15.1 Congruent triangles

Activity 1

- * Take a graph paper or square paper.
- * Make two triangles ABC and PQR as shown in the figure.
- * Cut the figures and fold them. Find out if both triangles are equal or not.

Based on the discussion with your peer, analyze the conclusions you have drawn and discuss them.

In the given figure, $\angle BAC = \angle QPR$, $\angle ABC = \angle PQR$ and $\angle BCA = \angle QRP$, $AB = PQ$, $BC = QR$, and $AC = PR$. Here, $\angle BAC$ and $\angle QPR$, $\angle ABC$ and $\angle PQR$, $\angle BCA$ and $\angle QRP$ are corresponding angles. Sides AB and PQ , BC and QR , AC and PR are corresponding sides. Therefore, $\triangle ABC$ and $\triangle PQR$ congruent.



When two triangles have the same shape and size, they are called congruent. In congruent triangles, corresponding sides and angles are equal. In the figure triangles ABC and PQR are congruent. Symbolically, it can be written $\triangle ABC \cong \triangle PQR$

15.2 Test of congruent triangles

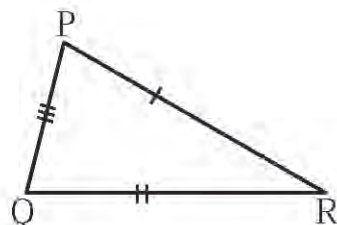
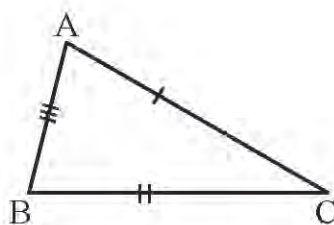
On what condition can $\triangle PQR$ be constructed congruent with $\triangle ABC$? Discuss.

a Side Angle and Side SAS :

Activity 2

First make a triangle ABC as shown in the figure.

Construct triangle PQR with sides PQ and QR and the included angle PQR , which are respectively equal in order to sides AB , BC , and the included angle ABC of triangle ABC .



Now, fill in the given table with the measurements of the remaining angles and sides, and complete the missing entries to draw a conclusion:

Bases	In triangle ABC	In triangle PQR	Result
Conditions of construction	$AB =$	$PQ =$	$AB = PQ$
	$\angle ABC =$	$\angle PQR$	$\angle ABC = \angle PQR$
	$BC =$	$QR =$	$BC = QR$
Parts of examination	$AC =$	$PR =$	
	$\angle BAC =$	$\angle QPR =$	
	$\angle ACB =$	$\angle PRQ =$	

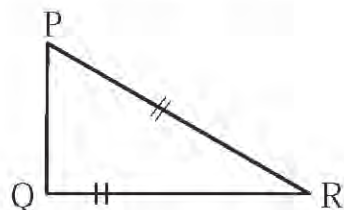
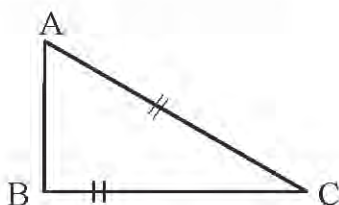
Conclusion: here, two sides and the included angle of $\triangle ABC$ are equal to the two sides and the included angle of $\triangle PQR$. The other corresponding sides and angles are also _____. So, the $\triangle ABC$ and $\triangle PQR$ are _____ triangles.

If two sides and the angle between them of one triangle are equal to the corresponding two sides and the angle between them of another triangle, then the triangles are congruent. it is known as SAS axiom.

(b) Right angle hypotenuse Side(RHS)

Activity 3

Construct a right angle triangle ABC with an angle at B as shown in the figure
Construct a right angled triangle PQR with hypotenuse PR and side QR equal to hypotenuse AC and side BC respectively of triangle ABC.



Now, fill in the given table with the measurements of the remaining angles and sides, and complete the missing entries to draw a conclusion:

Bases	In triangle ABC	In triangle PQR	Result
Conditions of construction	BC =	QR =	BC = QR
	$\angle ABC = 90^\circ$	$\angle PQR = 90^\circ$	$\angle ABC = \angle PQR$
	AC =	PR =	AC = PR
Parts of examination	AB =	PQ =	
	$\angle CBA =$	$\angle RQP =$	
	$\angle ACB =$	$\angle PRQ =$	
Conclusion : Here, In right angled $\triangle ABC$ and $\triangle PQR$, right angle, hypotenuse and a side of $\triangle ABC$ are equal to the right angle, hypotenuse and a side of $\triangle PQR$. The other corresponding sides and angles are also _____. So, the $\triangle ABC$ and $\triangle PQR$ are _____ triangles.			

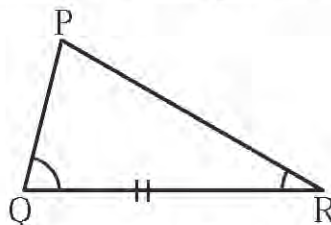
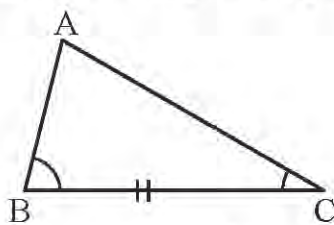
If the hypotenuse and one side of a right angled triangle are equal to the hypotenuse and one side of another right angled triangle, then the triangles are congruent by RHS.

c Angle-Side-Angle ASA

Activity 4

Make a triangle ABC as shown in the figure.

Construct a triangle $\triangle PQR$ such that two of its angles $\angle PQR$ and $\angle PRQ$ are equal to the angles $\angle ABC$ and $\angle ACB$ respectively, of triangle ABC, and side QR between these angles is equal to the side BC of triangle ABC.



Now, fill in the given table with the measurements of the remaining angles and sides, and complete the missing entries to draw a conclusion:

Bases	In triangle ABC	In triangle PQR	Result
Conditions of construction	$\angle ABC =$	$\angle PQR =$	$\angle ABC = \angle PQR$
	$BC =$	$QR =$	$BC = QR$
	$\angle ACB =$	$\angle PRQ =$	$\angle ACB = \angle PRQ$
Parts of examination	$AC =$	$PR =$	
	$AB =$	$PQ =$	
	$\angle CAB =$	$\angle RPQ =$	

Conclusion : Here, two angles and the included side of $\triangle ABC$ are equal to the two angles and the included side of $\triangle PQR$. The other corresponding sides and angles are also _____. So, the $\triangle ABC$ and $\triangle PQR$ are _____ triangles.

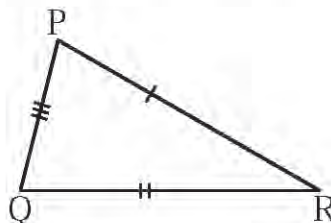
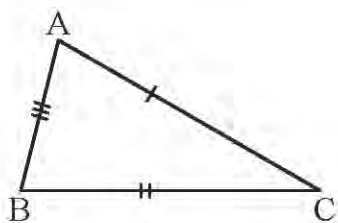
If two angles and a side between them of one triangle are equal to the two angles and a side between them of another triangle, then two triangles are congruent by ASA.

d Side- Side-Side SSS

Activity 5

Make a triangle ABC as shown in the figure.

Construct a triangle PQR such that its three sides, PQ, QR, and PR are respectively equal in length to the three sides AB, BC, and AC of triangle ABC.



Use protractor to measure the angles and complete the given table.

Bases	In triangle ABC	In triangle PQR	Result
Conditions of construction	AB =	PQ =	AB = PQ
	BC =	QR =	BC = QR
	AC =	PR =	AC = PR
Parts of examination	$\angle BAC =$	$\angle QPR =$	
	$\angle CBA =$	$\angle RQP =$	
	$\angle ACB =$	$\angle PRQ =$	

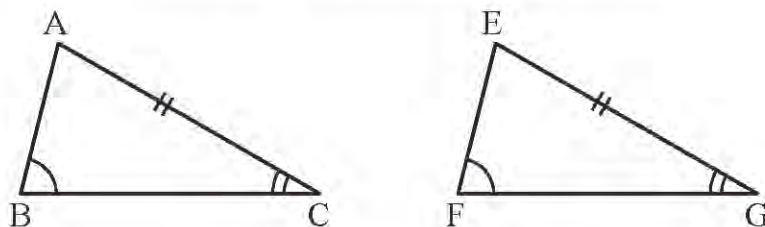
Conclusion : Here, all sides of $\triangle ABC$ are separately equal to all the sides of $\triangle PQR$. The corresponding angles are also _____. So, the $\triangle ABC$ and $\triangle PQR$ are _____ triangles.

If the three sides of one triangle are individually equal to the three sides of another triangle, then we can say that the triangles are congruent according by SSS.

e Angle-Angle-Side AAS

Activity 6

Construct a triangle EFG with $\angle EFG$, $\angle FGE$, and a side EG such that they are respectively equal to $\angle ABC$, $\angle ACB$, and a side AC of $\triangle ABC$.



Now, fill in the given table with the measurements of the remaining angles and sides, and complete the missing entries to draw a conclusion:

Bases	In triangle ABC	In triangle EFG	Result
Conditions of construction	$\angle ABC =$	$\angle EFG =$	$\angle ABC = \angle EFG$
	$\angle ACB =$	$\angle FGE =$	$\angle ACB = \angle FGE$
	$AC =$	$EG =$	$AC = EG$
Parts of examination	$BC =$	$FG =$	
	$AB =$	$EF =$	
	$\angle CAB =$	$\angle GEF =$	

Conclusion: Here, two angles and a non-included side of $\triangle ABC$ are equal to the corresponding two angles and a non-included side of $\triangle PQR$. The corresponding sides and angles are also _____. So, the $\triangle ABC$ and $\triangle PQR$ are _____ triangles.

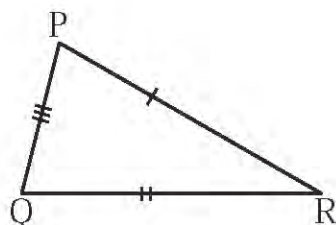
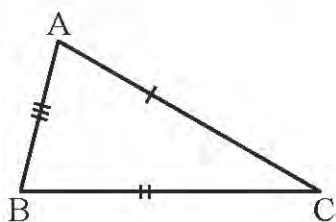
If a side of one triangle along with an adjacent angle, is equal to a side of another triangle, along with its adjacent angle, then the triangles are considered congruent when their corresponding parts are equal in order, i.e., when the remaining corresponding angle and the side opposite to it are also equal.

Questions to think: If any two sides and an angle of a triangle are equal to two sides and an angle of another triangle, those two triangles are also congruent, Analyze.

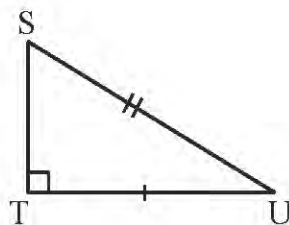
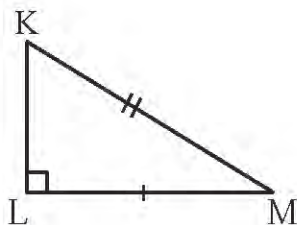
Example 1

1. Write the conditions of congruency for each of the following pairs of triangles with reasons?

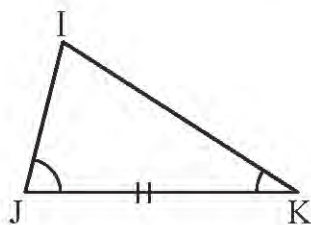
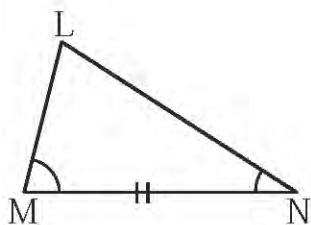
(a)



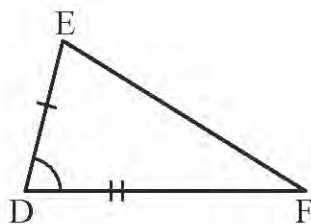
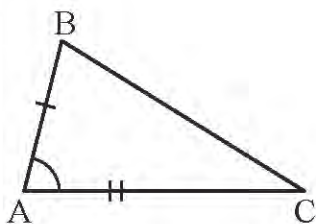
(b)



(c)



(d)



Solution

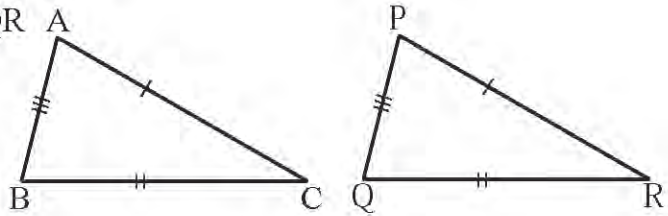
Here,

(a) Here, in $\triangle ABC$ and $\triangle PQR$

$$AB = PQ \text{ (S)}$$

$$AC = PR \text{ (S)}$$

$$BC = QR \text{ (S)}$$



Since all the three sides of the triangles $\triangle ABC$ and $\triangle PQR$ are equal to each other, the base of the triangle is corresponding to those triangles

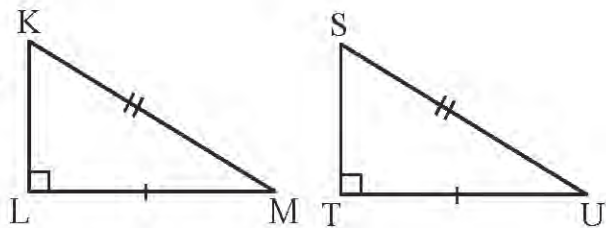
(b) Here, In $\triangle KLM$ and $\triangle STU$

$$\angle KLM = \angle STU = 90^\circ$$

$$MK = SU$$

$$LM = ST$$

$$\therefore \triangle KLM \cong \triangle STU \text{ [from RHS]}$$



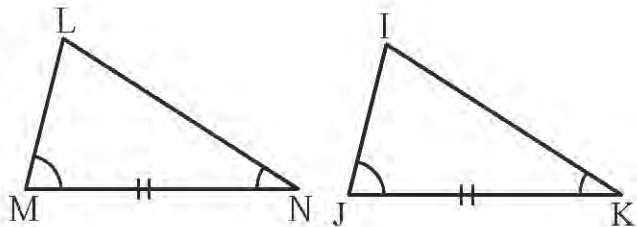
(c) Here, in $\triangle LMN$ and $\triangle IJK$

$$\angle LMN = \angle IJK$$

$$MN = JK$$

$$\angle LNM = \angle IKJ$$

$$\therefore \triangle LMN \cong \triangle IJK \text{ [from ASA]}$$



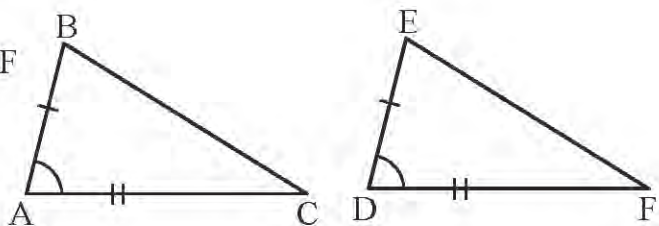
(d) Here, In $\triangle ABC$ and $\triangle DEF$

$$AB = DE$$

$$\angle BAC = \angle EDF$$

$$AC = DF$$

$$\therefore \triangle ABC \cong \triangle DEF \text{ [from ASA]}$$



Example 2

The given pair of triangles are congruent. Find the value of x and determine the measurement of the remaining angles and sides.

Solution

Here, triangles ABC and XYZ are congruent.

$$\angle A = \angle X = 35^\circ, \angle B = \angle Y = 123^\circ \text{ and } \angle C = \angle Z = 22^\circ$$

Again, $AB = XY$

$$\text{or, } (1.7x - 1.3) \text{ cm} = (3x - 3.9) \text{ cm}$$

$$\text{or, } 3x - 1.7x = 3.9 - 1.3$$

$$\text{or, } 1.3x = 2.6$$

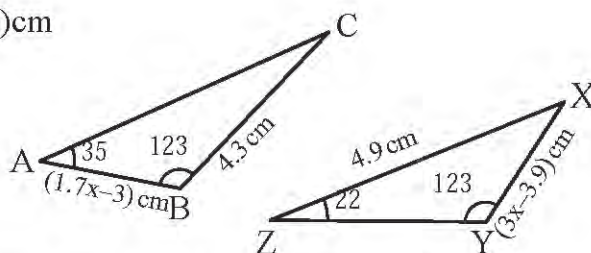
$$\therefore x = \frac{2.6}{1.3} = 2$$

$$\text{Therefore, } AB = 1.7x - 1.3 = 1.7 \times 2 - 1.3 = 2.1 \text{ cm}$$

$$XY = 3x - 3.9 = 3 \times 2 - 3.9 = 2.1 \text{ cm}$$

$$AC = XZ = 4.9 \text{ cm}$$

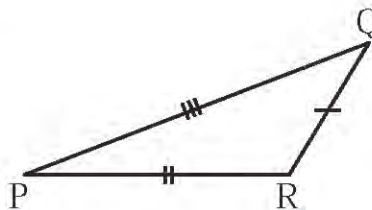
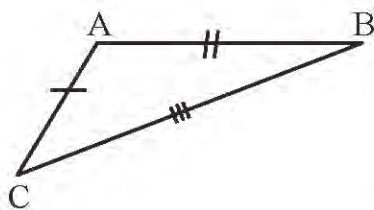
$$BC = YZ = 4.3 \text{ cm}$$



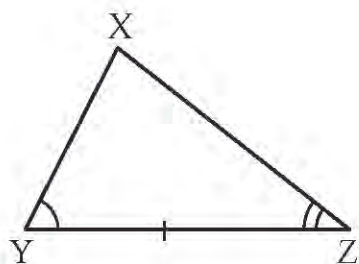
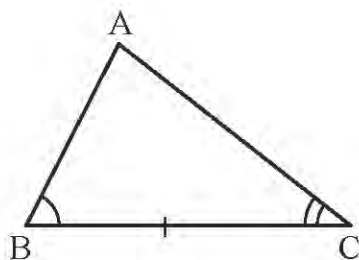
Exercise 15.1

1. Write the reason for congruency of following pair of triangle.

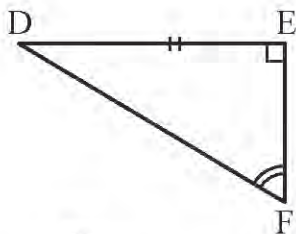
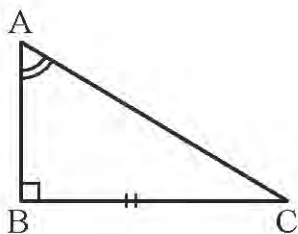
(a)



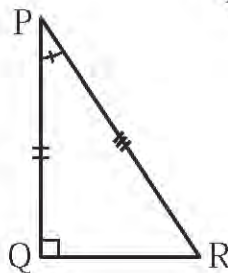
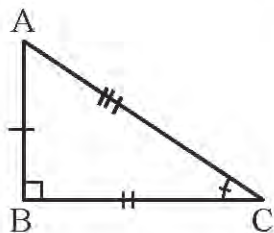
(b)



(c)

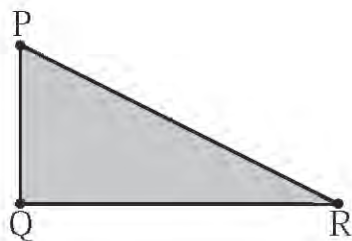
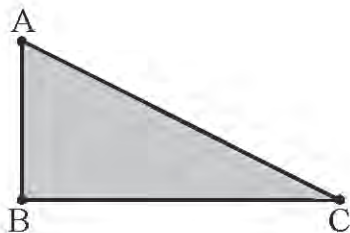


(d)

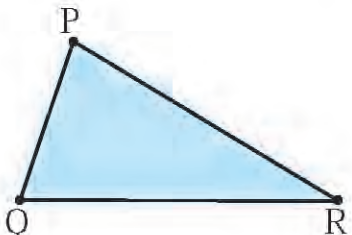
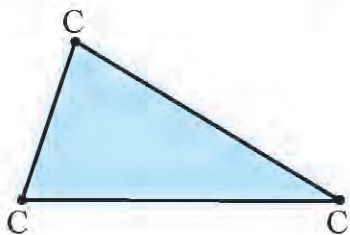


2. In the given condition, determine whether the triangles are congruent or not, and measure the angles and sides. If they are congruent, identify the corresponding angles and corresponding sides.

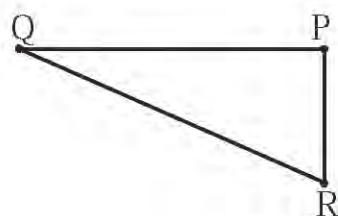
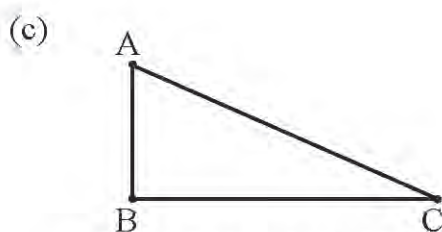
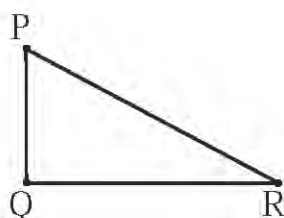
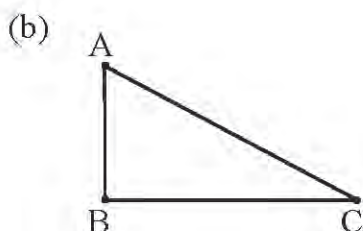
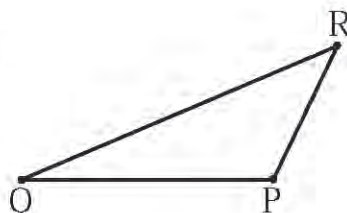
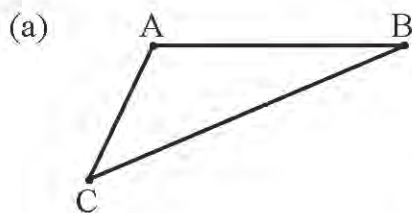
(a)



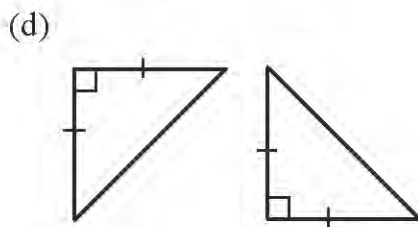
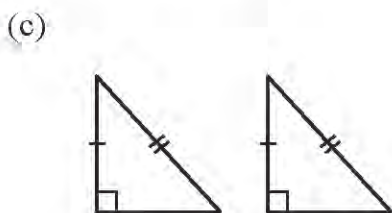
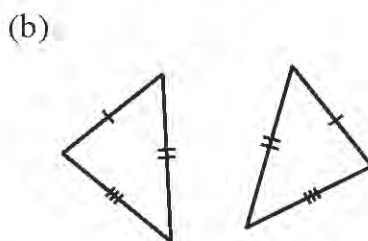
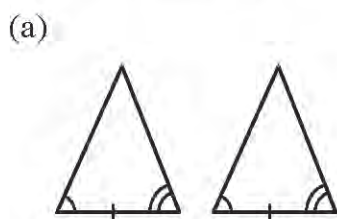
(b)



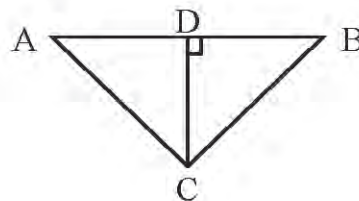
3. If $\triangle ABC$ and $\triangle PQR$ are congruent, what are the corresponding angles and corresponding sides? Write.



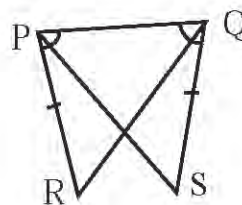
4. To make $\triangle ABC$ and $\triangle PQR$ congruent, which three corresponding sides must be equal? Write.
5. Write the reason for congruency of following pair of triangles.



6. In the figure, D is the midpoint of AB. CD is perpendicular to AB. Prove that $\triangle ACD \cong \triangle BCD$.

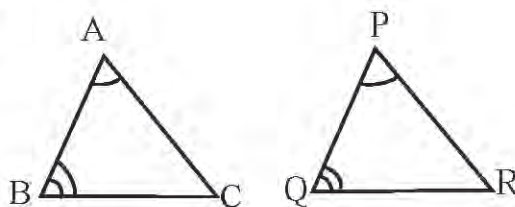


7. In the figure, $\angle RPQ = \angle PQS$ and $QS = PR$. Prove that $\triangle PQR \cong \triangle QPS$.

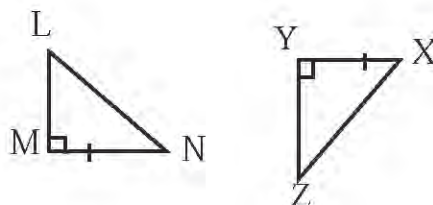


8. In $\triangle ABC$ and $\triangle PQR$, $\angle BAC = \angle QPR$ and $\angle ABC = \angle PQR$.

Which of the given condition should be added to make $\triangle ABC \cong \triangle PQR$?



9. In $\triangle LMN$ and $\triangle XYZ$, $\angle LMN = \angle XYZ = 90^\circ$ and $MN = XY$. Which of the given condition should be added in the figure to make $\triangle LMN \cong \triangle XYZ$? State the axiom of congruency?

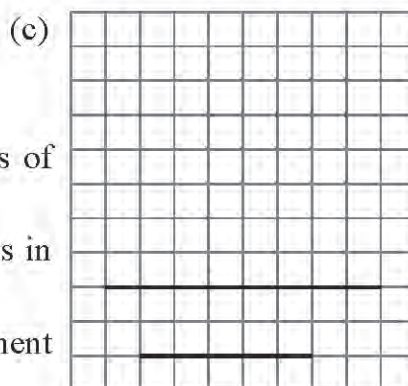
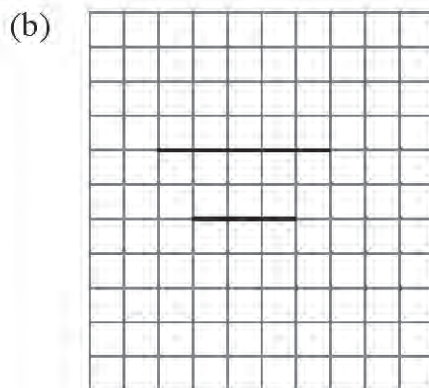
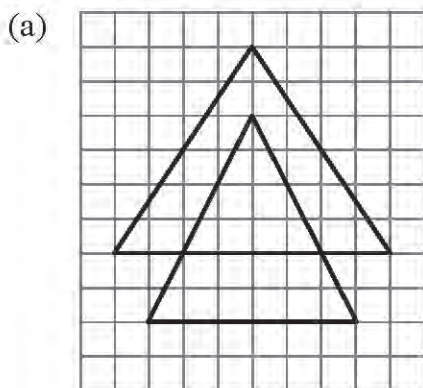


Answer: Show to your teacher.

15.3 Similar Figures

Activity 7

Given below is worksheet of three square papers. Draw the shape similar to worksheet a using line segment of worksheet b and d. Discuss the question below.



- Are all shapes the same?
- What is the relationship between angles of shapes?
- Are the measurement of different shapes in all figure same?
- What is the reason of different measurement of shapes?
- What are these shapes?

The shape created in the above worksheet (b) and (c) is similar to shape (a) but shape created in worksheet (b) is smaller than shape (a). The shape made in worksheet (c) is larger.

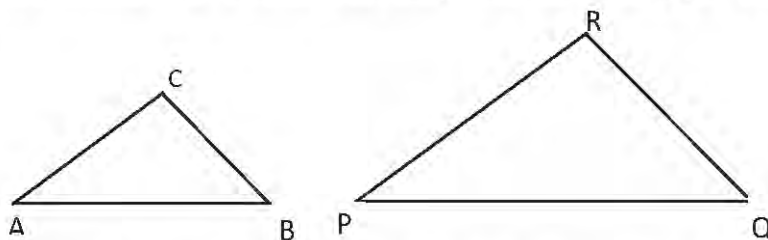
Two plane figures are said to be similar if they have the same shape. They may be small, large, or of equal size, but their shapes will be identical.

15.4 Similar Triangles

Activity 8

Take a photocopy paper and draw two line segments AB and PQ with different measurement.

Now, as shown in the figure, draw the angles of fixed measurement at points A and P, as well as the points B and Q, such that they are equal. Also, label the points of intersection of the respective sides as C and R, in order. What shape is formed by this construction? Are the two shapes of the same type? Discuss.



Now, cut out the two triangles you have made and place them on top of each other. Discuss the following questions based on the overlapping triangles:

- Are the shape of the two triangles the same?
- Which angles of triangle ABC are equal to the angles of triangle PQR? Determine and indicate.
- Are AB and PQ, BC and QR, as well as AC and PR equal to each other?
- Measure all the side of both triangles.
- Find the ratio of the sides opposite the equal angle in triangle ABC and triangle PQR.
- What relationship among the ratios of the corresponding sides of the equal angles?

The given triangle ABC and PQR are similar triangle. Where, $\angle BAC = \angle QPR$,

$$\angle ABC = \angle PQR \text{ and } \angle ACB = \angle PRQ \text{ then, } \frac{BC}{QR} = \frac{AC}{PR} = \frac{AB}{PQ}.$$

Here, corresponding angles are

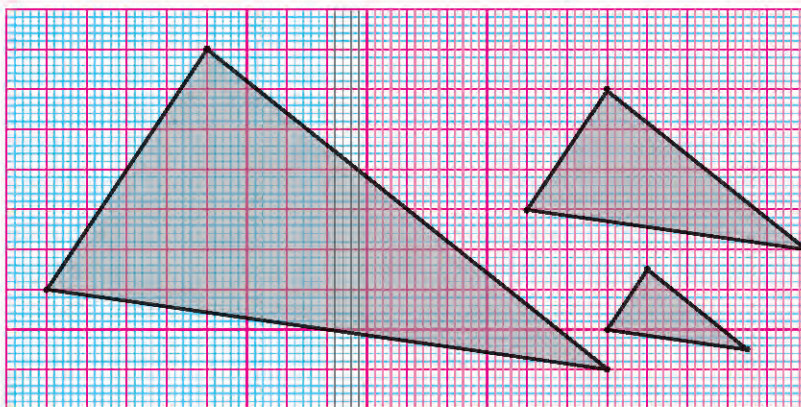
- $\angle A$ and $\angle P$
- $\angle B$ and $\angle Q$
- $\angle C$ and $\angle R$

The opposite sides of equal angles are the corresponding sides which are BC and QR, AC and PR, AB and PQ.

In a similar triangle, equal angle are called corresponding angles and opposite sides of equal angles are called corresponding sides. When the corresponding angles of two triangles are equal and the corresponding sides are proportional, then those two triangles are called similar triangles. Similar triangles are denoted by $\triangle ABC \sim \triangle PQR$.

Activity 9

- (a) Each student should draw the following triangles on a chart paper and cut them to make three triangles.

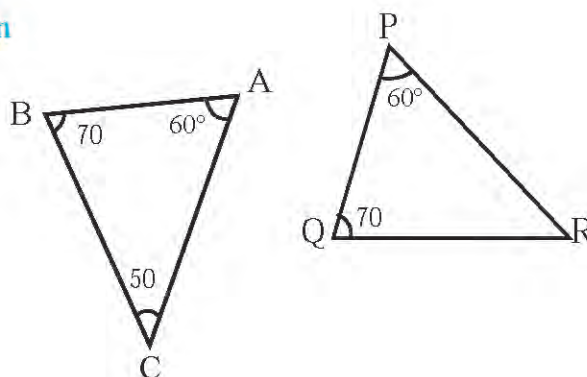


- (b) Paint each of the three triangles with different color.
- (c) Collect all the triangles in a box.
- (d) Each person, take out one pair of triangle without looking from the box. Match that pair to determine if they form congruent triangle or a similar triangle.
- (e) Sit congruent pair in one side and similar pair in another side. Whichever group will have more will win.
- (f) Repeat this game 5 or 6 times.
- (g) How do you come to the conclusion that the pair of triangles you have withdrawn each time are congruent or similar? Prepare the report about it.

Example 1

In the given $\triangle ABC \sim \triangle PQR$, then

- Write down the name of corresponding angles and sides.
- Find the value of $\angle PRQ$.



Solution

Here, $\triangle ABC \sim \triangle PQR$.

- Therefore,

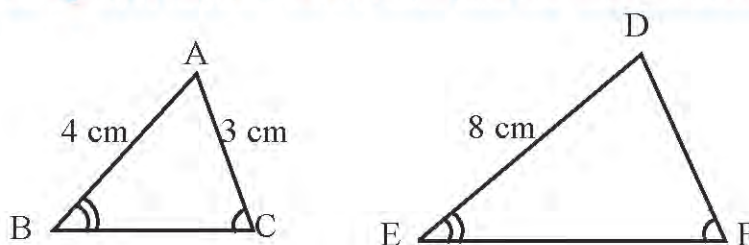
Corresponding angles: A and P, B and Q, so C and R

Corresponding sides: BC and QR, AC and PR so AB and PQ.

Thus: $\angle BCA = \angle PRQ = 50^\circ$.

Example 2

In the given figure if $\triangle ABC \sim \triangle DEF$, then find the measurement of DF.



Solution

Here, $\triangle ABC \sim \triangle DEF$ and $\angle B = \angle E$, $\angle C = \angle F$

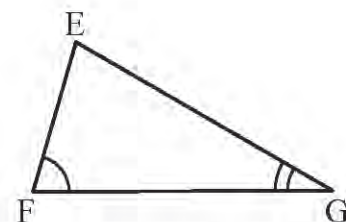
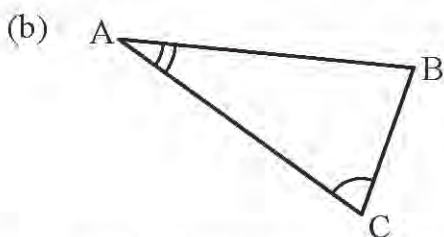
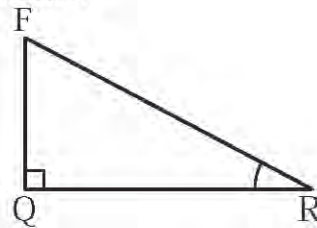
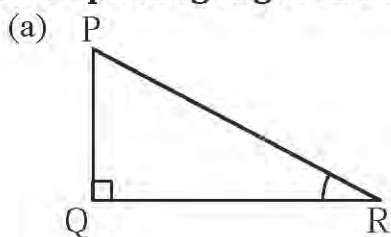
$$\frac{AB}{DE} = \frac{AC}{DF} \quad [\because \text{Corresponding sides of similar triangles are proportional}]$$

$$\frac{4}{8} = \frac{3}{DF}$$

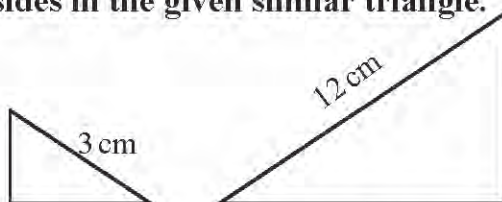
$$\therefore DF = 6 \text{ cm}$$

Exercise 15.2

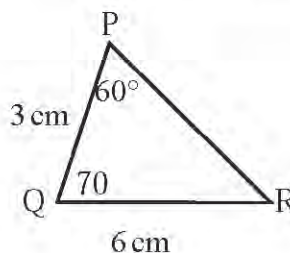
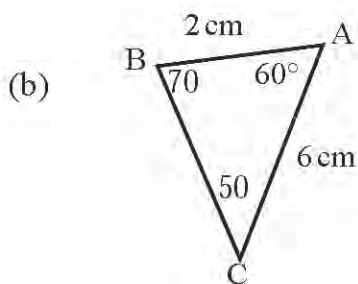
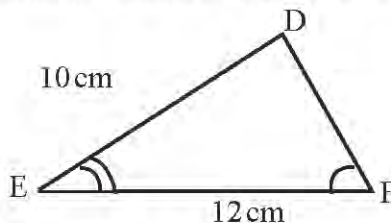
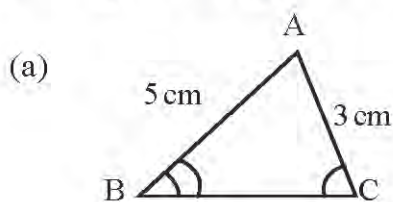
1. If the given pair of triangles are similar, then find their corresponding angles and sides with their name.



2. If the ratio of corresponding sides of Δ and ΔPQR is 1:1, then what types of triangle are they?
3. Find the ratio of corresponding sides in the given similar triangle.

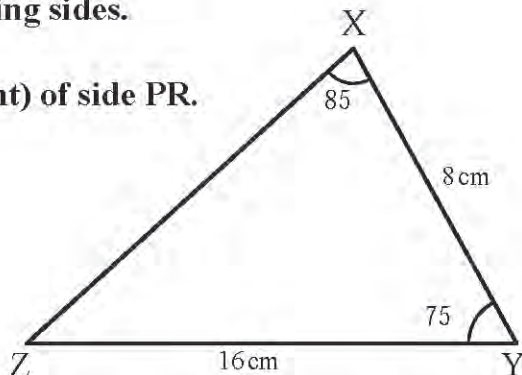
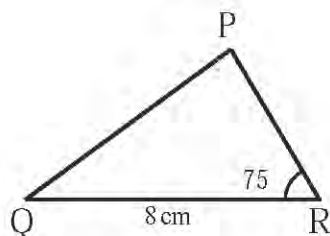


4. If the given triangles are similar, find the value of unknown sides.

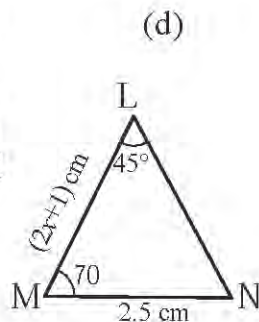
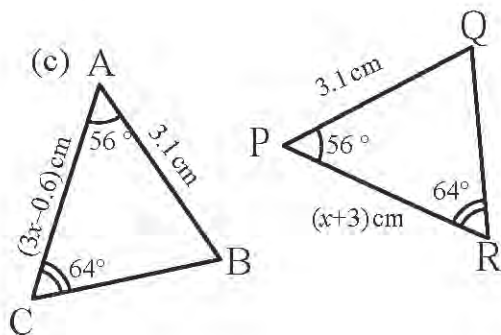
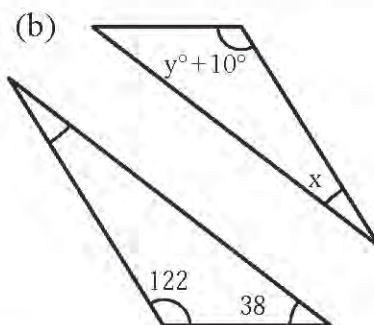
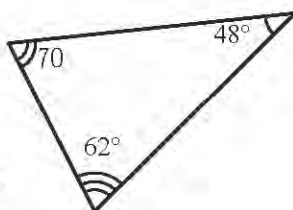
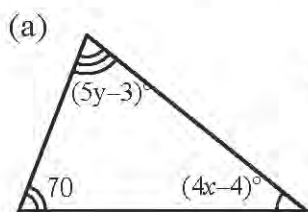


5. If $\triangle PQR \sim \triangle XYZ$ in the adjoining figure, answer the following questions.

- Find the ratio of corresponding sides.
- Find the value of $\angle PQR$.
- Find the length (measurement) of side PR.



6. Find the values of x and y from the given similar triangles and then find out the value of unknown sides and angle.

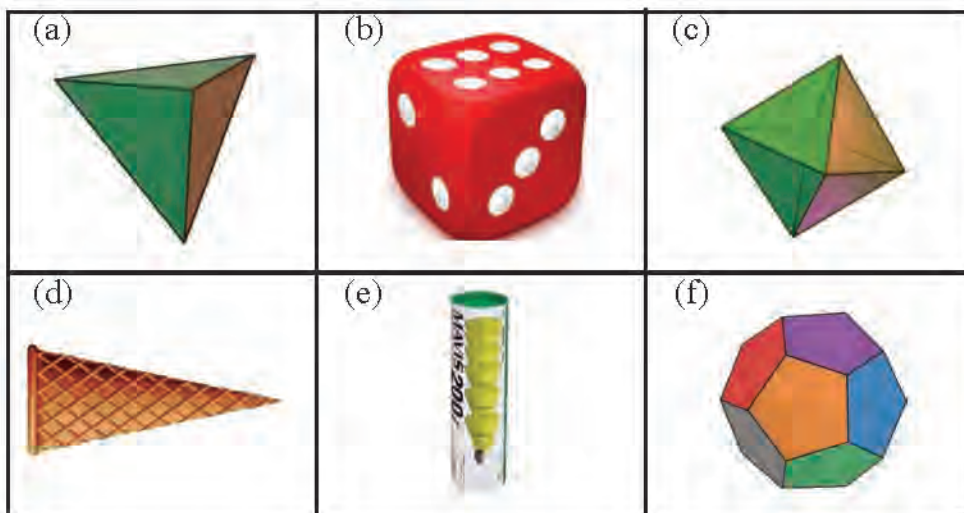


Answers

1. Show to your teacher
2. Similar triangle
3. 1:4
4. (a) BC = 6 cm, DF = 6 cm
5. (a) 1:2 (b) 20°
6. (a) $x = 13^\circ$, $y = 13^\circ$
- (b) $x = 20^\circ$, $y = 112^\circ$
- (c) 4 cm
- (d) $x = 1$
- (c) $x = 1.8$

16.0 Review

- (I) The figure of solid objects are given below. Observe them and find out their geometrical features by discussing in the classroom, note them and then present in classroom.



- (II) Present the number of edges, surfaces, and vertices of the above solid objects in a table.
- (III) Write is the relationship among the edges, surfaces and vertices of the above solid objects? Write.

16.1 Triangular base prism and pyramid

A Triangular prism

Activity 1

The figure below is a triangular prism. Observe the solid object and discuss the following questions.

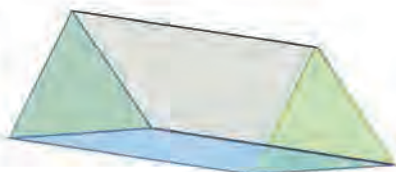
- (a) How many plane surfaces does it have? What are their shapes?

- (b) How many edges, surfaces, and vertices are there in the prism? What is the relationship among them?
- (c) Which surfaces of a triangular prism are congruent?



Activity 2

With the help of the teacher, construct a hollow model of a triangular prism from a juice pipe or wheat straw. Present in the classroom by posting different colored paper to separate the surfaces of the prism.



A prism is a solid made of plane surfaces. It has two types of surfaces. The upper and lower parallel and congruent surfaces are called the base of the prism. The surfaces except the bases are called lateral surface. The name of the prism is given to the shape of its base. For example, triangular prism (has triangular bases), rectangular prism (has rectangular bases).

B Pyramid

Activity 3

The shapes of solid objects are given below. Observe them and discuss the asked questions in pairs.



- (i) You might have seen shapes or solid objects like above? If so, where have you seen them?
- (ii) What are the shapes of surface in the above solid object? How many surfaces are combined?
- (iii) What is the relationship between its surface edges and vertices?
- (iv) Which surfaces are congruent to each other?

The given solid shapes are pyramids. The base of all pyramids is polygon. In addition to the base, the pyramid has triangular lateral surface. When the base of a pyramid is a regular polygon, the faces of the pyramid are congruent to each other. One common vertex exists for all triangular faces. In addition, the height is perpendicular to the base surface.

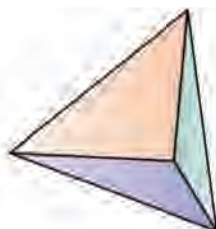
A pyramid is a solid object that has a polygonal base and triangular faces that meet at a common point or vertex. Pyramids are classified based on the shape of their base, such as triangular pyramid having triangular base, rectangular pyramid having rectangular base, or square pyramid, having square base.

Activity 4

Construct a pyramid-shaped structure using juice pipe or wheat straw with the help of a teacher. Paste different colored paper sheets to distinguish the surface of the pyramid and present in the class room.



Rectangular based
pyramid



Triangular based
pyramid



Square based
pyramid

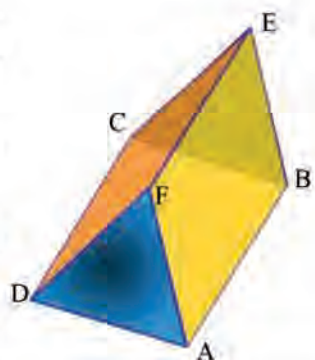


Hexagonal based
pyramid

Example 1

The figure below is a triangular base prism. Look at the figure and answer the following questions.

- Write the name of all the surfaces of the prism with a triangular base.
- Which surfaces are congruent in a prism with a triangular base?
- Write about the relationship between edges, surfaces and vertices in a prism with a triangular base?

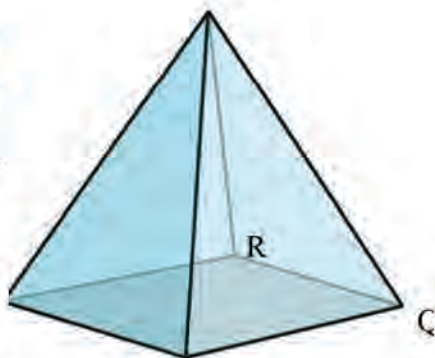


Triangular base prism

Example 2

The figure below is a rectangular base pyramid. Look at the figure and answer the following questions.

- Write the name of all the surfaces of triangular base pyramid.
- Which surfaces are congruent in triangular base pyramid?
- Write about the relationship among edges, surfaces, and vertices in rectangular base pyramid.



Rectangular base pyramid

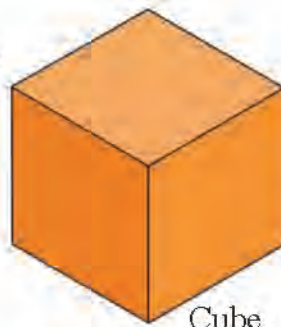
16.2 Nets of Solid objects

A. Cube

Activity 5

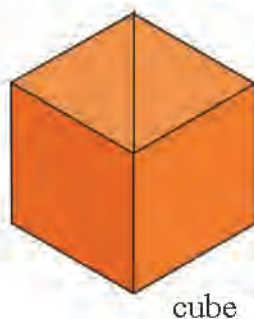
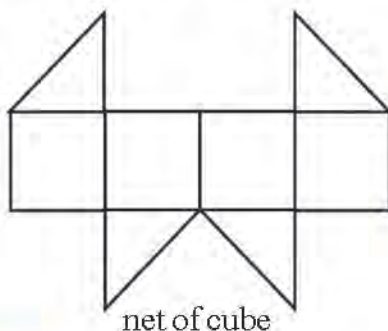
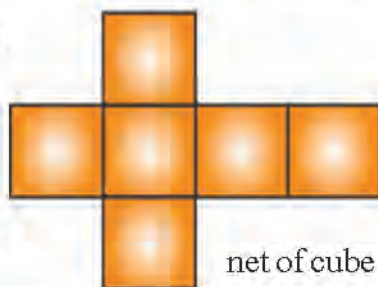
A figure of cube made by folding a paper is given below. Discuss the following question in group.

- How many faces, edges, and vertices are there in a cube?
- If we unfold it, what would it look like?
- Search for other net to make a cube.
- Construct a cube from the net provided.



A cube has 6 faces, 12 edges and 8 vertices. If we unfold it, it will look like the net shown in the figure on the right.

A cubic lattice other than that shown in the right figure is given below. The cube has also been constructed using the given net.



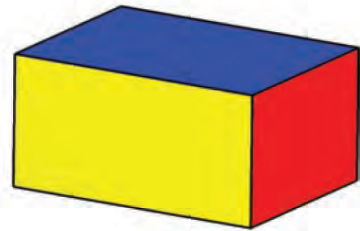
B. Cuboid

Activity 6

A figure of a cuboid made by folding a paper is given. Discuss the following questions in group.

- How many surfaces, edges, and vertices does a cuboid have?

- (ii) If you unfold this, what does it look like?
- (iii) What is the difference between net of a cube and a cuboid ?
- (iv) Are there other nets to make a cuboid besides this one? Search it.
- (v) Construct a cuboid using the net provided.

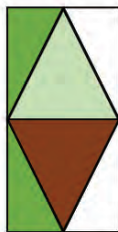


C. Tetrahedron

Activity 7

A figure of a tetrahedron made by folding a paper is given. Discuss the following questions in group.

- (i) What are the number of faces, edges, and vertices of the tetrahedron?
- (ii) If you unfold this, what does it look like?
- (iii) Are there other nets to make a tetrahedron besides this one? Search it.
- (iv) Construct a tetrahedron using the net provided.



D. Cone

Activity 8

It is a solid cone made by folding a paper. Discuss the following questions in group.

- (i) Define a cone. What are the number of surface, edges, and vertices of the cone?



- (ii) If you unfold this, what does it look like?
- (iii) Are there other nets to make a cone besides this one? Search it.
- (iv) Construct a cone using the net provided.

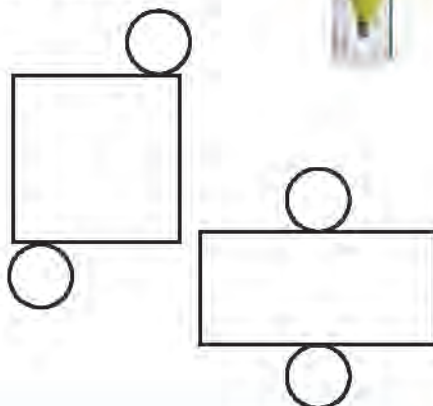


E. Cylinder

Activity 9

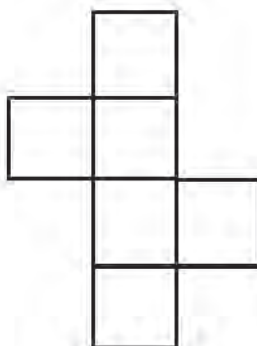
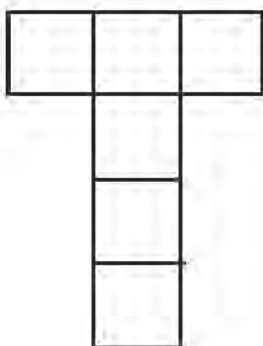
A figure of the cover of badminton cock is given. This is the model of a cylinder. Discuss the following questions based on this in group.

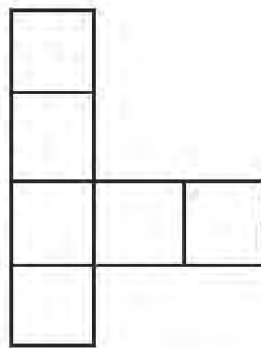
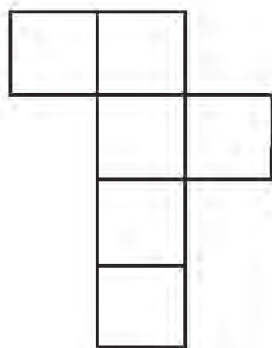
- (i) Define a cylinder. What are the number of surfaces, edges, and vertices of the cylinder?
- (ii) What would it look like if we covered all of its parts with a paper and then took it out again?
- (iii) Are there other nets to make a cylinder besides this one? search it.
- (iv) Construct a cylinder using the net provided.



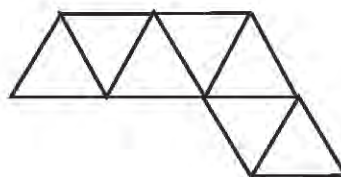
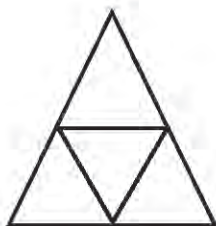
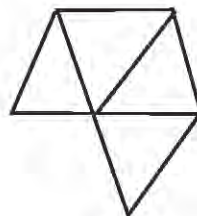
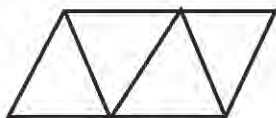
Exercise 16

1. Among the given nets, test which one forms a cube.





2. Among the given nets, test which one forms a tetrahedron.



Project work

- (i) Make a triangular prism and pyramid by using juice pipes or coins or wheat straw and present them in the classroom.
- (ii) Make a cube and cuboid net and then prepare cube and cuboid. What difference and similarity do you find between their nets, write in a chart paper and present in classroom.
- (iii) Make a solid model of cylinder, cone and tetrahedron and present them in the classroom.

17.0 Review

Take one red dice and one blue dice. Draw a graph board as shown in the figure.

Play this game with your friend in pairs. The first player rolls both red and blue dice.

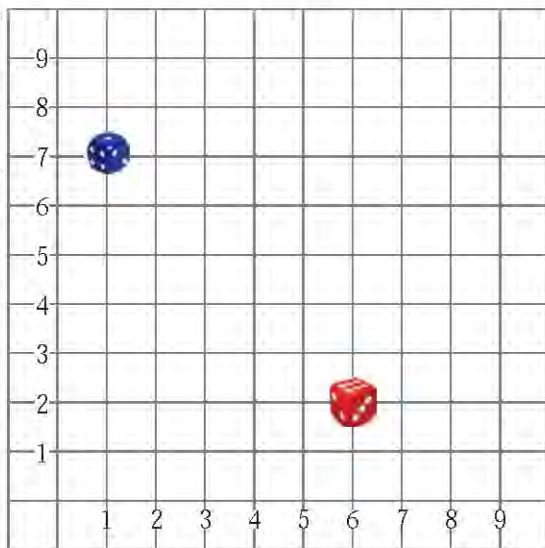
Mark a point of intersection of vertical and horizontal line so that the number shown by red dice and blue dice are on horizontal and vertical line respectively.

Again, roll both dice to get another point. Join these two points with a straight line.

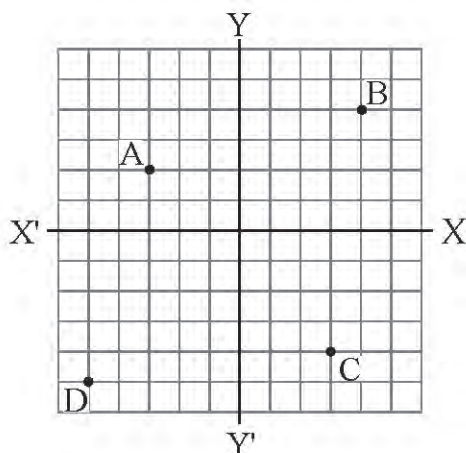
The second player also rolls both red and blue dice. Mark a point of intersection of vertical and horizontal line so that the number shown by red dice and blue dice are on horizontal and vertical line respectively. Similarly, roll both dice again to find another point and join these two points with a straight line.

Here, out of two lines, the longer one gets 2 points and if the two lines are equal, both get 1/1 point. Play this game for 5 times. The one who gets the highest score will be the winner.

After the game, discuss the score in a group of 4-5 persons and answer the following questions:



- What is XOX' called?
- What is YOY' called?
- How many units should be moved right and then up to reach from point O to B?
- What are the coordinates of points O, A, B, C and D?
- Plot the point (5, -5) on the graph.



Now, find the coordinates of X-axis and Y-axis points

17.1 Distance between two points

Activity 1

$A(x_1, y_1)$ and $B(x_2, y_2)$ are the two given points. How can we find the distance between these two points? In the given figure XOX' and YOY' are X-axis and Y-axis respectively.

Take two points $A(x_1, y_1)$ and $B(x_2, y_2)$

Draw $AP \perp OX$, $BQ \perp OX$ and $AC \perp BQ$

$$\therefore OP = x_1, AP = y_1,$$

$$OQ = x_2, BQ = y_2,$$

$$CQ = AP = y_1,$$

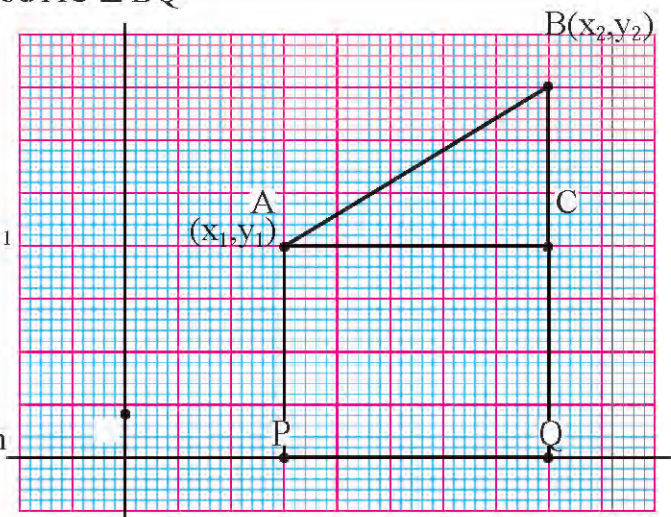
$$AC = PQ = OQ - OP = x_2 - x_1$$

$$BC = BQ - CQ = y_2 - y_1$$

Now,

From right-angled triangle $\triangle ABC$, by Pythagoras theorem

$$\begin{aligned} AB^2 &= AC^2 + BC^2 \\ &= (x_2 - x_1)^2 + (y_2 - y_1)^2 \end{aligned}$$

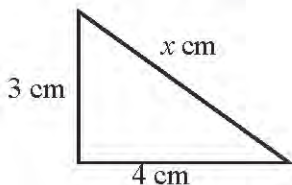


Then, the distance between two points A and B = $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

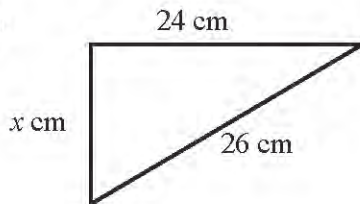
Example 1

Find the value of x from the given figures.

(a)



(b)



Solution:

(a) Here, $p = 3$ cm, $b = 4$ cm,
 $h = x = ?$

Now, we know that

According to the Pythagoras theorem

$$h^2 = p^2 + b^2$$

$$\text{or, } x^2 = 3^2 + 4^2$$

$$\text{or, } x^2 = 9 + 16$$

$$\text{or, } x^2 = 25$$

$$\text{or, } x = \sqrt{25} \text{ cm} = 5 \text{ cm}$$

b) Here, $p = 24$ cm,

$b = x = ?$ $h = 26$ cm

$$\therefore x = 5 \text{ cm}$$

According to the Pythagoras theorem

$$h^2 = p^2 + b^2$$

$$\text{or, } 26^2 = 24^2 + x^2$$

$$\text{or, } x^2 = 26^2 - 24^2$$

$$\text{or, } x^2 = 676 - 576$$

$$\text{or, } x = \sqrt{100} \text{ cm} = 10 \text{ cm}$$

$$\therefore x = 10 \text{ cm}$$

Example 2

If the centre of a circle is A(4,6) and P(10,8) be any point on its circumference, find the radius of the circle.

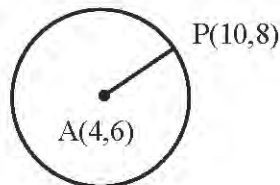
Solution

Here, A(4,6) is the centre of the circle and P(10,8) be any point on its circumference.

$$x_1 = 4, x_2 = 10, y_1 = 6 \text{ and } y_2 = 8$$

$$AP = ?$$

$$\begin{aligned} \text{Now, AP} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(10 - 4)^2 + (8 - 6)^2} \\ &= \sqrt{6^2 + 2^2} = \sqrt{40} = 2\sqrt{10} \text{ unit} \end{aligned}$$



Example 3

If the point P is 5 units from the X-axis and point Q is 6 units from the Y-axis, find the distance between P and Q.

Solution

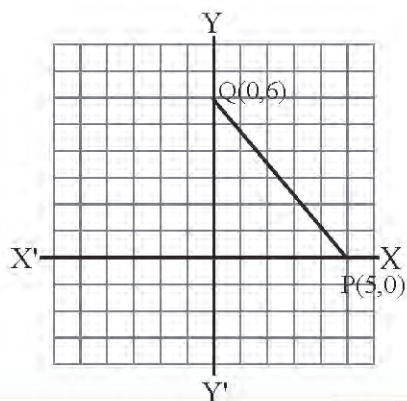
Here the point P is 5 units from the X-axis, Hence, the coordinates of P is (5, 0) and point Q is 6 units from the Y-axis. So, coordinates of Q is (0, 6). (because in X-axis, $y = 0$ and in Y-axis, $x = 0$)

Now, $(x_2, x_1) = (5, 0)$ and $(y_2, y_1) = (0, 6)$

We know that,

$$\begin{aligned}PQ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\&= \sqrt{(0 - 5)^2 + (6 - 0)^2} \\&= \sqrt{(25 + 36)} = \sqrt{61} \text{ unit}\end{aligned}$$

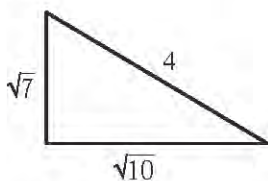
$\therefore$ The distance between P and Q is $\sqrt{61}$ units.



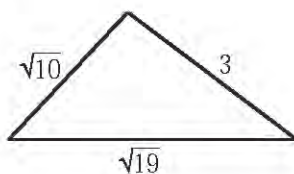
Exercise 17.1

1. Use Pythagoras theorem and identify whether the following triangles are right-angled triangle or not:

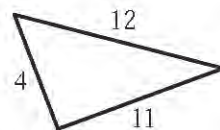
(a)



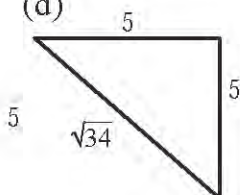
(b)



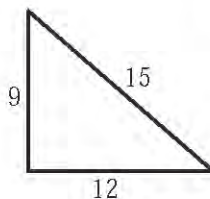
(c)



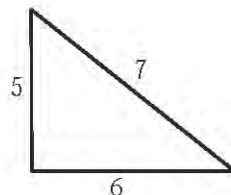
(d)



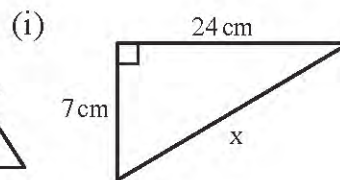
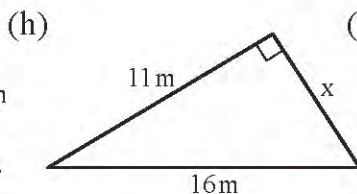
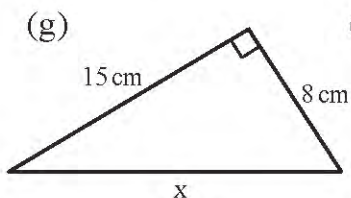
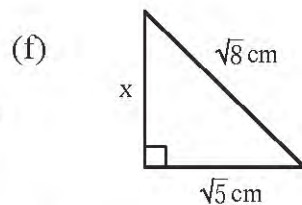
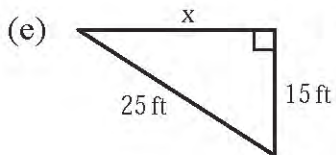
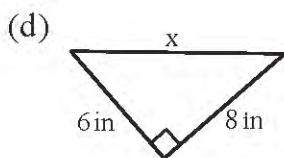
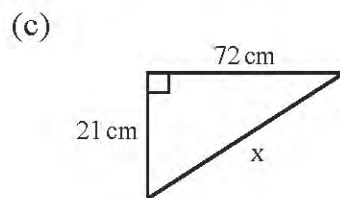
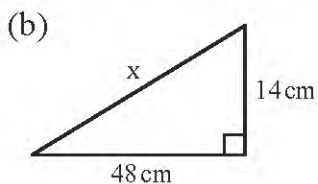
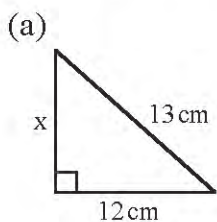
(e)



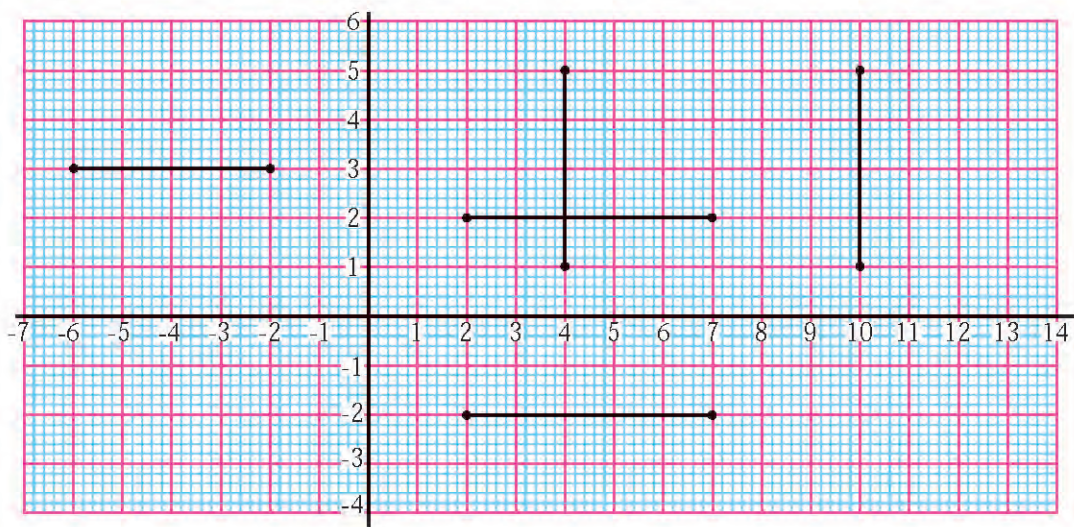
(f)



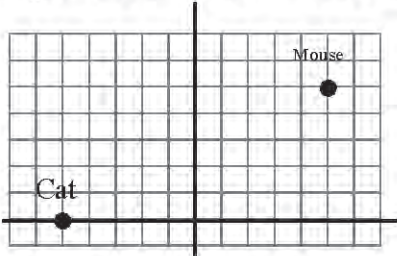
2. Find the value of x from the following right-angled triangles.



3. Find the distance between two points given.



4. Find the distance between two points given below:

(a) $(4, -7)$ and $(-1, 5)$ (b) $(-3, 4)$ and $(4, 3)$ (c) $(1, -2)$ and $(5, -6)$
 (d) $(1, 7)$ and $(1, 1)$ (e) $(2, 7)$ and $(4, 9)$ (f) $(-8, 7)$ and $(-3, 4)$
 5. If the point A intersects X-axis at -8 and point B intersects Y-axis at 6 , find the distance of AB.
 6. The position of cat and mouse are given in the graph paper, Find the coordinates of their position and find the distance between them?
- 
7. Prove that the points A $-4, 0$, B $-4, -4$, C $2, -4$ and D $2, 0$ are the vertices of a rectangle.
 8. When presented on the maps, if the coordinates of A and B are $4, 7$ and $7, 3$ respectively, what is the distance between these two points on the map? If 1 unit is equal to 55 km, find the actual distance between A and B.
 9. Prove that the points P $(1, 6)$, Q $(4, 1)$ and R $(-4, 3)$ are the vertices of an isosceles triangle.
 10. If the points A $2, -1$, B $3, 4$, C $-2, 3$ and D $-3, -2$ are the vertices of a rhombus ABCD, then find the distance of its diagonals AC and BD.
 11. If the points P $9, 12$ and Q $1, 6$ are on the circumference of the circle, what will be the radius of that circle? Do the point $-7, 0$ lie on the circumference of the circle?
 12. Find the distance from the origin O to point A and point B, where A = $(-7, 7)$ and B $(7, -7)$.
 13. If the distance between P $0, 6$ and Q $a, 0$ is 6 units, what is the value of a?

Answers

Show to your teacher.

18.0 Review

Activity 1

Look at the given pictures. Have you seen these shapes around your house? Where have you seen those figures? Discuss with your friend.

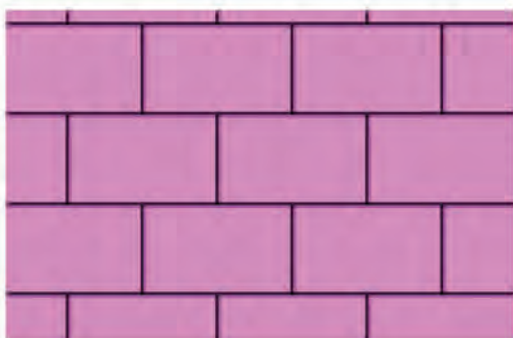


Figure (a)



Figure (b)



Figure (c)



Figure (d)

How the shapes in the above pictures are made? Differentiate which one belongs to tessellation and which are not.

18.1 Regular Tessellation

Activity 1

The following figures show the tessellations made up of equilateral triangles, squares, and regular hexagons. Observe these tessellations and make some pieces of equilateral triangles, squares, and regular hexagons by cutting colourful paper. Assemble these pieces into three different chart papers as shown in the figure in a perfect manner without overlapping and letting a space between them. Then discuss the tessellation that you made with your friends and demonstrate it in your class

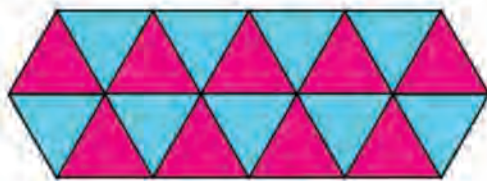


Figure (a)

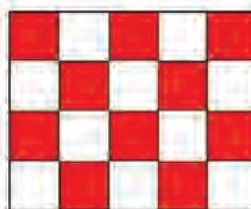


Figure (b)



Figure (c)

The above tessellations are composed of regular polygons. Tessellations are named using numbers. For example, figure (a) shows the tessellation made up of equilateral triangles. It is called 3.3.3.3.3.3 tessellation. Similarly figure (b) shows the tessellation made up of squares which is called 4.4.4.4 tessellation. On the other hand, in figure (c) the regular hexagons are arranged to make tessellation and named as 6.6.6 tessellation.

Here,

To name the tessellation, take any vertex where the polygons meet. On this point, the joining each of the polygon side is numbered respectively, from small to large and the symbol (.) is assigned just after number of sides of polygons joining the points.

The tessellation made up of regular congruent polygons is called regular tessellation. For examples: tessellations made from equilateral triangles, squares.

18.2 Semi-regular Tessellation

Activity 2

As shown in the figure, make a tessellation by arranging fig (a) equilateral triangles and squares fig (b) regular polygons and equilateral triangles. Then, discuss and demonstrate the tessellation you made, among the friends in your class and draw the conclusion.



Figure (a)

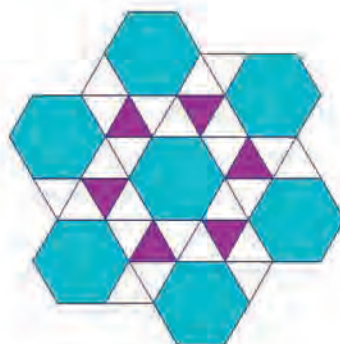


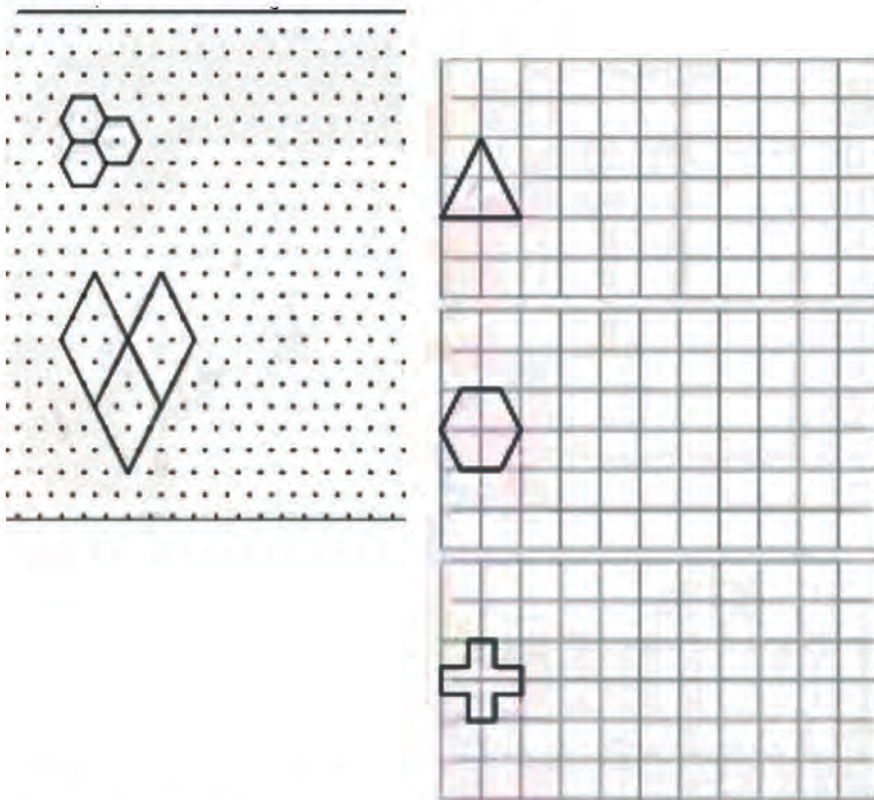
Figure (b)

These tessellations shown above are semi-regular tessellations. Here in fig. (a) each corner is named as 3.3.4.3.4. Similarly in fig. (b), the tessellation is 3.3.3.3.6

If the tessellation is made using two or more than two regular polygons, it is called semi-regular tessellation. For examples, tessellation made from equilateral triangle and square; equilateral triangle, and regular hexagon etc.

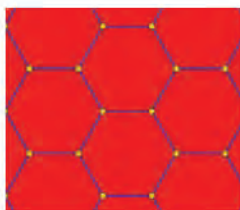
Exercise 18.1

1. Fill the tessellation in the given graph.

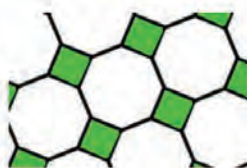


2. Add 2-3 round of tessellation pattern in the given regular and semi-regular tessellation.: fig. a fig. b fig c

(a)



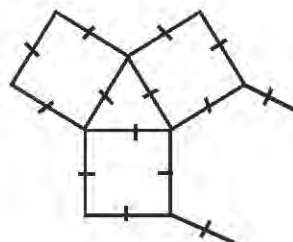
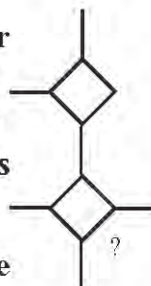
(b)



(c)



3. Name the tessellation of above question no. (2) by using the tessellation number.
4. The figure given alongside is the semi-regular tessellation, which is made using one square and another regular polygon. What is the name of the regular polygon? Make a tessellation by using this regular polygon.
5. The given figure is a tessellation made from three regular polygons. If one is equilateral triangle, second is square, what is the name of third regular polygon? Make a tessellation with the help of that polygon.



Project work on mini-exhibition

Divide all students of your class into an appropriate groups. Make one of the following tessellations by each group using a thick paper. Organize a mini-exhibition of tessellation. Invite all the students of the school to attend the exhibition.

- | | | | |
|-----------------|---------------|-------------|---------------|
| (a) 3.3.3.3.3.3 | (b) 4.4.4.4 | (c) 6.6.6 | (d) 3.3.3.3.6 |
| (e) 3.3.3.4.4 | (f) 3.3.4.3.4 | (g) 3.4.6.4 | (h) 3.6.3.6 |
| (i) 3.12.12 | (j) 4.6.12 | (k) 4.8.8 | |

Answers

Show to your teacher.

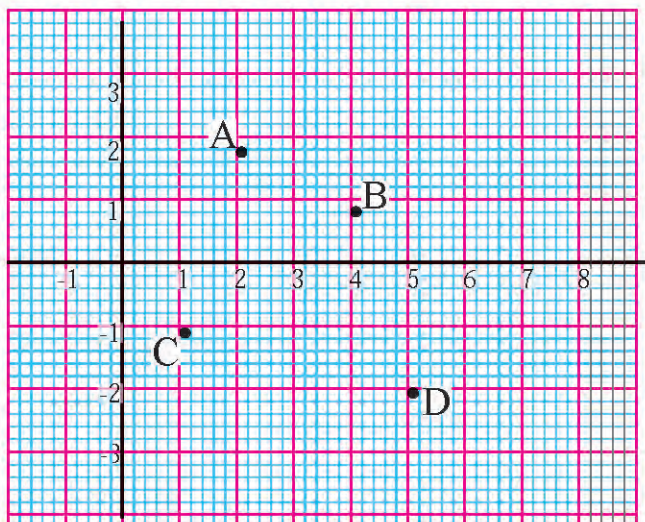
Lesson 19

Transformation

19.0 Review

Points A, B, C and D are shown in the graph paper below.

- What are the co-ordinates of the given points A, B, C and D?
- Write the co-ordinate of image A', B', C' and D' of the points A, B, C and D while reflecting in X-axis.
- What are the co-ordinate of the image of the points A, B, C, and D while reflecting in Y-axis?



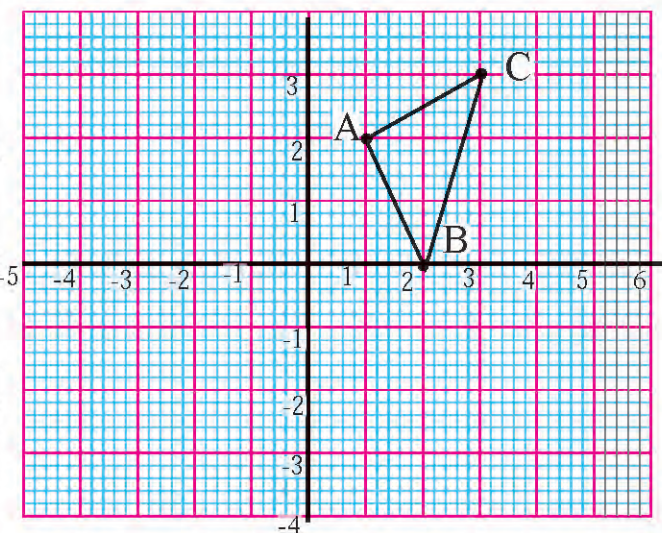
19.1 Reflection

Activity 1

$\triangle ABC$ is given in the adjoining graph paper.

Perform the activities under the given instructions.

- Write the co-ordinate of vertices A, B and C of triangle ABC.



- (b) Reflect the vertices A, B, and C in X-axis.
- (c) Write the co-ordinate of image A', B', and C'.
- (d) Now, compare the co-ordinate of vertices of $\triangle ABC$ and vertices of image $\triangle A'B'C'$ and write conclusion.

Here, in $\triangle ABC$, A(1,2), B(2,0) and C(3,3). After reflection, image A'(1,-2), B'(2,0) and C'(3,-3) is obtained.

While reflecting $\triangle ABC$ in X-axis, the X-coordinate of vertices A, B, and C remains same and sign of the Y- coordinate is changed after reflection. A point on X-axis has the same coordinate while reflecting in Y-axis.

Activity 2

Perform the following activities in the group of two students.

- (a) Reflect $\triangle ABC$ from graph of activity 1 in Y-axis.
- (b) Write the coordinate of image A'', B'' and C''.
- (c) Now, compare the coordinate of $\triangle ABC$ and $\triangle A''B''C''$ and write the conclusion.

Here, in $\triangle ABC$, A (1,2), B(2,0) and C(3,3). After reflecting, image A'' (-1, 2), B''(-2,0) and C''(-3,3) is obtained.

While reflecting $\triangle ABC$ in Y- axis, the sign of X- coordinate of vertices A, B and C is changed and Y- coordinate remain same after reflection.

Question to think: What are the coordinate of image while reflecting in Y-axis if a vertex of triangle is in Y-axis?

Example 1

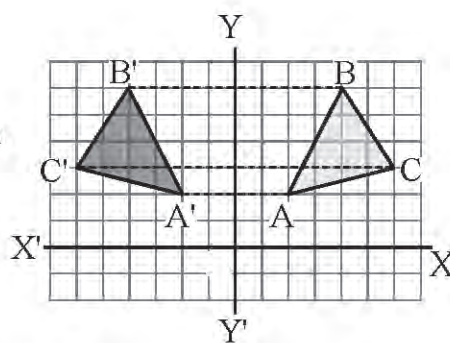
If A 2, 2 , B 4,6 and C 6,3 are the vertices of a triangle then,

- Show $\triangle ABC$ in a graph paper.
- Reflect $\triangle ABC$ in Y-axis and show image $\triangle A'B'C'$ in graph paper.
- Write the coordinate of image $\triangle A'B'C'$.

Solution

Here, A (2,2), B(4,6) and C(6,3) are the co-ordinates of $\triangle ABC$.

- The coordinates of $\triangle ABC$ are shown in the adjoining graph paper.
- Image $\triangle A'B'C'$ is also shown in the same graph paper.

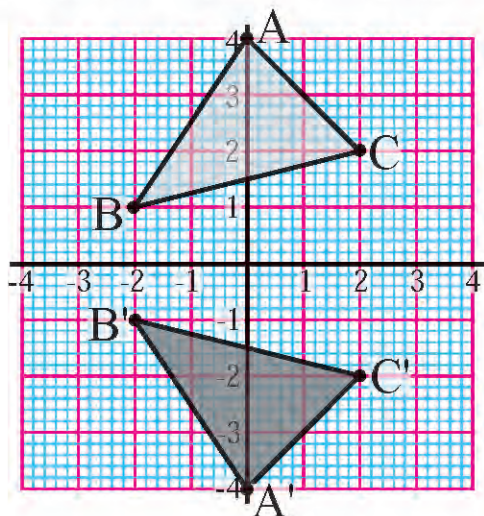


- The co-ordinates of image $\triangle A'B'C'$ are A'(-2,2), B'(-4,6) and C' (-6,3).

Exercise 19.1

- Write the coordinates of the image of the following co-ordinates after reflection in X-axis using graph paper.
 - A (1, 2)
 - M (2, 3)
 - C (4, 5)
 - D (6, 6)
 - E (5, 4)
 - F (2, 5)
 - G (9, 8)
 - H (3, 9)
- Write the points of QN 1 in graph paper after reflecting in Y-axis.
- Reflect the point P(5,-6) in Y-axis.
 - Find the co-ordinates of P'.
 - Find the length of PP'.

4. $P(4,3)$, $Q(7,3)$ and $R(4,-3)$ are the vertices of a right angled triangle. Draw the triangle and its image in the graph paper after reflecting in y-axis.
5. $A 2,-2$ $B 8,3$ and $C 5,-2$ are the vertices of a triangle. Draw the triangle and its image in the graph paper after reflecting in Y-axis.
6. Plot $A(-2,3)$, $B(-5,2)$ and $C(-4,5)$ in graph paper and then find the image $\Delta A'B'C'$ after reflecting about Y-axis. Again, reflect the final image $\Delta A'B'C'$ with Y-axis and present in the graph paper.
7. In the adjoining graph, the image $\Delta A'B'C'$ of ΔABC while reflecting about x-axis is given. Write the co-ordinates of image $\Delta A'B'C'$.



Answers

- | | | | |
|--------------------|------------------|------------------|------------------|
| 1.(a) $A'(1, -2)$ | (b) $M'(-2, -3)$ | (c) $C'(4, 5)$ | (d) $D'(-6, -6)$ |
| (e) $E'(-5, 4)$ | (f) $F'(-2, -5)$ | (g) $G'(9, 8)$ | (h) $H'(-3, 9)$ |
| (i) $I'(-10, -12)$ | (j) $W'(7, -8)$ | | |
| 2. (a) $A'(-1, 2)$ | (b) $M'(2, 3)$ | (c) $C'(-4, -5)$ | (d) $D'(6, 6)$ |
| (e) $E'(5, -4)$ | (f) $F'(2, 5)$ | (g) $G'(-9, 8)$ | (h) $H'(3, 9)$ |
| (i) $I'(10, 12)$ | (j) $W'(-7, 8)$ | | |
| 3.(a) $P'(-5, -6)$ | (b) 10 | | |

4 to 7 : Show to your teacher.

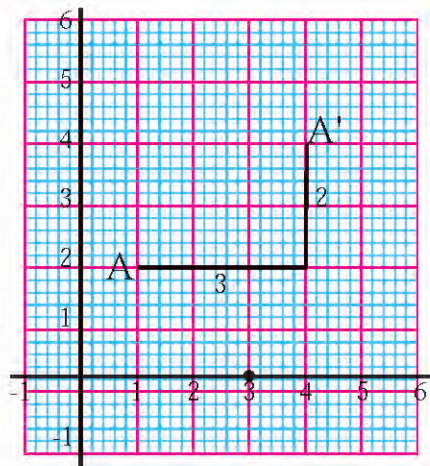
19.2 Translation

Activity 1

In the given figure, the point A is translated into the point A'.

The rule of translation is 3 unit right and 2 units upward.

It is represented as $(3,2)$. Point A is shifted 3 units right and then 2 units up. According to this, A $(1,2)$ is translated into A' $(4,4)$.



If the given point or object are shifted or transformed into the given direction with certain distance, it is called as translation.

It will be written + for right translation, - for left translation, + for upward translation and - for downward translation for any co-ordinates.

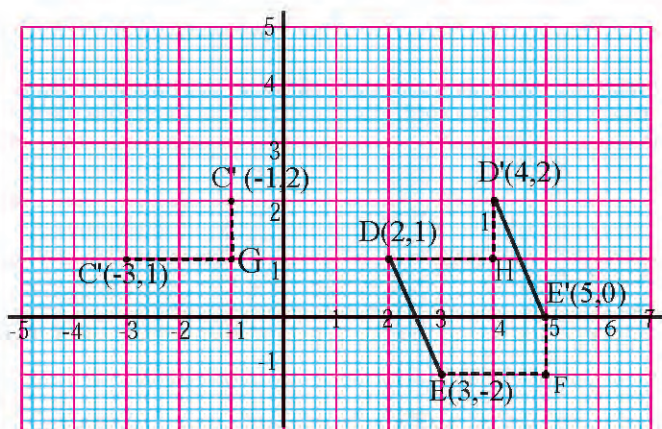
Example 1

A) Translate the point C, 2 units right and 1 unit upward.

B) Translate the line DE, 2 units right and 1 unit upward.

Solution

If point C $(-3,1)$ is shifted 2 units right, then it reaches at G and if it is shifted 1 unit upward, it reaches at C' $(-1,2)$. This can be written as C $(-3,1)$ translation C' $(-1,2)$. Similarly, if D $(2,1)$ is translated 2 units right and 1 unit upward,



then it will be D' $(4,2)$. Also, if E $(3,-1)$ is translated 2 units right and 1 unit upward, then, it will be E' $(5,0)$. Finally D'E' is the translated image of line DE.

Example 2

$\triangle ABC$ is given in the figure alongside, which is translated 5 units right and 4 units upward to obtain the image $\triangle A'B'C'$. Find the co-ordinate of the image.

Solution

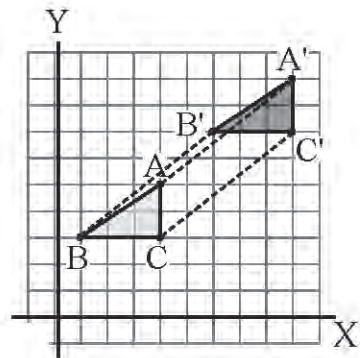
Let us observe the coordinate of vertices of $\triangle ABC$ and $\triangle A'B'C'$.

$\triangle ABC$	$\triangle A'B'C'$
-----------------	--------------------

$A(4,5)$	$A'(9,9)$
----------	-----------

$B(1,3)$	$B'(6,7)$
----------	-----------

$C(4,3)$	$C'(9,7)$
----------	-----------

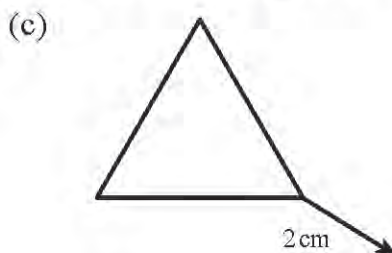
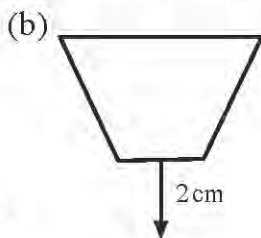
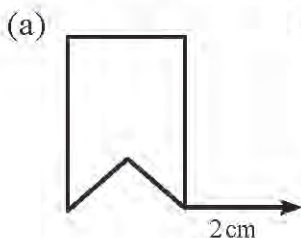


Let us see the X and Y co-ordinates before and after translation.

It is added 5 into the X co-ordinate of all three vertices after translation. Similarly, it is added 4 into the Y coordinate of translation. The image triangle after translation is shown by shading.

Exercise 19.2

1. Translate the following shapes under the given direction and magnitude.



2. Plot the point $(4,-5)$ in graph paper and then translate 3 units right and 4 units upward and present it.

3. Plot the following co-ordinate in graph and then translate 3 units left and 3 units upward and present in graph paper.
(a) (4,9) (b) (-3,6) (c) (-2,2) (d) (-5,5)
(e) (2,3) (f) (4,-7) (g) (-4,8) (h) (-5,-6)
4. Translate $P(-1,-3)$ and $Q(4,5)$ 2 units right and 3 units upward, then present in graph paper.
5. Plot the vertices $A(1,0)$, $B(4,5)$ and $C(7,-2)$ of $\triangle ABC$ in graph paper and translate it 3 units right and 5 units downward, then present in graph paper.
6. Plot the points (4, 6), (7,5), (5,1) and (2,2) in graph paper. Then, translate it 4 units right and 5 units downward and present in graph.
7. Plot the point $A(4, 1)$ in graph paper and translate it 5 units right and 4 unit upward. Again, translate the image point 2 units right and 5 units downward and present in graph paper.
8. By what units, the point $(-3,5)$ should be translated to obtain $(4,5)$? Show in graph paper.
9. Translate the line AB with $A(4,4)$ and $B(-5,7)$ taking the translation that translates $P(3,-2)$ into $Q(2,-1)$.

Project work Transformation of triangle

1. Draw different type of triangles (Equilateral, Isosceles, Scalene, Acute angled, Right angled and Obtuse angled) in graph paper.
2. Reflect each triangle about X- axis and Y- axis.
3. Translate each triangles 3 unit's right and 4 units upward.
4. Prepare the presentation of these activities.
5. Present in the classroom.

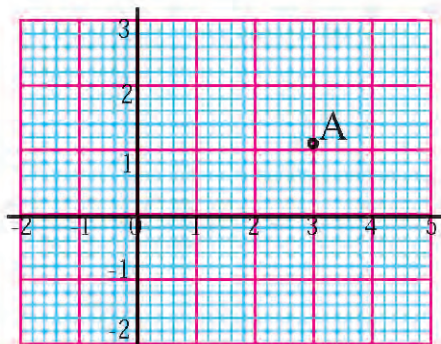
Answer: Show to your teacher.

19.3 Rotation

Activity 1

Let each students take small piece of paper write the following angle in such piece.

45° clockwise, 45° anti-clockwise,
 90° Clockwise and 90° anti-clockwise.



Play this game in the group of 4-5 students. Fold and keep these pieces in a box before playing the game. Shake the box. Tie an eraser with a thread. Paste the thread at a point on the side of paper. The game will start from 0° . Take a paper randomly from the box. Rotate the eraser as written in the paper. Repeat it for 5 times. The group with highest angle in the fifth attempt will be the winner.

Discuss the things that are required for rotation based on the above game and present them.

The following three things are necessary for rotation.

- Centre of rotation
- Angle of rotation
- Direction of rotation

The direction of hand of the clock is called the negative direction of rotation and the direction opposite to the hand of the clock is called the positive direction of rotation.

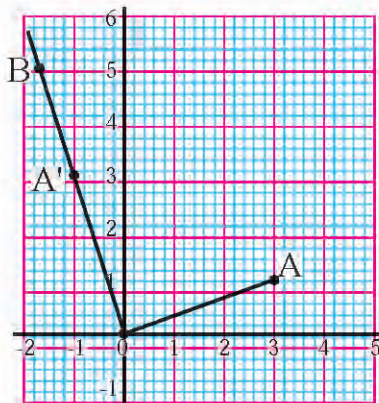
In the above game, the point where thread is pasted is the center of rotation. The angle to be rotated (45° , 90° or 180°) is the angle of rotation and +or- are the direction of rotation.

Activity 2

Sit in pairs. How can we rotate a point A about origin (0,0) through 90° angle in opposite direction of hand of a clock? Discuss in the group.

The conclusion is as follows.

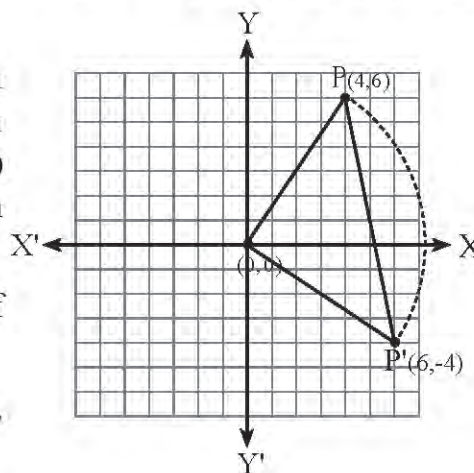
- Join A and O since the center of rotation is O (0,0).
- Make an angle BOA in anticlockwise direction from center O.
- Take an arc equal to OA and cut OB from O.
- Represent the intersected point as A', which is the required image.



Activity 3

How can we rotate a point P(4,5) center at origin O(0,0) through an angle - 90° ? Discuss in group.

If a point P(4,6) is rotated in the direction of hands of clock or negative direction through 90° in graph, it is formed P'(6,4) from P(4,6), which can be shown in graph as:



- Join P and O since the centre of rotation is origin O(0,0).
- Here the angle of rotation is -90° . So, draw the angle of 90° with OP.
- Take an arc equal to PO and cut in the line where -90° is made.
- The image of P(4,6) is P'(6,-4) and $\angle POP' = 90^\circ$ when rotated through $+90^\circ$. What will be the image of the about point P(4,6), if it is rotated through $+90^\circ$? Guess and discuss.

Example 1

Present a triangle with vertices A (6,6) , B (4,5) and C(6,2) in a graph paper. Rotate the triangle about origin (0,0) through (A) $+90^\circ$ and (B) 180° .

Solution:

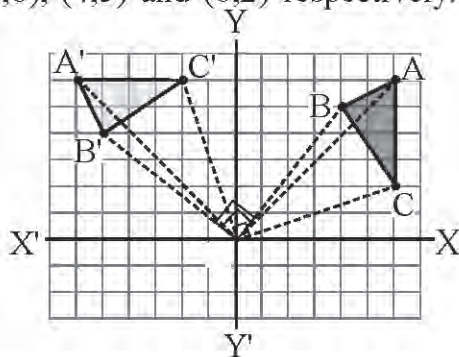
Here, the vertices of triangle ABC are (6,6), (4,5) and (6,2) respectively.

Now, the points A, B and C are rotated through 90° and the image triangle is shaded in the graph paper.

Where,

$$A(6,6) \longrightarrow A'(-6,6), B(4,5) \longrightarrow B'(-5,4)$$

$$\text{and } C(6,2) \longrightarrow C'(-2,6)$$



Exercise 19.3

- Plot the following points in graph and rotate each points centre about origin through 90° .
(a) (-4,7) (b) (4,-7) (c) (5,9) (d) (3,0) (e) (-4,-8)
(f) (2,-5) (g) (10,-10) (h) (0,6) (i) (0,0) (j) (-9,-9)
- Plot the following points in graph and rotate each points centre about origin through -90° .
(a) (-4,7) (b) (4,-7) (c) (5,9) (d) (3,0) (e) (-4,-8)
(f) (2,-5) (g) (10,-10) (h) (0,6) (i) (0,0) (j) (-9,-9)
- What relation did you find in the above questions 1 and 2? Mention as an formula.
- Draw a square with vertices A(0,0) ,B(3,0), C(3,3) and D (0,3) in a graph paper and rotate it centre about origin (0,0) through (A) $+90^\circ$ and(B) -90° and present in the graph paper.

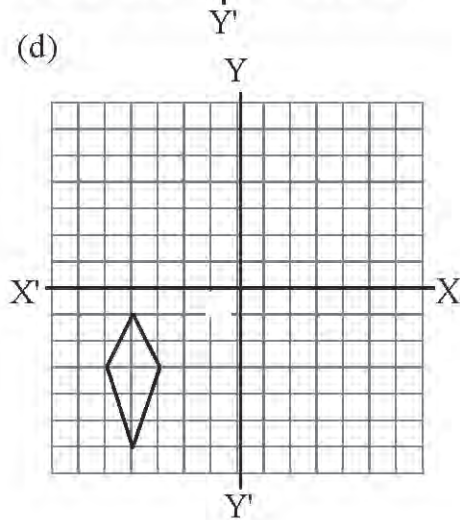
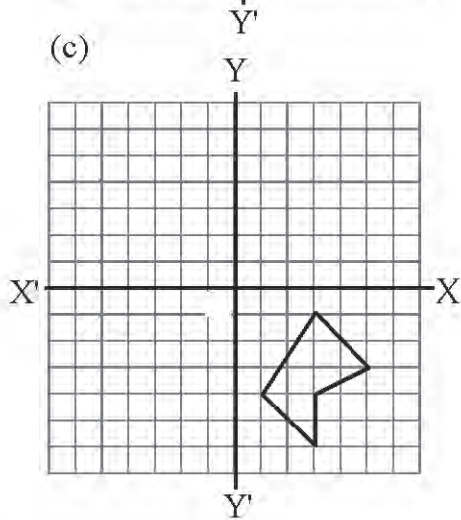
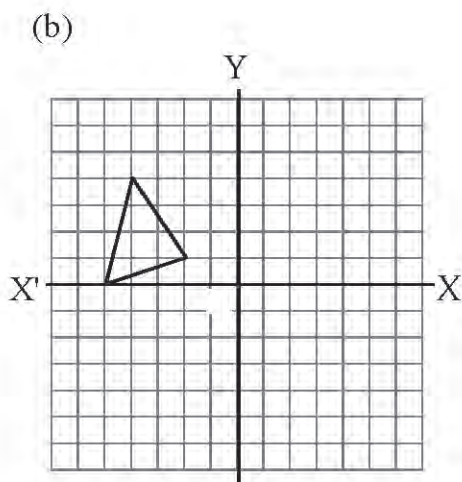
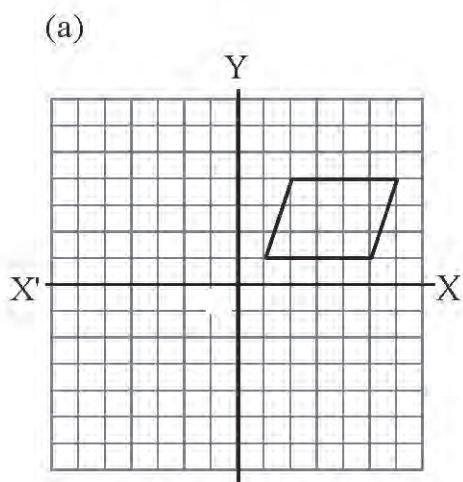
5. Draw a geometrical shape from the following vertices and rotate it centre about origin $0,0$ through A 90° and B -90° , then present in separate graph paper.

(i) $(2,7), (3,3)$ & $(6,7)$

(ii) $(3,2), (-2,2), (6,5)$ & $(1,5)$

(iii) $(10,6)$ & $(12,6)$

6. Rotate the following figures centre about origin through $+90^\circ$ and -90°

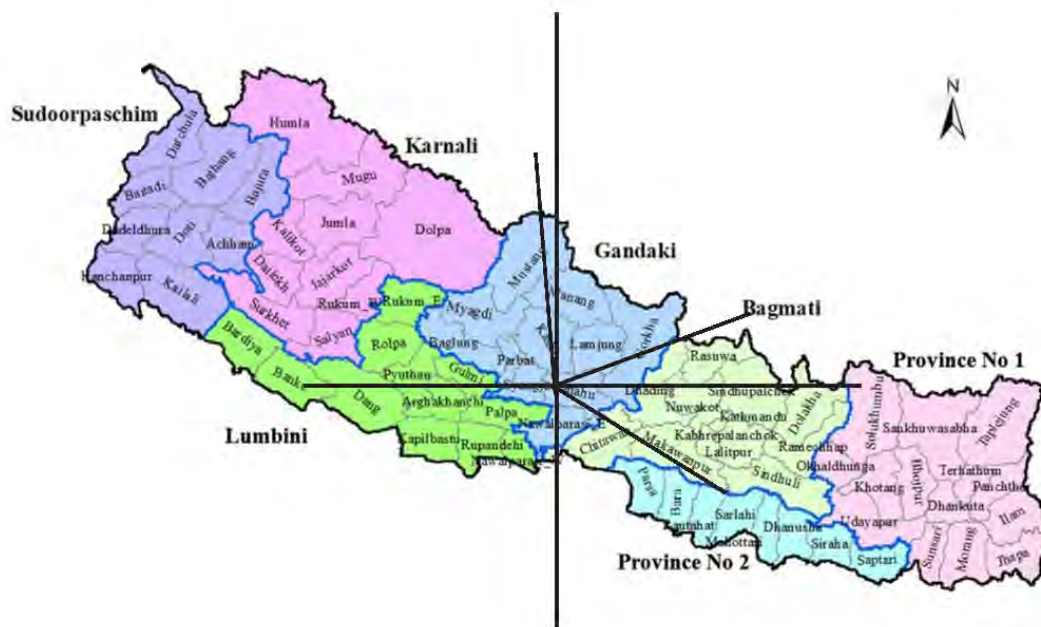


Answers

Show to your teacher

20.0 Review

- (a) The map of Nepal is given below. Taking the letter 'T' from Tanahun as the centre let's connect the first letter of the any six districts. Find the direction of those six districts from the place of letter 'T' of Tanahu. Discuss whether it is correct or incorrect.

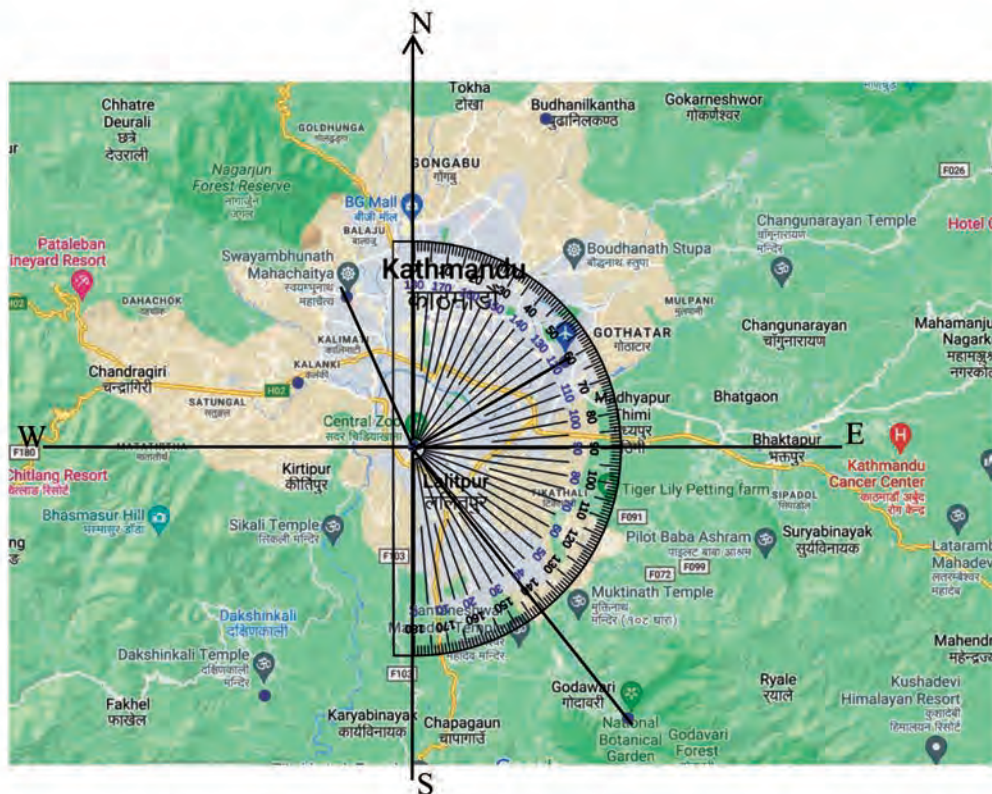


In the above map of Nepal taking letter 'T' Tanahun as the centre,

- 'M' of Manang in north direction.
 - 'G' of Gorkha in north east direction.
 - 'M' of Makwanpur in south east direction. Similarly, in which direction other districts lie form the letter 'T' of Tanahu?
- (b) By using a protractor, find the bearing in degree of 'G' of Gorkha district from the 'T' of Tanahu district.

20.1 Bearing

Activity 1



The map of Kathmandu valley is given above. From the central zoo, find the bearing of international airport, National Botanical Garden, Godabari and Swayambhunath.

Here, the bearing of international airport is 060° from the central zoo. Similarly, the bearing of National Botanical Garden is 142° . Bearing of Swayambhunath is 335° . Similarly, find the bearing of Kalanki, Bhudhanilkantha, Dakchinkali temple and so on from the central zoo.

The direction in which an object or place is sighted may be specified by giving angle in degrees that the direction makes with north line in clockwise is called the bearing of the object or place. Three digits are used to represent it. It is used in aeroplanes and ships by pilots and sailors respectively. The bearing of international airport from the central zoo is given above.

Example 1

The figure on the right side is a compass on the screen of two aeroplanes at the flight time. On the basis of this, find the bearing of aeroplanes.

Solution

Here, the first aeroplane is flying with the bearing of 050° . Similarly, the second aeroplane is flying with the bearing of 340° .



Example 2

From the place P,

- The bearing of a place is 60° , find that place.
- The bearing of a place is 330° , find that place.

Solution

Here, the line PN is drawn at P which represents the north direction.

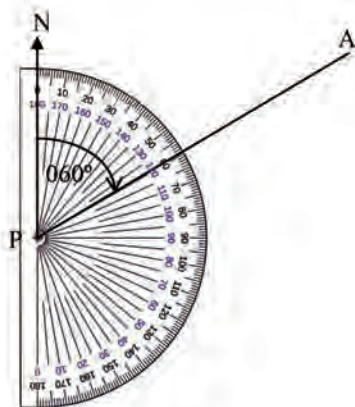


Figure (a)

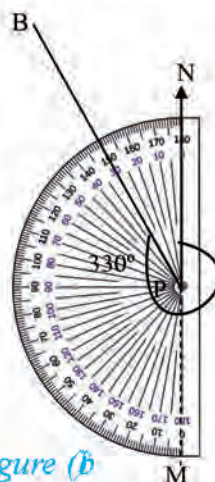


Figure (b)

- (a) PA is drawn making 60° with PN. Hence, the bearing of A from a point P is $\angle NPA = 060^\circ$.
- (b) Similarly, to draw BP through the point P making 330° with PN, the acute angle $\angle NPB$ should be equal to $= 360^\circ - 330^\circ = 30^\circ$. Hence, the bearing of B from the point P, the reflex angle $\angle NPB = 330^\circ$.
- i.e. produced NP, straight to the Point M so that, $\angle NPM + \angle MPB = 180^\circ + 150^\circ = 330^\circ$.
- Hence, the bearing of B from the point P the reflex angle $\angle NPB = 330^\circ$.

Example 3

If the bearing of a place S from the place R is 050° , what is the bearing of place R from the place S? Find out it.

Solution

Here,

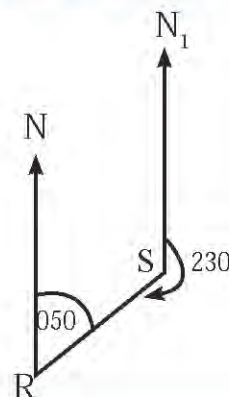
The bearing of S from the place R is 050°

$$\angle NRS + \angle N_1SR = 180^\circ$$

$$\text{Or, } 050^\circ + \angle N_1SR = 180^\circ$$

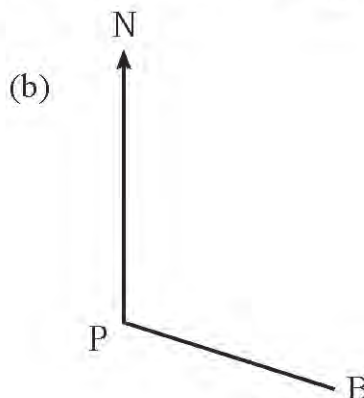
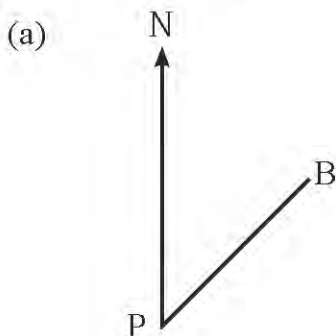
$$\text{Or, } \angle N_1SR = 180^\circ - 050^\circ = 130^\circ$$

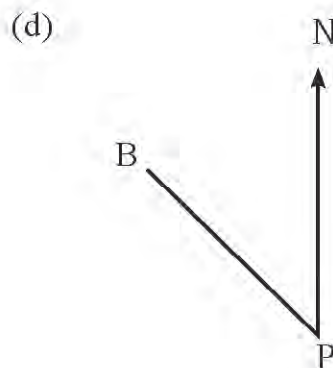
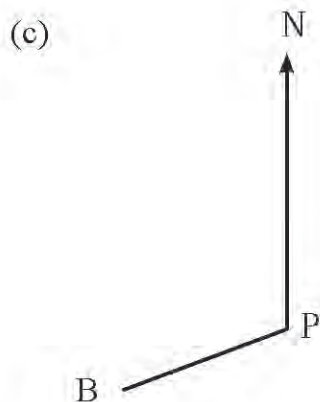
$$\text{The bearing of R from the place S} = 360^\circ - 130^\circ = 230^\circ$$



Exercise 20.1

1. Measure the angle in the given figure and write the bearing of place B from the place P.

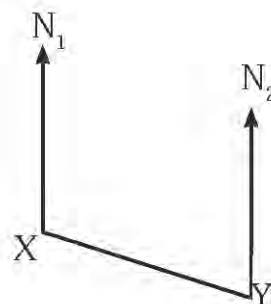
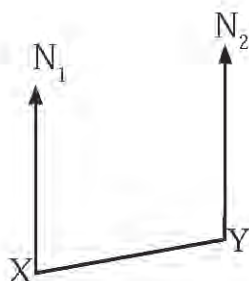
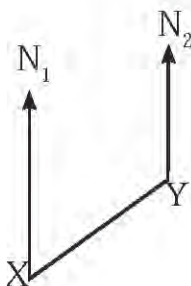




2. Find a place from a point A in the given bearings below.

(a) 030° (b) 090° (c) 125° (d) 260° (e) 285° (f) 340°

3. In the given figure, with the help of protractor, find the bearing of Y from the place X. Also find the bearing of X from Y.



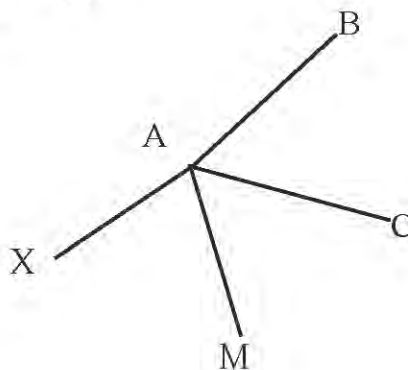
4. In the given condition, find the bearing and draw the figure:

- If the bearing of your school from your home is 045° , then the bearing of your house from the school.
- In a classroom, if the bearing of the place of Sangita from the place of Rita is 100° , the bearing of Rita's place from Sangita's.
- The bearing of the temple of village from your school is 190° , the bearing of the school from the temple.
- The bearing of Kathmandu from your district headquarter is 265° , the bearing of your district headquarter from Kathmandu.

- (e) The bearing of health post from your house is 065° , the bearing of your house from the health post

5. On the basis of the adjoining figure, answer the following questions:

- (a) What is the bearing of B from the point A?
- (b) What is the bearing of C from the point B?
- (c) What is the bearing of M from the point A?
- (d) What is the bearing of X from the point A?
- (e) What is the bearing of C from the point A?



Answers

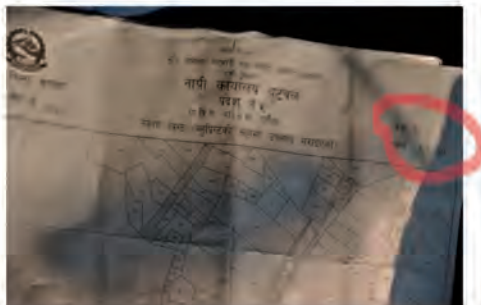
- 1 - 3 Show to your teacher.
4. (a) 225° (b) 280° (c) 010° (d) 085°
5. Show to your teacher.

20.2 Scale Drawing

Activity 1

Observe the given maps and discuss the questions given below in group.

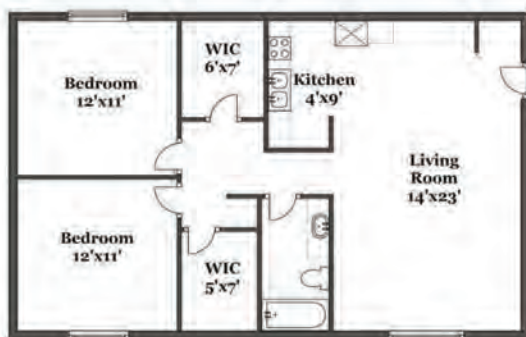
(a) Map of the plot



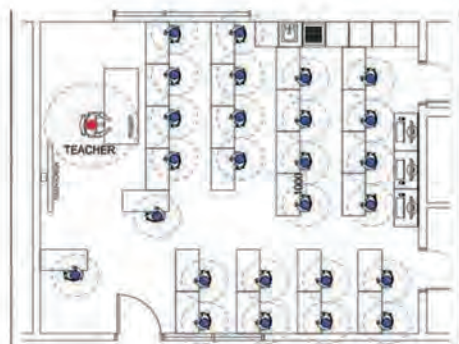
(b) Map of Nepal



(c) Map of the ground floor of a house



(d) Map of your class room



- In which maps is the scale mentioned? If yes, what scale is given?
- What is the difference when the scale is given or not given in map?
- Construct a map of football court using scale.

A scale is used to map an object or surface.

- Very big and small object can be drawn by using scale.
- By taking larger or smaller measurements as needed on the actual object and on the map, we have to construct the map where certain proportions are made.
- We can find the actual measurement of the object or surface, while we are preparing the map with certain scale.

Activity 2

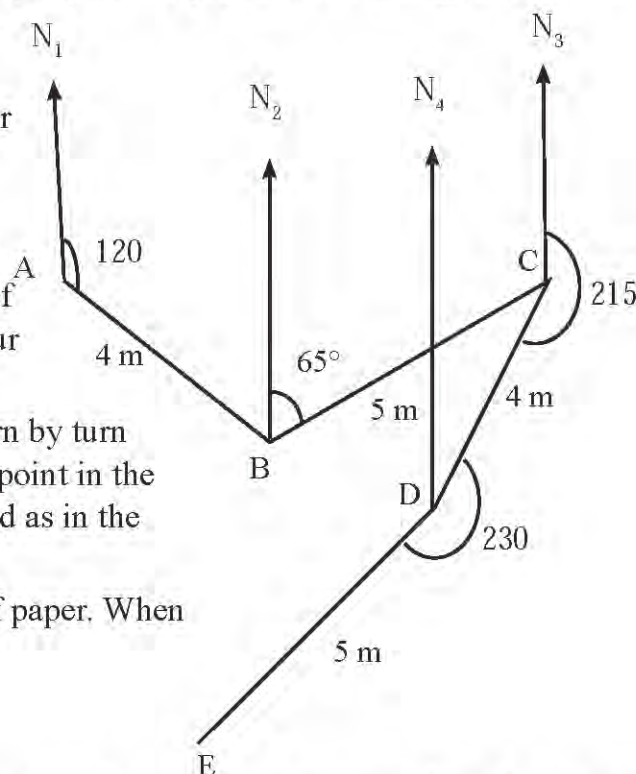
Activity to fly aeroplane,

Required material: Thread or rope, pin, protractor or compass which shows the direction, ruler or meter tape, piece of paper with bearing and distance and the boxes to put pictures.

Number of players:

You can make the group as per your need. The rules and the process:

- Bearing (in degree) and distance written pieces of paper are to be put in four different boxes.
 - Take a piece of paper turn by turn by each group. Find the point in the different places of ground as in the pieces of paper.
 - Give the second piece of paper. When the first task is done.
 - In this way, the group or person who prepare the aeroplane flying root first with correct bearing and scale, will be the winner of the game.
- For example:
- The bearing from point A to point B is 125° and the point B is at a distance of 4 meter.
 - The bearing from point B to point C is 065° and the point C is at a distance of 5 meter.
 - The bearing from point C to point D is 215° and the point D is at a distance of 4 meter.
 - The bearing from point D to point E is 230° and the point E is at a distance of 5 meter.



Example 1

The map is drawn using the scale $1 \text{ cm} = 500 \text{ m}$. The actual distance between the two places is 6 cm . What is the actual distance between the two places?

Solution

Here, scale used in map, $1 \text{ cm} = 500 \text{ m}$

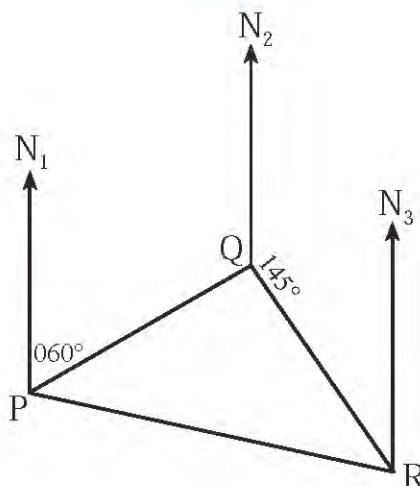
In map, the distance between two places = 6 cm

The actual distance between two places = $6 \times 500 = 3000 \text{ m} = 3 \text{ km}$

Example 2

1. An aeroplane flies 600 miles in the bearing of 060° from the initial place P to another place Q. Again, it flies 800 miles in the bearing of 145° from the place Q to place R then,

- (a) What is the actual distance between initial place (P) and final place (R)?
(Scale: $1 \text{ cm} = 200 \text{ miles}$)
- (b) Find the bearing of initial place (P) from final place (R).



Solution

Here, the initial place of aeroplane = P and final place = R

The scale in map, $1 \text{ cm} = 200 \text{ miles}$

In map, the distance between P and R = 4.5 cm

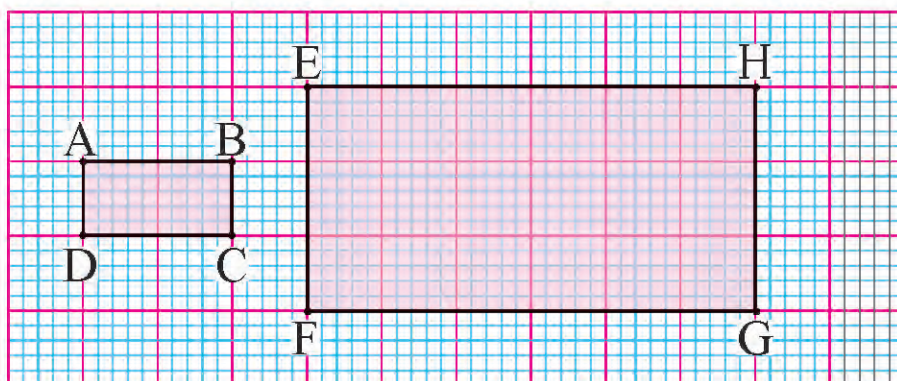
- (a) The actual distance between P and R = $4.5 \text{ cm} \times 200 = 900 \text{ miles}$
- (b) Again, in the figure, taking the measurement of the angle $\angle N_3RP$ by protractor $\angle N_3RP = 078^\circ$

The bearing of initial place from final place = $360^\circ - 078^\circ = 282^\circ$

Hence, the bearing of initial place (P) from final place (R) is 282° .

Exercise 20.2

- The scale drawing of rectangle EFGH is rectangle ABCD in the adjoining figure. What is the scale used here?



- Using the graph, draw the triangle such that the given triangle has the following:

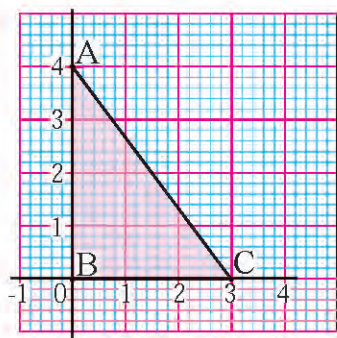


Figure (a)

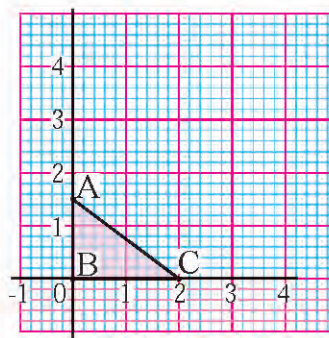
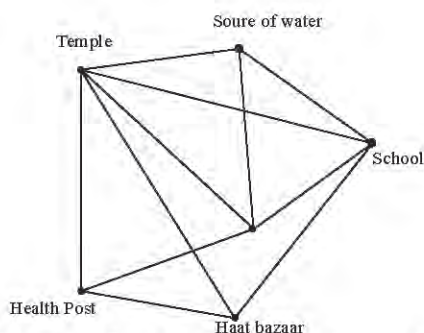


Figure (b)

- The size is half of the given triangle
- The size is double of the given triangle
- What is the relationship on corresponding sides and angles of given triangles and triangle drawn using scale?
- What about the scale when the size was made half and doubled?
- What should be the scale to draw ten times smaller? What should be the scale to draw ten times bigger?

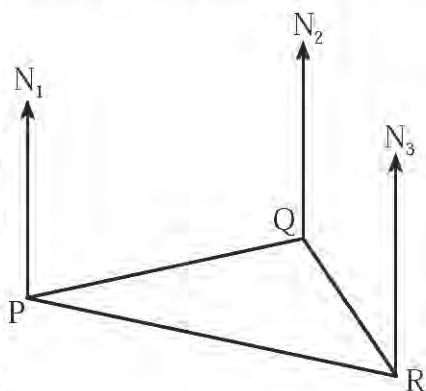
3. Find the actual distance between the two place on the basis of the following questions.
 - (a) The distance between two places in map is 7 cm (Scale: 1 cm = 750 m)
 - (b) The distance between two places in map is 6.5 cm (Scale: 1 cm = 1000 miles)
 - (c) The distance between two places in map is 3 cm (Scale: 1 cm = 350 m)
 - (d) What should be the scale to draw ten times smaller? What should be the scale to draw ten times bigger?
4. Measure the sides of given regular hexagon. It is drawn in the scale of 1:200. Find the length of sides of hexagon.
5. Draw the map of a triangular rural municipality having measurement: 15 km, 20 km and 25 km. Use a suitable scale.
6. Madhav walks 5 km in the bearing of 030° from the place A to reach place B. Again, he walks 3 km in the bearing of 140° from the place B to reach place C. Finally, he returned straight from C to A then,
 - (a) Let's do the scale drawing by taking suitable scale.
 - (b) What is the scale distance from place C to place A?
 - (c) When he returned straight from C to place A, then what is the actual distance covered by him?
 - (d) Find the bearing of place A from place C.
7. A temple lies 500 meter south from the bus park of a city. A pond lies in the bearing of 030° from the temple. Pond lies in the bearing of 145° from the Bus Park. What is the actual distance between the pond and the temple? Use the scale 1 cm = 100m and show it.
8. Place A lies 400 meter west from the place B. The bearing of the place C from the place A is 050°. The bearing of the place C is 290° from the place B, then.
 - (a) Let's do the scale drawing by taking 1 cm = 40 m.
 - (b) What is the actual distance between the place B and place C?
 - (c) Find the bearing of place B on the basis of place C.
9. In the figure, the important places of a village are mentioned. If 1 cm = 150 m, use the ruler and find the actual distance of the following places from the post office:

- (a) Temple
- (b) Source of water
- (c) Health post
- (d) Market place (Haat bazaar)
- (e) School



10. Renuka has prepared a map scale of $1 \text{ cm} = 500 \text{ m}$ for the triangular path she walk every morning. n the basis of the map answer the following asked questions:

- (a) What is the distance Renuka covers each day?
- (b) What is the actual distance between the initial place and the final place?
- (c) Find the bearing of place Q from the place P.
- (d) Find the bearing of place R from the place Q.
- (e) Find the bearing of initial place P from the final place R.



Project Work

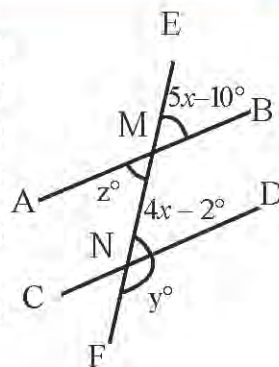
- (a) Take the measurement of the plot your house is construct the map of the plot. Take the scale according to the size of paper and size of plot. On the basis of the scale taken by you, find the area of plot.
- (b) Draw the measurement and design of the rooms of the first floor of your house, taking suitable scale.
- (c) Let's present the measurement of plot and design of the house to the members of family. Note their feedback, if necessary, improve the map or design.

Answers

1. 1:3 2. Show to your teacher 3. (a) 5.25 km
 (b) 6500 mile (c) 1.05 km 4 -10 Show to your teacher

Mixed Exercise

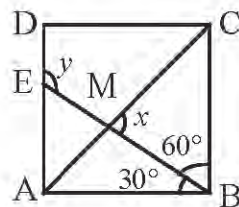
1. In the figure, the lines AB and CD are intersected by a transversal line EF at the points M and N respectively. Where, $\angle EMB = 5x - 10^\circ$, $\angle END = 4x - 2^\circ$, $\angle FND = y^\circ$ and $\angle AMN = z^\circ$. Observe the given figure and answer the following questions.



- Write two pairs of alternate angles and corresponding angles.
- Are the measurement $\angle BMN$ and $\angle MND$ equal?
- Find the value of x , y and z .

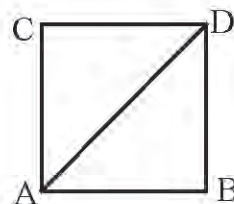
2. In the figure, a square ABCD having length 4 cm are given. Where, $\angle ABE = 30^\circ$, $\angle BMC = x$, and $\angle DEM = y^\circ$. Observe the given figure and answer the following questions.

- Write the formula to find the exterior angle of square.
- Find the values of x and y .
- Construct the square ABCD by using compass.

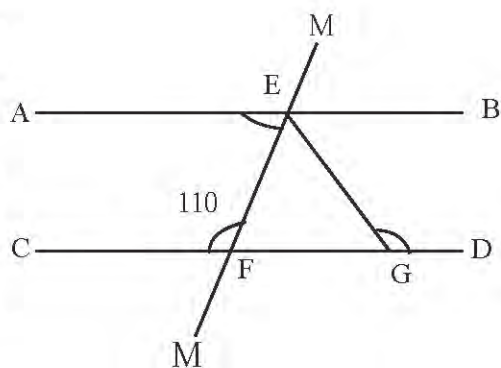


3. In the figure, ABCD is a square whose length is 8 cm and AC is a diagonal.

- Write the formula to find the interior angle of the square.
- Prove that: $\triangle ABD \cong \triangle ACD$
- What is the area of $\triangle ABC$? Calculate it.



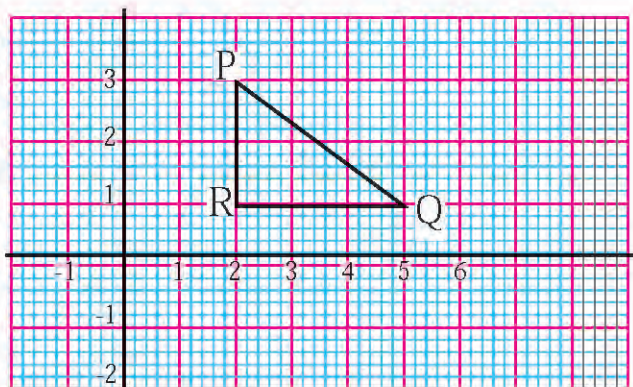
4. **On the basis of the given figure and answer the following questions:**



- From figure, write the name of corresponding angle equal to $\angle MEA$.
- If $\angle CFE = 110^\circ$, what is the value of x ? Find it.
- If $EF = FG$, then, what is the value of y ? Find it.

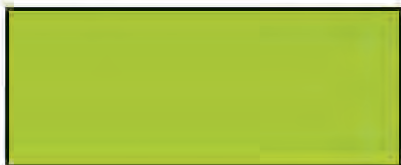
5. **Answer the following questions based on the given figure:**

- Draw the ΔPQR in graph as given in the figure. Reflect the ΔPQR in Y-axis and write the coordinates of the vertices of ΔPQR after reflection.



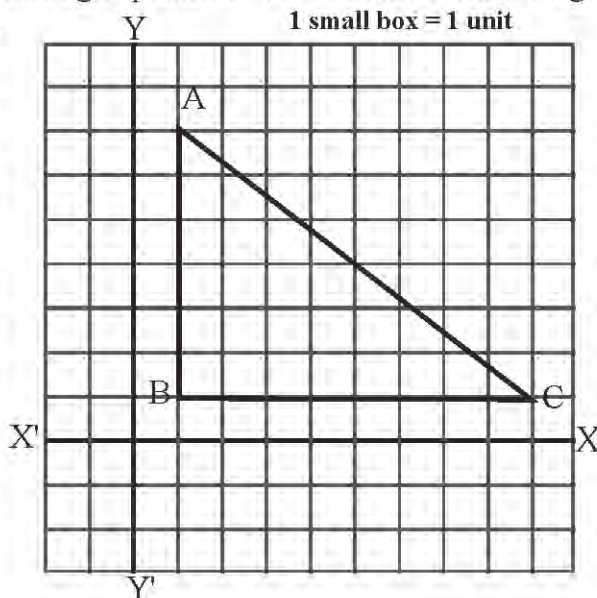
- In the given ΔPQR , when the coordinates of P, Q and R are doubled, then what is the length of $P'Q'$ of $\Delta P'Q'R'$? Is the length of $P'Q'$ double of the length of PQ ? Write the reason with calculation.

6. The given rectangle and parallelogram are equal in area. The length and breadth of rectangle are 8 cm and 5 cm respectively. The length of base of parallelogram is 10 cm.



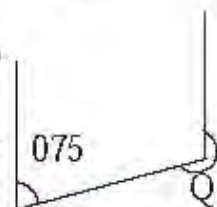
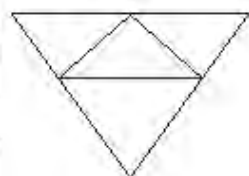
- Construct the rectangle ABCD with the help of compass.
 - What is the height of parallelogram? Find it out.
 - Does the area double, when both the height and the length of base of parallelogram are doubled? Write with reason.
7. In the given graph, A (1, 7), B (1, 1) and C (9, 1) are the vertices of $\triangle ABC$.

- Find the coordinates of the vertices of the image triangle $\triangle ABC$ under the rotation through positive 90° about centre origin. Present both triangles on the same graph.
- What is the length of side AC? Use Pythagoras theorem to find it.
- All the congruent triangles are similar but all the similar triangles are not congruent. Justify this statement with example.



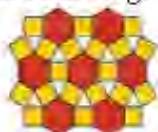
8. In the given figure, net of solid object formed by four equilateral triangles are given.

- Write the name of solid formed by the given net.
- How many linear axes of symmetry can be drawn in an equilateral triangle at most? Give example with reason.
- In the given figure, the bearing of point Q from point P is 075° . What is the bearing of point P from Q? Find it out.



9. In the given figure, four samples of tessellation are mentioned.

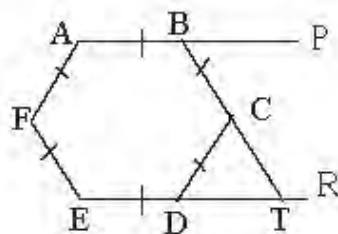
- Define regular and semi-regular tessellation.



- Distinguish the regular and semi-regular tessellation from the above four tessellations.
- Write the relationship between regular polygon and the regular tessellation.

10. In the adjoining figure, $AP \parallel ER$ and $AB = BC = CD = DE = EF = FA$.

- In the figure, what is the geometrical name of ABCDEF? Write it.
- Find the value of $\angle CDT$.
- Prove that: $\triangle CDT$ is an equilateral triangle.



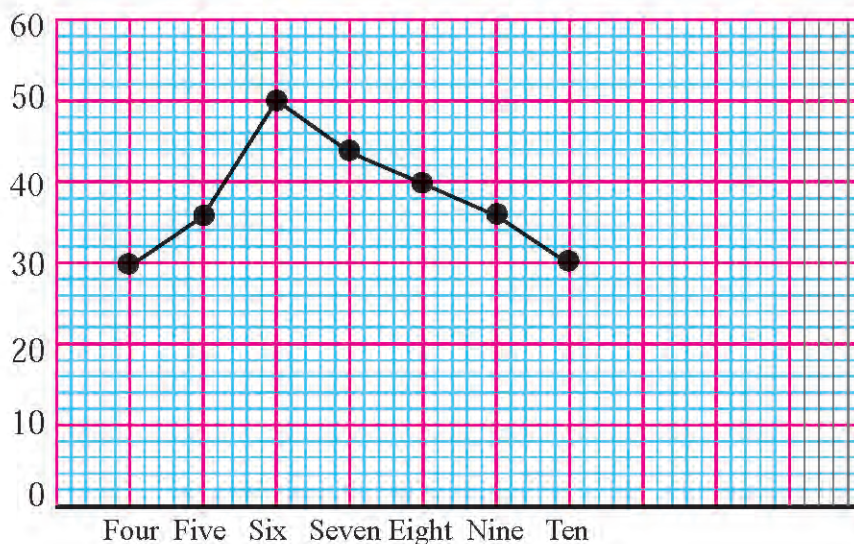
Answers

- (a) and (b) Show to your teacher (b) $x = 8^\circ$, $y = 150^\circ$, $z = 30^\circ$
- (i) $x = 75^\circ$, $y = 120^\circ$ 3. (a) and (b) Show to your teacher (c) 32cm^2
- Show to your teacher. 5. (b) $x = 70^\circ$ (c) $y = 125^\circ$
- 6 - 7 Show to your teacher 8. (a) tetrahedon (b) 3 (c) 255
- Show to your teacher 11. (a) regular hexagan (b) 60°
- (c) Show to your teacher

21.0 Review

The given line graph is prepared on the basis of the number of books obtained by Mahendragram Secondary School, in 2079. Study the line graph and discuss the following questions:

The details of the books obtained by Mahendragram Secondary School (2079)

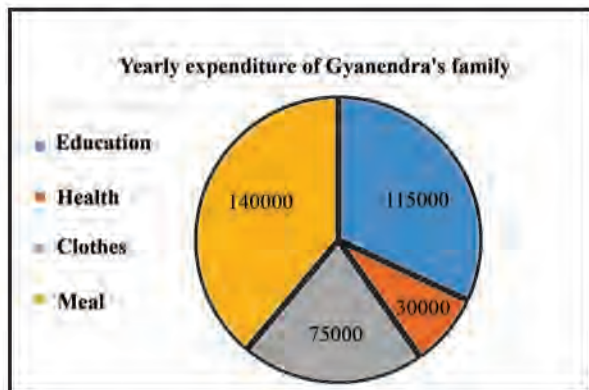


- How many books were received in each class?
- Which class received more number of books?
- Which class received less number of books?
- Which classes received the equal numbers of books?
- How many books were received in total?
- Construct the frequency distribution table on the basis of line graph.
- Construct the bar graph on the basis of frequency table prepared above in (f).

21.1 Pie Chart

Activity 1

The details of yearly expenditure of Gyanendra's family is presented in the given pie chart. Discuss the following questions based on it.



- What is the yearly expenditure of Gyanendra's family in education?
- What is their expenditure on health, clothes and food? Find it.
- Which headings have maximum expenditure and minimum expenditure?
- What is the expenditure percentage in each heading?
- What is the monthly expenditure on food if they are doing equal expenditure in each month of that year?
- What is the yearly income of the Gyanendra's family if they have saved Rs. 2,35,000 yearly in a bank after deducting the above mentioned expenditure?

Question to think: Why the pie chart is used in the discussion of the above activity 1?



It is easy to take and give the information from the pie chart but how can we construct it?

Activity 2

A pie chart prepared on the basis of yearly expenditure of Gyanendra's family is given in activity 1. Observe the prepared pie chart. How is it constructed? Discuss in group. On the basis of the group discussion, construct the pie chart using the given data.

Details of the yearly expenditure of Aappa's family

Education	Health	Clothes	Food
Rs. 96,000	Rs. 40,000	Rs. 64,000	Rs. 1,20,000

The pie chart in activity 1 is prepared on the basis of details of yearly expenditure of Gyanendra's family. The expenditure is on health, clothes and food in his family. The complete angle is 360° at the centre of circle. How the total expenditure should be divided into 360° with their heading?

Total expenditure of Gyanendra's family = Rs. 3,60,000. Let's divide it equally into 360° .

$$\text{Rs. } 3,60,000 = 360^\circ$$

$$\text{Rs. } 1 = \frac{360^\circ}{3,60,000}$$

$$\text{Expenditure on education} = \text{Rs. } 1,15,000 = \frac{360^\circ}{3,60,000} \times 1,15,000 = 115^\circ$$

Test whether the angle 115° is formed or not in education in the above pie chart? Let's verify it.

$$\text{Expenditure on health} = \text{Rs. } 30,000 = \frac{360^\circ}{3,60,000} \times 30,000 = 30^\circ$$

$$\text{Expenditure on clothes} = \text{Rs. } 75,000 = \frac{360^\circ}{3,60,000} \times 75,000 = 75^\circ$$

$$\text{Expenditure on food} = \text{Rs. } 1,40,000 = \frac{360^\circ}{3,60,000} \times 1,40,000 = 140^\circ$$

Similar to the above process, construct the pie chart of yearly expenditure of Aappa's family

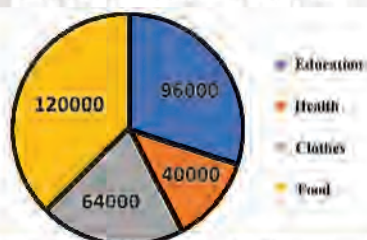
Total expenditure of Aappa's family = Rs. 3,20,000

$$\text{Rs. } 3,20,000 = 360^\circ$$

$$\text{Rs. } 1 = \frac{360^\circ}{3,20,000}$$

Titles	Expenditure	Angle at the centre
Education	Rs. 96000	$= \frac{360^\circ}{320000} \times 96000 = 108^\circ$
Health	Rs. 40,000	$= \frac{360^\circ}{320000} \times 40000 = 45^\circ$
Clothes	Rs. 64,000	$= \frac{360^\circ}{320000} \times 64000 = 72^\circ$
Food	Rs. 1,20,000	$= \frac{360^\circ}{320000} \times 120000 = 135^\circ$
Total	Rs. 3,20,000	

Details of the yearly expenditure of Aappa's family



If any data is equal to the area of a circle, different titles are presented in different sectors, then the picture is called a pie chart.

The following steps should be followed while presenting the given data in a pie chart.

Step 1 Find the total value of the given data.

Step 2 Equating the total value of the given data with 360° , calculate the size of the angle in each title.

$$\frac{360^\circ}{\text{Total frequency}} \times \text{value of each title}$$

Step 3 Taking a suitable radius, construct a circle and draw a radius.

Step 4 With the help of protractor, taking the radius as the base line construct the angle in each title respectively.

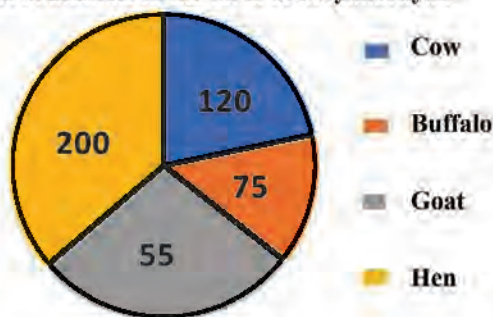
Step 5 Put the different colours in different sectors. Show the index of the expenditure of colour.

Step 6 Give the title of pie chart. Now, the pie chart is ready.

Example 1

1. The pie chart is prepared on the basis of the details of animals on Satyanarayan's farm. Study the pie chart and answer the following questions.

Details of animals in the farm of Satyanarayan



- (a) Find the number of animals and their types in the farm of Satyanarayan?
(b) Which animals are in maximum number and minimum number?
(c) What is the total number of animals all together?
(d) What percentage of cow, buffalo, goat and hen are in the farm?

Solution: Here,

- (a) In Satyanarayan farm, there are 120 cows, 75 buffaloes, 155 goats and 200 hens.
(b) The hens are in maximum number and buffaloes are in minimum number.
(c) Total number of all animals = $120 + 75 + 155 + 200 = 550$

$$\text{Cow} = \frac{120}{550} \times 100\% = 21.8\%$$

$$\text{Buffalo} = \frac{75}{550} \times 100\% = 13.6\%$$

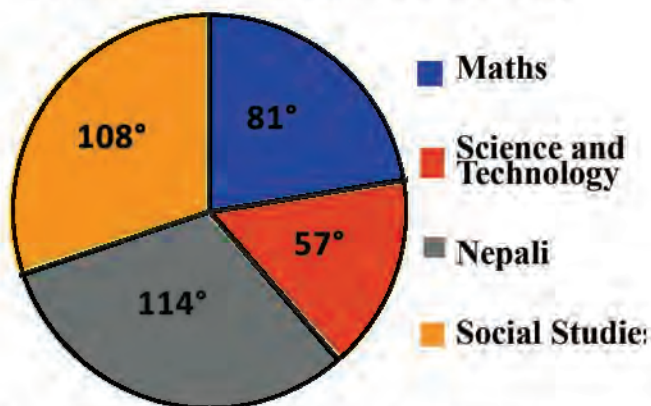
$$\text{Goat} = \frac{155}{550} \times 100\% = 28.2\%$$

$$\text{Hen} = \frac{200}{550} \times 100\% = 36.4\%$$

Example 2

A question which subject do you like? is asked to the 480 students of Janata Secondary School. The following pie chart is prepared on the basis of the answers obtained from them. Study the pie chart and answer the following asked questions.

Details of favorite subjects of students



- Find the number of students who like each subject?
- What is the difference between the number of students in the most and least favourite subjects? Find.
- What is the percentage of students who like Mathematics and Science and Technology?

Solution

Here,

Total number of students = 480

Now, $360^\circ = 480$

$$\therefore 1^\circ = \frac{480}{360} = \frac{4}{3}$$

$$(a) \text{ The number of students who like Nepali } = 114^\circ = 114 \times \frac{4}{3} = 152$$

$$\text{The number of students who like Maths } = 81^\circ = 81 \times \frac{4}{3} = 108$$

$$\text{The number of students who like Science and Technology } = 57^\circ = 57 \times \frac{4}{3} = 76$$

$$\text{The number of students who like Social studies } = 108^\circ = 108 \times \frac{4}{3} = 144$$

(b) Maximum number of students who like Nepali = 152

Minimum number of students who like Science and Technology = 76

Difference = $152 - 76 = 76$

(c) Percentage of students who like Mathematics = $\frac{108}{480} \times 100\% = 22.5\%$

Percentage of students who like Science = $\frac{76}{480} \times 100\% = 15.8\%$

Example 3

The number of students who are studying from grade 5 to 8 in Gyan Jyoti Secondary School are given below in the table below. Present it in pie chart.

Grade	5	6	7	8	Total
Number of students	45	55	40	60	200

Solution

Here,

Total number of students = 200

Now, 200 students = 360°

$$1 \text{ student} = \left(\frac{360^\circ}{200}\right) = \left(\frac{9}{5}\right)$$

$$\text{Now, 45 students of grade 5} = \left(\frac{9}{5}\right) \times 45 = 81^\circ$$

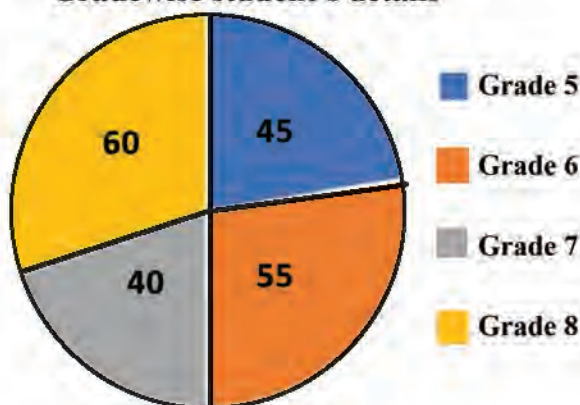
$$55 \text{ students of grade 6} = \left(\frac{9}{5}\right) \times 55 = 99^\circ$$

$$40 \text{ students of grade 7} = \left(\frac{9}{5}\right) \times 40 = 72^\circ$$

$$60 \text{ students of grade 8} = \left(\frac{9}{5}\right) \times 60 = 108^\circ$$

Representing above information in the pie chart.

Gradewise student's details



Example 4

The detail of distribution of Dipendra's monthly expenditure in different titles and saving amount is given in the table below. Present it in a pie chart.

Title	Food	Health	Miscellaneous	Saving	Total
sum of money (in percent)	20	15	35	30	100

Solution

Total income = 100%

100% income = 360°

$$1\% \text{ income} = \left(\frac{360}{100}\right) = \left(\frac{18}{5}\right)$$

$$\text{Now, in food} = 20\% = \left(\frac{18}{5}\right) \times 20 = 72^\circ$$

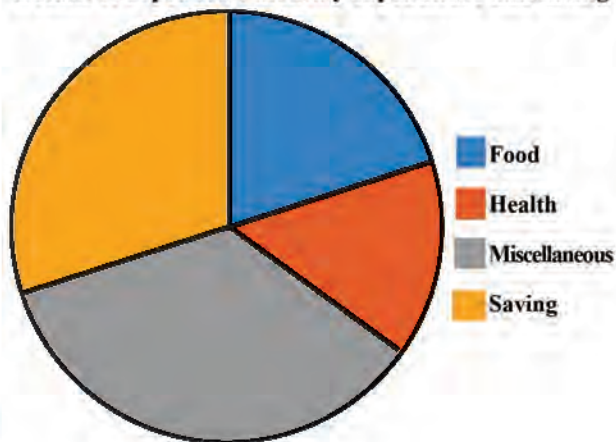
$$\text{In health} = 15\% = \left(\frac{18}{5}\right) \times 15 = 54^\circ$$

$$\text{In miscellaneous} = 35\% = \left(\frac{18}{5}\right) \times 35 = 126^\circ$$

$$\text{In saving} = 30\% = \left(\frac{18}{5}\right) \times 30 = 108^\circ$$

Representing above information in the pie chart,

Details of Dipendra's monthly expenditure and saving



Example 5

The students of Yuba Secondary School are going for an educational tour. They are asked where they would like to visit; Pokhara, Lumbini, Illam or Kathmandu. The responses of the students are given in the table below. Present it in a pie chart.

The place for educational tour	Pokhara	Lumbini	Illam	Kathmandu	Total
Number of student	75	55	81	42	253

Solution: Here,

Total number of students = 253

253 student = 360°

$$1 \text{ student} = \left(\frac{360}{253} \right)$$

Now, 75 students who likes Pokhara = $\left(\frac{360}{253} \right) \times 75 = 106.7^\circ = 107^\circ$ (Round off)

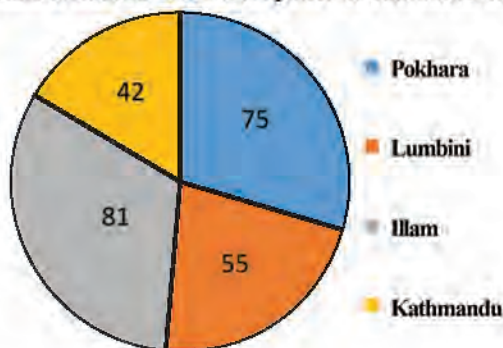
55 students who likes Lumbini = $\left(\frac{360}{253} \right) \times 55 = 78.3^\circ = 78^\circ$ (Round off)

81 students who likes Illam = $\left(\frac{360}{253}\right) \times 81 = 115.3^\circ = 115^\circ$ (Round off)

42 students who likes Kathmandu = $\left(\frac{360}{253}\right) \times 42 = 59.8^\circ = 60^\circ$ (Round off)

Representing above information in pie chart,

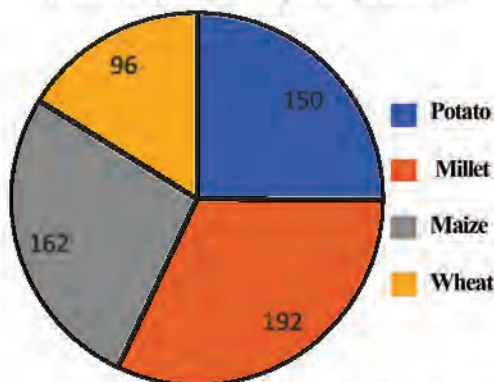
Details of students who like the places for educational tour



Exercise 21.1

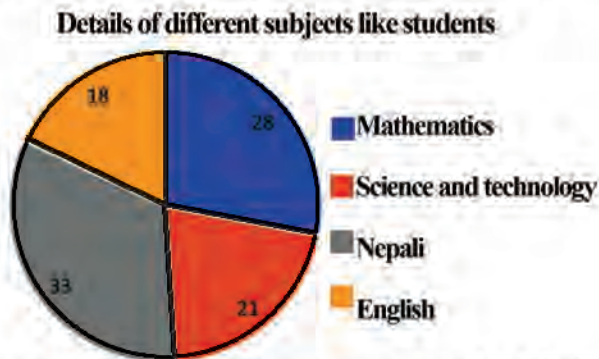
1. In the given figure, the data of main crops of the family of Dolpa district are given. Answer the following questions on the basis of the pie chart.

The main crops of the family of Dolpa district



- (a) How many families are there having Potato and millet as the main crops?
- (b) How many families are there having maize and wheat as the main crops?
- (c) How many families are there in the above data? Find it.

2. The pie chart prepared on the basis of the favourite subjects of 300 students of class 8 of an Aki Secondary School is given in the figure. Observe the pie chart and answer the following questions.

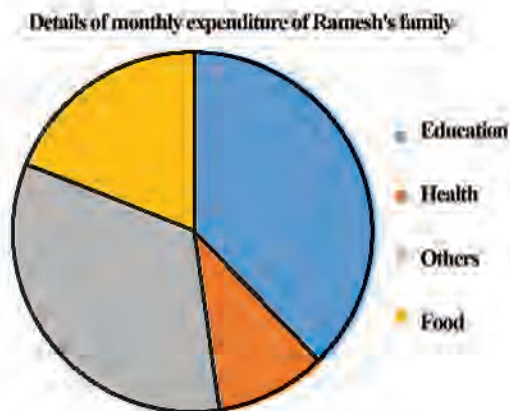


- (a) What is the number of students who like Mathematics?
 - (b) What is the number of students who like English?
 - (c) Find the total number of students who like Mathematics and English.
 - (d) Find the total number of students except the ones who like Mathematics, Science and Technology and Nepali.
3. In the adjoining figure, the details of monthly expenditure of Sarswati Cooperative Limited is given in the pie chart. If the expenditure of that cooperative in the month of Mangsir is Rs. 54, 000 answer the following questions.



- (a) What is the expenditure in salary?
- (b) What is the expenditure in stationary?
- (c) What is the expenditure in others? Find it.
- (d) How much more in the expenditure in salary than in stationary? Find.

4. In the adjoining figure, the monthly expenditure of Ramesh's family is presented in the pie chart. If the expenditure of that family for food in a month is Rs. 8,500, answer the following questions.



- (a) What is the expenditure in education?
(b) What is the expenditure in health?
(c) What is the expenditure in others?
(d) What is the total monthly expenditure of Ramesh's family? Find it.
5. The most liked extra-curricular activities and the number of students of class 8 are given in the table below. Present the table in a information of the pie chart.

Drama	Comedy	Dance	Sports
7	8	9	10

6. The marks obtained by the students of class 8 of Saraswati Secondary School in mathematics are given in the table below. Present the information the table in a pie chart.

Distinction division	First division	Second division	Third division
40	56	32	16

7. The details of Pemba's monthly expenditure are given in the table below. Present the information the table in a pie chart.

Title	Food	Health	Education and transport	Communication and house rent
Percentage	30%	15%	32%	23%

8. The income of a family is given in the table below. Present the informations in the table in a pie chart.

Title	Salary	Agriculture	Business	House rent
Percentage	30%	10%	45%	15%

Project work

Ask your friends which one game they like more among volleyball, football and cricket. Tabulate the obtained information. On the basis of the table, make a pie chart and present in the class.

Answers

1. (a) 150, 192 (b) 162, 96 (c) 600
2 -8 Show to your teacher.

21.2 Mean, Median and Mode of Individual Series

a Mean

Activity 3

12 students of class 8 went to a cafeteria to have tiffin. They ordered the tiffin according to their interest. Later on, they asked the price. The shopkeeper gave the price list separately. Those 12 students had taken the tiffin having the price equal as given below.

65, 55, 45, 75, 78, 56, 90, 85, 57, 62, 80, 78

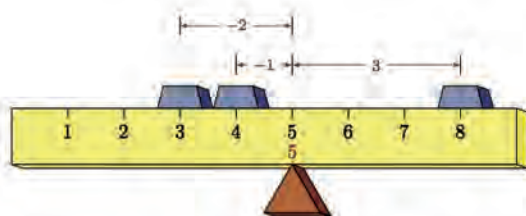
Discuss on the following questions and beased on data given above. write your conclusion.

- (a) How many rupees worth of tiffin did the person who ate the most eat? What did they eat for tiffin?
- (b) How many rupees worth of tiffin did the person who ate the least eat? What did they eat for tiffin?
- (c) What is the total amount of tiffin for all 12 friends?
- (d) If they have decided to pay equal amounts, what is the amount for each?
- (e) If they are paying equal amount, what is called the individual paying amount?

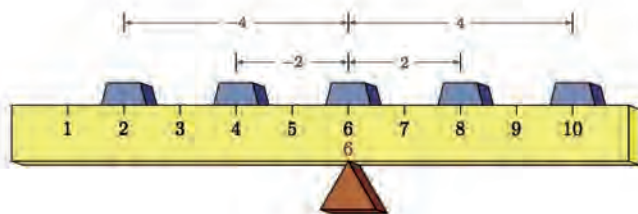
The quotient obtained by dividing the sum of the given data by the number of terms is called mean of the given data. It is also called arithmetic mean or only mean. It is denoted by $\bar{X}$. If the given data is denoted by X , the sum of data is denoted by $\sum X$. If the total number of data is 'n', the mean is

$$(\bar{X}) = \frac{\sum X}{n}.$$

The mean is more clear from the given figure.



3, 4 and 8 are the first data given. 5 is the mean of 3, 4 and 8. In the first figure, the fulcrum just put below 5 creates balance.



2, 4, 6, 8 and 10 are given as the second data. 6 is the mean of 2, 4, 6, 8 and 10. In the second figure, the fulcrum just put below 6 it creates balance.

Example 1

The marks obtained by the students in their first terminal examination in mathematics are given below.

4, 12, 13, 21, 12, 12, 10

Solution

The sum of the obtained marks (ΣX) = $4 + 12 + 13 + 21 + 12 + 12 + 10 = 84$

Total number of students (n) = 7

Average marks = mean ($\bar{X}$) = ?

We know that,

$$\text{Average marks} = \text{mean } (\bar{X}) = \frac{\Sigma X}{n} = \frac{84}{7} = 12$$

Therefore, average marks = mean ($\bar{X}$) = 12

Example 2

The score obtained by the 5 houses in inter-house volleyball competition organized by a school are given as 12, 15, 18, a and 14. If their mean score is 15 then calculate the value of a .

Solution

The sum of the scores $(\Sigma X) = 12 + 15 + 18 + a + 14 = 59 + a$

Total number of houses $(n) = 5$

Mean score $(\bar{X}) = 15$

We know that,

$$\text{Mean } (\bar{X}) = \frac{\Sigma x}{n}$$

$$\text{or, } 15 = \frac{59+a}{5}$$

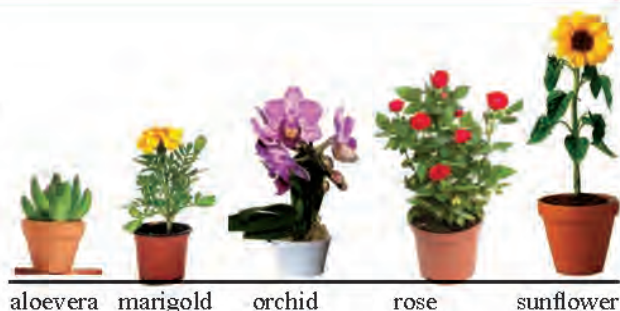
$$\text{or, } 75 = 59 + a \quad \therefore a = 16$$

5 Median

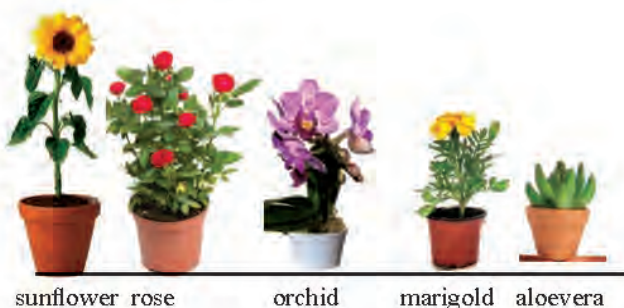
Activity 4

Let's discuss on the following two conditions in the group:

- (i) In the given figure, six vases are kept in the garden of a school, on the basis of the height of plants in ascending order.



Now, let's put the vases on the basis of the height of the plant in descending order.



- Which plant is in the middle position when the plants are kept on both ways?
- How many plants are there before and after the middle position plant?
- What is called the middle value?
- If the height of the plants with their pots are as: Sunflower 115 cm, Rose 95 cm, Orchid 85 cm, Marigold 75 cm and Aloe vera 50 cm, which height divides the height of plants in two equal parts?

(ii) If Dhupi is kept in the descending order as in the figure,



- Which plant is in the middle?
- On this condition, how can we find the height of the plant which is in the middle position?
- The height of the Dhupi with its pot is 125 cm. Which height divides the height of plants in two equal parts?

When we put aloe vera, marigold, orchid, rose and sunflower either in ascending order of their heights or descending order of their heights, orchid lies in the middle. There are two plants before and two plants after it. Orchid lies in the middle, so its height is called median. Again, according to the height of Dhupi, the plants are kept in descending order. Then it is difficult to say which plant is in the middle. If we consider the rose is in the middle, there are two plants before it and three plants after it. If we consider orchid is in the middle, there are three plants before it and two plants after it. In such condition, from the middle of the rose and orchid there are Dhupi, sunflower and rose before it and orchid, marigold and aloe vera after it. That position is called the position of median. The value in between the height of Rose and

orchid is $\frac{95+85}{2} = 90$ is the required median height of the plant.

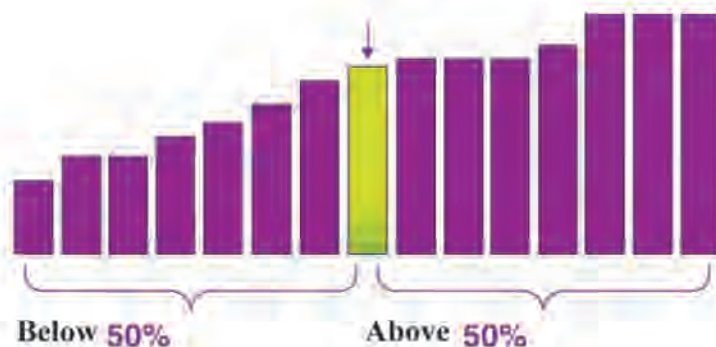
Condition i	Condition ii
Total number of plants (n) = 5	Total number of plants (n) = 6
The position of the middle = third position (after ascending or descending order)	The position of the middle = middle value of third and fourth position (after ascending or descending order)
The relationship between the total number of plants and the position of median	The relationship between the total number of plants and the position of median
The position of median = position of third place = $\frac{(5+1)}{2}$ term = $\frac{n+1}{2}$ term	The position of median = the value in between the $\frac{n}{2}$ th position and $(\frac{n}{2} + 1)$ th
Median (M_d) = the value of $\frac{n+1}{2}$ term.	Median (M_d) = the value of $\frac{n}{2}$ position and $(\frac{n}{2} + 1)$ is the mathematical mean.

The value which divides the given data into two equal parts is called median. It is denoted by M_d . If there are 'n' values in the given data,

(a) If n is odd, median (M_d) = the position of $\frac{n+1}{2}$ term .

(b) If n is even, median (M_d) = $\frac{\text{the value of } \frac{n}{2}^{\text{th}} \text{ item} + \text{the value of } (\frac{n}{2} + 1)^{\text{th}} \text{ item}}{2}$

We can make the median more clear from the given figure



Example 3

Find the median from the data related to the number of pencils Grade 4 students possess with them.

4, 12, 13, 11, 8, 15, 10

Solution

Here, writing the given data in ascending order,

4, 8, 10, 11, 12, 13, 15

Total number of students (n) = 7

Median (M_d) = ?

We know that,

Being 'n' is odd, Median (M_d) = the value of (M_d) = $\frac{n+1}{2}$ item

the value of $= \frac{7+1}{2}$ item

the value of 4th item.

∴ Therefore, the value of median (M_d) = 13.

Example 4

If the weights of 8 students are: 45kg, 42kg, 44kg, 50kg, 45kg, 48kg, 40kg and 43kg respectively, find the median.

Solution

Here, writing the given data in ascending order,

40kg, 42kg, 43kg, 44kg, 45kg, 45kg, 48kg, 50kg

Total number of students (n) = 8

Median (M_d) = ?

We know that,

Being, 'n' is even, median (M_d) = $\frac{\text{the value of } \frac{n}{2} \text{ item} + \text{the value of } (\frac{n}{2} + 1) \text{ item}}{2}$

$$= \frac{\text{the value of } \frac{8}{2} \text{ item} + \text{the value of } (\frac{8}{2} + 1) \text{ item}}{2}$$

$$= \frac{\text{the value of 4}^{\text{th}} \text{ item} + \text{the value of 5}^{\text{th}} \text{ item}}{2}$$

$$= \frac{44 + 45}{2} = 44.5$$

∴ Hence, median = 44.5

Alternative method,

Median (M_d) = the value of $= \frac{n+1}{2}$ item

$$= \text{the value of } = \frac{8+1}{2} \text{ item} = \text{the value of 4.5 item}$$

$$\therefore \text{Therefore, median } (M_d) = \frac{\text{the value of 4}^{\text{th}} \text{ item} + \text{the value of 5}^{\text{th}} \text{ item}}{2} = \frac{44 + 45}{2} = 44.5$$

(c) Mode

Activity 5

The sizes of shoes used by the present students of class 8 from Sagarmatha Secondary School are given below.

5, 6, 6.5, 7, 5.5, 5, 6, 6, 5.5, 5, 6, 6, 7, 6, 5, 6.5, 7

On the basis of the data given above, answer the following questions.

- Which size of shoes are used by the maximum number of students?
- What is called the maximum time repeated value?

In any data, the maximum time repeated value is called mode. In the above data maximum time repeated size of shoes is 6. So, 6 is the mode value of the given data. Mode is denoted by (M_o).

Example 5

If the height of 10 students of class 8 are 145cm, 149cm, 140cm, 148cm, 142cm, 149cm, 142cm, 155cm, 150cm and 149cm, find the mode.

Solution

Here, writing the given data in ascending order,

140cm, 142cm, 142cm, 145cm, 148cm, 149cm, 149cm, 149cm, 150cm, 155cm.

Model height (M_o) = ?

We know that,

The height of 3 students is 149cm. So, maximum time repeated height is 149cm

Hence, model height (M_o) = 149 cm

Exercise 21.2

1. Calculate the arithmetic mean of the given data.

- (a) 15, 13, 18, 16, 14, 17, 12
- (b) 84, 91, 88, 94, 91, 105, 98, 85
- (c) 45, 35, 37, 32, 47, 38, 39, 36, 34, 37
- (d) 105, 108, 112, 106, 120, 108, 112, 110, 100
- (e) 5ft, 4.8ft, 5.1ft, 4.8ft, 5ft, 4.9ft, 4.7ft, 4.9ft

2. Find the value of m from the following data.

- (a) $\Sigma X = 77 + m$, $n = 10$ and median $(\bar{X}) = 8$
- (b) $\Sigma X = 117$, $n = 8 + m$ and median $(\bar{X}) = 13$
- (c) $\Sigma X = 40 + m$, $n = 4 +$ and median $(\bar{X}) = 5$
- (d) $\Sigma X = 264 + 24a$, $n = 11 + a$ and median $(\bar{X}) = m$
- (e) The mean of 3, 6, m, 9 and 10
- (f) The mean of 42, 48, 36, 39, m, 45, 42 and 32.

3. Find the median from the following data.

- (a) 27, 29, 18, 25, 32, 21, 26
- (b) 250, 282, 211, 190, 235, 284, 237, 217, 245, 257, 281
- (c) 34, 46, 49, 38, 56, 86, 68, 35
- (d) 5.9ft, 5.2ft, 6.1ft, 7.2ft, 6.5ft, 5.4ft
- (e) 112kg, 104kg, 108kg, 109kg, 111kg, 109kg, 114kg, 112kg, 110kg, 113kg

4. a If x , $x + 2$, $x + 5$, $x + 7$ and $x + 8$ are in ascending order and their median is 15. Find the value of x .

- b If 5, 17, $x - 5$, $x - 7$, $x - 12$, 50 and 58 are in ascending order and their median is 28. Find the value of x .**

5. Calculate the mode from the following data.

- (a) 2, 3, 3, 2, 4, 5, 6, 3, 3, 5, 5, 4, 3, 2
- (b) 3, 7, 9, 8, 8, 9, 8, 6, 5, 8
- (c) 29cm, 34cm, 29cm, 26cm, 55cm, 34cm, 35cm, 40cm, 34cm, 56cm
- (d) 120, 125, 130, 125, 120, 135, 120, 140
- (e) 99kg, 135kg, 182kg, 49kg, 189kg, 196kg, 78kg, 192kg, 182kg, 113kg

Project Work

- (a) Measure the height of 8 items which are available in your house and fill up this information in a table. On the basis of the table, find mean and median; and present it in your classroom.
- (b) What are the pages of your textbooks? On the basis of the data, what will be the pages of one textbook? Find it out and present in classroom.
- (c) Write a report exploring about the uses of the condition of mean and mode.

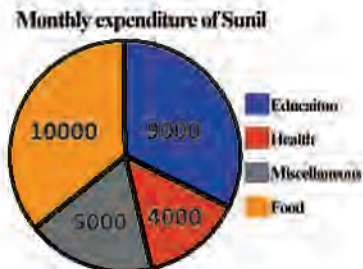
Answers

- | | | | | | |
|----|--------|---------|----------|----------|--------------|
| 1. | (a) 15 | (b) 92 | (c) 38 | (d) 109 | (e) 4.9 ft |
| 2. | (a) 3 | (b) 1 | (c) 5 | (d) 24 | (e) 7 |
| | (f) 36 | | | | |
| 3. | (a) 26 | (b) 245 | (c) 47.5 | (d) 6 ft | (e) 110.5 kg |
| 4. | (a) 10 | (b) 21 | | | |
| 5. | (a) 3 | (b) 8 | (c) 34cm | (d) 120 | (e) 182 kg |

Mixed Exercise

1. The monthly expenditure of Sunil in food, education, health and miscellaneous are shown in the adjoining pie chart,

- Which title of Sunil's expenditure shows the highest expenditure?
- What is his average expenditure according to the titles? Find it out.
- What is Sunil's income if his monthly saving is Rs.7000?



2. Study the following table and solve the given problems.

Class	V	VI	VII	VIII	Total
No. of students	53	43	46	50	192

- Present the above table in pie chart.
 - What is the average student of each class?
 - What percentage of students is in each class?
3. 80 girl students are asked about the games they like most. The following table is prepared on the basis of their answer.

Details of sports	Girls students
Badminton	40
Table tennis	10
Volleyball	20
Cricket	10
Total	80

- Which two games are liked by equal number of girl students?
- Present the above information in pie chart.

4. **The monthly expenditure of Renish is given below.**

Expenditure	Amount (Rs.)
Transportation	1,000
Education	4,000
Health	2,000
Cloths	3,000
Total	10,000

- (a) Present the above information in pie chart.
- (b) The average expenditure of Renish is 2,500. If the average expenditure of each title is to be made Rs. 2,000, how much equal amount should be reduced in education and health? Calculate.
5. **The weight in kg of 10 students of class 8 are given below.**
- 32, 35, 38, 42, 42, 47, 34, 36, 44, 40
- (a) What is the mode value of the above data? Write it.
- (b) Find the average weight (in kg) of students from the above data.
- (c) Are the median and mean values equal? Compare.
6. **The marks obtained in mathematics by 12 students of class 8 in their first terminal examination are given below.**
- 23, 30, 25, 26, 24, 28, 29, 28, 31, 33, 34, 28
- (a) Find the mean marks obtained by class 8 students.
- (b) Find the median marks of the students.
- (c) Find the mode from the above data.
7. **The weight in kg of the children who came for treatment in health post is given below.**
- 22, 20, 15, 21, 18, 19, 18
- (a) What is the average weight of children? Find it out.
- (b) What is the median weight of children? Find it.
- (c) 2 more children came for the treatment on the same day. What should be their weight to be the median weight as in question (b)

- (d) 2 more children came for the treatment on the same day. What should be their weight so that the median weight as in question (a)?

8. Find the mean, median and mode from the given data:

(a) 15, 20, 18, 16, 18, 16, 18

(b) 75, 80, 68, 95, 68, 46, 38, 45

Answer

1. (a) Food (b) Rs. 7000 (c) Rs = 4,20,000
2. (b) Show to your teacher (b) 48 (c) 27.6%, 22.4%, 24%, 26%
3. Show to your teacher.
4. (a) Show to your teacher (b) Rs. 1000
5. (a) 42 (b) 39 (c) yes
6. (a) 28.25 (b) 28 (c) 28
7. Show to your teacher
6. (a) mean = 17.3, median = 18 mode = 18
(b) mean = 64.4, median = 68 mode = 68