

Secondary Education Examination  
Specification Grid, 2078

Grade: 12

Subject: Mathematics (Mat. 402)

SN	Content Area	Working hour	Competency level																							
			Knowledge (16%)				Understanding (24%)						Application (40%)						Higher Ability (20%)							
			MCQ		SAQ		MCQ		SAQ		LAQ		MCQ		SAQ		LAQ		MCQ		SAQ		LAQ			
			No. of Question	Marks	No. of Questions	Marks	No. of Questions	Marks	No. of Questions	Marks	No. of Questions	Marks	No. of Questions	Marks	No. of Questions	Marks	No. of Questions	Marks	No. of Questions	Marks	No. of Questions	Marks	No. of Questions	Marks	Area wise Marks	No. of Questions
1	Algebra	44	2	2	2	10	5	5	1	5	1	8	2	2	4	20	1	8	2	2	1	5	1	8	20	MCQ: 2 SAQ: 2 LAQ: 1
2	Trigonometry	12																							6	MCQ: 3 SAQ: 2 LAQ: 1
3	Analytic Geometry	20																							9	
4	Vector	12																							6	MCQ: 1 SAQ: 1
5	Statistics & Probability	12																							6	
6	Calculus	48																							22	MCQ: 4 SAQ: 2 LAQ: 1
7	Computational methods or Mechanics	12																							6	MCQ: 1 SAQ: 1
Total		160	12				18						30						15				75	MCQ: 11 SAQ: 8 LAQ: 3		

Question format plan								
S.N.	Types of Questions	Marks per question	Number of questions				Total number of questions	Total Marks
			Knowledge	Understanding	Application	Higher Ability		
1.	Multiple Choice Question	1	2	5	2	2	11	11
2.	Short Answer Question	5	2	1	4	1	8	40
3.	Long Answer Question	8	0	1	1	1	3	24
<b>Grand Total</b>			<b>4</b>	<b>7</b>	<b>7</b>	<b>4</b>	<b>22</b>	<b>75</b>

**Note:**

- *Appropriate extra time will be provided for the handicapped students and the alternative questions to the figure-based questions should be prepared for blind students.*
- *Questions should be prepared by giving the context and one question may have more than one sub-questions.*
- *Application and higher ability questions can be made by relating the other content areas.*
- *Questions should be made by addressing all the sub-areas of content.*
- *At least one multiple choice question should be asked from each area (Trigonometry, Analytic geometry and Vector).*

## Secondary Education Examination

### Model question-2080

Subject: Mathematics (Mat.402)

Grade: XII

Full Marks: 75

Time: 3 Hrs.

Attempt all the questions.

#### Group A: Multiple Choice Questions ( $1 \times 11 = 11$ )

**Rewrite the correct option in your answer sheet.**

1. What is the number of permutations of  $n$  different things, taken  $r$  at a time while each thing may be repeated any number of times in any permutation? [K]  
a.  $n!$                       b.  $n^r$                       c.  $(n-1)!$                       d.  $\frac{n!}{(n-r)!}$
2. What is the sum of first 5 odd natural number? [U]  
a. 15                      b. 25                      c. 42                      d. 255
3. In a triangle ABC,  $\angle C = 30^\circ$ ,  $b = \sqrt{3}$  and  $a = 1$ . What type of triangle is ABC? [HA]  
a. Right angled                      b. Isosceles  
c. Isosceles right-angled                      d. Scalene
4. What is the value of  $k$  so that the length of the tangent from  $(5,4)$  to the circle  $x^2 + y^2 + 2ky = 0$  is 1. [U]  
a. -5                      b. -4                      c. 4                      d. 5
5. Given  $\vec{a} \cdot \vec{b} = 48$ ,  $|\vec{a}| = 15$  and  $|\vec{b}| = 4$ , what is the value of  $|\vec{a} \times \vec{b}|$ ? [U]  
a. 12                      b. 36                      c. 48                      d. 60
6. What is the probability of getting 53 Friday or Saturday in a leap year? [A]  
a.  $\frac{7}{7}$                       b.  $\frac{2}{7}$                       c.  $\frac{3}{7}$                       d.  $\frac{2}{53}$
7. What is the derivation of  $\tanh^{-1}x$ ? [K]  
a.  $\frac{2}{-x^2}$ ,  $|x| < 1$                       b.  $\frac{2}{-x^2}$ ,  $|x| > 1$   
c.  $\frac{2}{x^2-1}$ ,  $|x| < 1$                       d.  $\frac{2}{\sqrt{x^2-1}}$ ,  $|x| > 1$

8. Which one of the following is equal to  $\lim_{x \rightarrow 0} \frac{3x - \sin x}{2x}$ ? [U]

- a. 3
- b. 3
- c. 1.5
- d. 1

9. Which one of the following represents the equation of tangent to the curve  $y^2 = 4x$  at the point (1, 2)? [A]

- a.  $x + y + 1 = 0$
- b.  $x - y + 1 = 0$
- c.  $x + y - 1 = 0$
- d.  $x - y - 1 = 0$

10. The volume of a sphere is increasing at the rate of  $25 \text{ cm}^3/\text{sec}$ . At what rate the radius is increasing at the instant when the total surface area of a sphere is  $10\pi \text{ cm}^2$ ? [HA]

- a.  $\frac{\pi}{4} \text{ cm/sec}$
- b.  $\frac{4}{\pi} \text{ cm/sec}$
- c.  $\frac{5}{2\pi} \text{ cm/sec}$
- d.  $100\pi \text{ cm/sec}$

11. The system of equation  $2x + 3y = 5$  and  $x - y = 0$  has ... [U]

- a. no solution
- b. infinitely many solution
- c. one solution
- d. finitely many solution

OR

A car moving with a velocity of  $15 \text{ m/s}$  has a uniform acceleration of  $2 \text{ m/s}^2$ . If it moves for  $2.5 \text{ sec}$ , find its final velocity. [U]

- a.  $10 \text{ m/s}$
- b.  $15 \text{ m/s}$
- c.  $20 \text{ m/s}$
- d.  $24 \text{ m/s}$

## Group "B"

(5 × 8 = 40)

12. In the expansion of  $(x + a)^n$ ,  $n$  is a positive integer.

a. Write the general term of the expansion. [1K]

b. If  $a$  is replaced by  $(-a)$ , what is the general term of the expansion? [1K]c. How many terms are there in the expansion of  $(x + a)^n$ ? [1K]d. Write the expansion of  $e^x$ . [1K]e. State the relation between  $C(n, r)$ ,  $C(n, r-1)$  and  $C(n+1, r)$  for  $r \leq n$  [1K]13.a. Use De-moivre's theorem to solve  $x^3 = -1$ . [2A]

b. Solve the following system of equations by using row equivalent method: [3A]

$$2x - y + z = -1, \quad x - 2y + 3z = 4 \quad \text{and} \quad 4x + y + 2z = 4$$

14.a. In any triangle ABC, prove that:  $bc \cos^2 \frac{A}{2} + ca \cos^2 \frac{B}{2} + ab \cos^2 \frac{C}{2} = s^2$  where  $s$  is semi-perimeter of the triangle. [2U]b. The foci of the hyperbola coincide with the foci of the ellipse  $\frac{x^2}{25} + \frac{y^2}{9} = 1$ , find the equation of the hyperbola if its eccentricity is 2. [3U]

15. a. A man running a race course notes that the sum of his distance from the two flag posts is always 10 m and the distance between the flag posts is 8m. Illustrate and interpret the situation with diagram. [3HA]

b) If A,B,C &amp; D are any four points, show that:

$$\overrightarrow{AB} \cdot \overrightarrow{CD} + \overrightarrow{BC} \cdot \overrightarrow{AD} + \overrightarrow{CA} \cdot \overrightarrow{BD} = 0 \quad [2HA]$$

16. The following table shows hours of time spent by five students of grade 12:

Hours spending on study book (X)	5	7	9	10	11
Hours spending on playing game (Y)	5	4	3	2	1

a. Estimate the hours spending on playing game who spend 6 hours in study books. [3A]

b. Find the coefficient of correlation between X and Y. [2A]

17. a.  $f'(x)$  and  $g'(x)$  are derivatives of the functions  $f(x)$  and  $g(x)$ . Write the relation between
 $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$  and  $\lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$  when both expressions give  $\frac{0}{0}$  form. [1K]
b. Write the derivative of  $\sinh x$  with respect to  $x$ . [1K]c. What is the expression equal to  $\int \frac{1}{x^2 + a^2} dx$ ? [1K]

d. Write a differential equation of first order and first degree. [1K]

e. Write a difference between derivative and antiderivative? [1K]

18.a) Evaluate:  $\int \frac{dx}{2+\cos x}$  [3A]

b) Solve:  $\sec^2 x + \tan y dx + \sec^2 y \tan x dy = 0$  [2A]

19.a. Solve the following system of equations by Gauss seidel method: [2A]

$$3x + x_2 = 5$$

$$x - 3x_2 = 5$$

b. Using simplex method to maximize  $z = 5x - 3y$

subject to the constraints

$$3x + 2y \leq 6 \quad [3A]$$

$$x - 3y \leq 4$$

$$x \geq 0, y \geq 0$$

OR

a. A bullet of mass 25gm moving 250m/s penetrates into a tree trunk & is then brought to rest in 0.02 seconds. Find the distance of penetration of the tree-trunk. [3A]

b. Find two like parallel forces acting at a distance of 2.5m apart, which are equivalent to a given force of 30N. The lines of action of one being at a distance of 50cm from the given force. [2A]

Group "C"

20.a) In how many ways can the letters of the word 'INTERVAL' be arranged so that: [3U]

i. all vowels are always together?

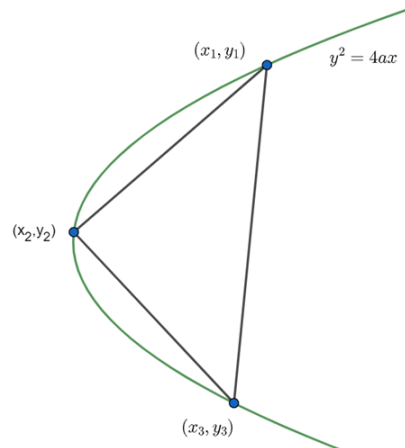
ii. the relative positions of the vowels and constants are not changed?

iii. the vowels may occupy only the odd positions?

b. Show that  $\log_e 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots$  to  $\infty$  [2U]

c. Prove by the method of induction that  $1.3 + 2.4 + 3.5 + \dots n(n+2) = \frac{n(n+1)(2n+7)}{6}$  [3U]

21.a. In the figure alongside triangle ABC is inscribed in a parabola  $y^2 = 4ax$ .



i. Express all the coordinates of triangle ABC in terms of x coordinates. [1A]

ii. Prove that the area of triangle ABC is  $\frac{1}{8a} |(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$  [2A]

b. In any triangle ABC,  $a(b \cos C - c \cos B) = b^2 - c^2$  [2A]

c. Prove that the unit vector perpendicular to each of the vectors  $2\vec{i} - \vec{j} + \vec{k}$  &  $3\vec{i} + 4\vec{j} - \vec{k}$  is  $\frac{-3\vec{i} + 5\vec{j} + 11\vec{k}}{\sqrt{155}}$  and the sine of an angle between them is  $\sqrt{\frac{155}{156}}$  [3A]

22.a. Illustrate derivative as a rate measure with suitable example. [2HA]

b. Prove that  $\int \frac{dx}{a + b \cos x} = \tan^{-1} \left[ \frac{4 \tan(\frac{x}{2}) - 3}{\sqrt{7}} \right] + \text{constant}$ , where a and b are positive constants. [3HA]

c. Does  $x \frac{dy}{dx} = \sqrt{x^2 + y^2} + y$  represent a homogeneous differential equation? give reason. Also show the equation. [3HA]

-The End-